OK to post!

Anusha Nagar, Homework 16, 10.30.2021

1 (a) $x(n) = x(n-1)\left(\frac{5}{3} - x(n-2)\right)$

(i) Convert to first-order system

$$\boxed{\begin{cases} x_2(n) = x_1(n-1) \\ x_1(n) = x_1(n-1)\left(\frac{5}{3} - x_2(n-1)\right) \end{cases}}$$

(ii) Equilibrium Points:

$$z = z\left(\frac{5}{3} - z\right)$$

$$z = \frac{5}{3}z - z^2$$

$$z^2 - \frac{2}{3}z = 0$$

$$z = 0,\ \frac{2}{3} \Rightarrow \boxed{(0,0),\ \left(\frac{2}{3}, \frac{2}{3}\right)}$$

(iii) Let $x_1(n-1) = z_1$ + $x_2(n-1) = z_2$

$$(z_1, z_2) \to \left(z_1\left(\frac{5}{3} - z_2\right), z_1\right)$$

$$J = \begin{pmatrix} f_{z_1} & f_{z_2} \\ g_{z_1} & g_{z_2} \end{pmatrix} = \begin{pmatrix} \frac{5}{3} - z_2 & -z_1 \\ 1 & 0 \end{pmatrix}$$

$(0,0) \Rightarrow J = \begin{pmatrix} \frac{5}{3} & 0 \\ 1 & 0 \end{pmatrix} \Rightarrow \lambda = \frac{5}{3}, 0$. Both not less than 1 in absolute value $\Rightarrow (0,0)$ is unstable

$\left(\frac{2}{3}, \frac{2}{3}\right) \Rightarrow J = \begin{pmatrix} 1 & -\frac{2}{3} \\ 1 & 0 \end{pmatrix} \Rightarrow \det \begin{bmatrix} 1-\lambda & -\frac{2}{3} \\ 1 & -\lambda \end{bmatrix} \Rightarrow (1-\lambda)(-\lambda) + \frac{2}{3}$

$$\lambda^2 - \lambda + \frac{2}{3}$$

$$\frac{1 \pm \sqrt{1 - 4(1)\left(\frac{2}{3}\right)}}{2}$$

$$= \frac{1}{2} \pm \frac{\sqrt{5/3}}{2} i$$

Absolute value: $\sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{5/3}}{2}\right)^2} \approx 0.817$

This is $<1 \Rightarrow \left(\frac{2}{3}, \frac{2}{3}\right)$ is stable equilibrium point

b) (i) $x(n) = x(n-1)(2 - x(n-2))$

$$\boxed{\begin{cases} x_2(n) = x_1(n-1) \\ x_1(n) = x_1(n-1)\left(2 - x_2(n-1)\right) \end{cases}}$$

(ii) $z = z(2-z)$

$z = 2z - z^2$

$z^2 - z = 0$

$z = 0, 1 \quad \Rightarrow \boxed{(0,0), (1,1)}$

(iii) $(z_1, z_2) \to \left(z_1(2-z_2),\ z_1\right)$

$$J = \begin{pmatrix} 2-z_2 & -z_1 \\ 1 & 0 \end{pmatrix}$$

$(0,0) \Rightarrow J = \begin{pmatrix} 2 & 0 \\ 1 & 0 \end{pmatrix} \Rightarrow \lambda = 0, 2$

Both not <1 in abs. value $\Rightarrow$ $(0,0)$ unstable

$(1,1) \Rightarrow J = \begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix} \Rightarrow \det \begin{vmatrix} 2-\lambda & -1 \\ 1 & -\lambda \end{vmatrix} = (2-\lambda)(-\lambda) + 1$

$\lambda^2 - 2\lambda + 1$

$(\lambda - 1)(\lambda - 1)$

$\lambda = 1, 1$

Since this is not <1,

$(1,1)$ is unstable

Orbk $(k, z, f, |N|, k_1, k_2)$

② $x(n) = x(n-1)(a - x(n-1))$

To first order:

$$\begin{cases} x_2(n) = x_1(n-1) \\ x_1(n) = x_1(n-1)(a - x_2(n-1)) \end{cases}$$

Equilibrium points:

$$z = z(a-z)$$

$$z = za - z^2$$

$$z^2 + z(1-a) = \emptyset$$

$$z[z + (1-a)] = \emptyset$$

$$z = \emptyset,\ a-1$$

$$(z_1, z_2) \Rightarrow (z_1(a-z_2), z_1)$$

$$J = \begin{pmatrix} a-z_2 & -z_1 \\ 1 & \emptyset \end{pmatrix} \Rightarrow \det \begin{bmatrix} a-z_2-\lambda & -z_1 \\ 1 & -\lambda \end{bmatrix}$$

$$((a-z_2)-\lambda)(-\lambda) + z_1$$

$$\lambda^2 - (a-z_2)\lambda + z_1 = \emptyset$$

$$\lambda = \frac{(a-z_2) \pm \sqrt{(a-z_2)^2 - 4(z_1)}}{2}$$

$$(\emptyset,\emptyset) \Rightarrow J = \begin{pmatrix} a & \emptyset \\ 1 & \emptyset \end{pmatrix} \Rightarrow \lambda = \emptyset,\ a$$

↳ $(\emptyset,\emptyset)$ stable when $a \leq 1$

$$(a-1, a-1) \Rightarrow J = \begin{pmatrix} -1 & 1-a \\ 1 & \emptyset \end{pmatrix} \Rightarrow \det \begin{pmatrix} -1-\lambda & 1-a \\ 1 & -\lambda \end{pmatrix}$$

$$(-1-\lambda)(-\lambda) - (1-a)$$

$$\lambda^2 + \lambda + (a-1) = \emptyset$$

$$\frac{-1 \pm \sqrt{1-4(a-1)}}{2}$$

$$\frac{-1 \pm \sqrt{5-4a}}{2} = -\frac{1}{2} \pm \frac{\sqrt{5-4a}}{2}$$

Stable when $\sqrt{\frac{1}{4} + \left(\frac{\sqrt{5-4a}}{2}\right)^2} < 1$

$$= \sqrt{\frac{1}{4} + \frac{5-4a}{4}} < 1$$

$$= \frac{\sqrt{6-4a}}{2} < 1 \qquad \sqrt{6-4a} < 2$$

$$6 - 4a < 4$$

$$2 < 4a$$

$\frac{1}{2} < a$ for stability

④ a) $x'(t) = x(t)\left[3-x(t)\right)\left(5-x(t)\right)$

$$F(x) = (3-x)(5-x)$$
$$= (15-8x+x^2) = \emptyset$$
$$\boxed{x = 3, 5}$$ (equilibrium points)

$$F'(x) = 2x-8$$

$x = 3 \Rightarrow F'(x) = -2 \Rightarrow$ stable! (negative)

$x = 5 \Rightarrow F'(x) = 2 \Rightarrow$ unstable (not negative)

plot(Dis1(F, y, y0, h, A)

h = 0.01, A = 20

b) $x'(t) = x(t)^2 (3-x(t))(5-x(t))(7-x(t))$

$$F(x) = x^2(3-x)(5-x)(7-x)$$

↳ Equilibrium: $x = \emptyset, \emptyset, 3, 5, 7$

$$F(x) = x^2(x^2-8x+15)(7-x)$$
$$= (x^4-8x^3+15x^2)(7-x)$$
$$= 7x^4 - 56x^3 + 105x^2 - x^5 + 8x^4 - 15x^3$$
$$= -x^5 + 15x^4 - 71x^3 + 105x^2$$

$$F'(x) = -5x^4 + 60x^3 - 213x^2 + 210x$$

$x = \emptyset \Rightarrow F'(x) = \emptyset \Rightarrow$ not negative $\Rightarrow$ unstable!

$x = 3 \Rightarrow F'(x) = -72 \Rightarrow$ stable!

$x = 5 \Rightarrow F'(x) = 100 \Rightarrow$ unstable

$x = 7 \Rightarrow F'(x) = -392 \Rightarrow$ stable!

> **read** "C://Users/an646/Documents/M15.txt"

> *Help15*( )

HW3(u,v,w), HW2(u,v) , Dis1(F,y,y0,h,A), ToSys(k,z,f,INI) **(1)**

> *#Problem 1 a*

>

> $Orbk\left(2, z,\ z[1]\cdot\left(\frac{5}{3} - z[2]\right), [0, .5], 1000, 1020\right)$

[0.6666666665, 0.6666666665] **(2)**

> *#Problem 1b*

> $Orbk(2, z, z[1]\cdot(2 - z[2]), [0, 0.5], 1000, 1020)$

[0.9914425065, 0.9413502159, 0.9494058147, 1.005088261, 1.055939882, 1.050566984, 0.9917983909, 0.9416461375, 0.9493691509, 1.004768507, 1.055640790, 1.050606960, 0.9921503588, 0.9419406453, 0.9493345412, 1.004452292, 1.055343328, 1.050644631, 0.9924984606, 0.9422337423, 0.9493019455] **(3)**

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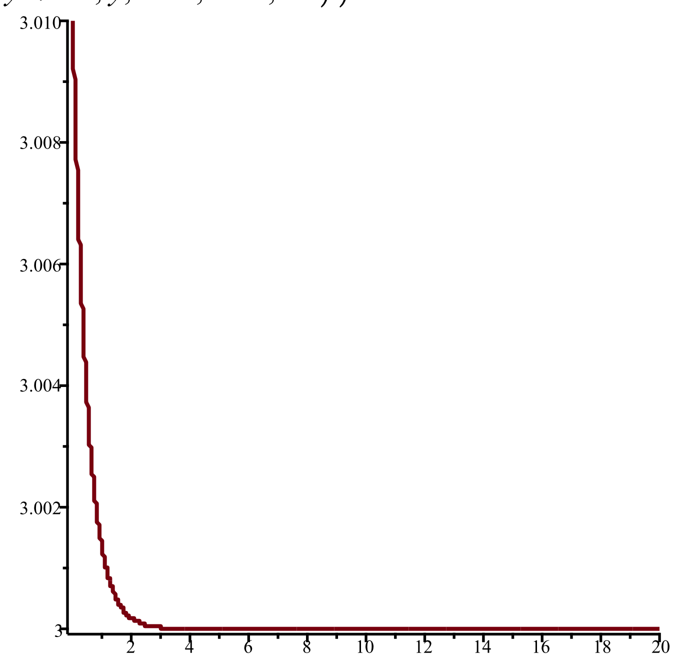
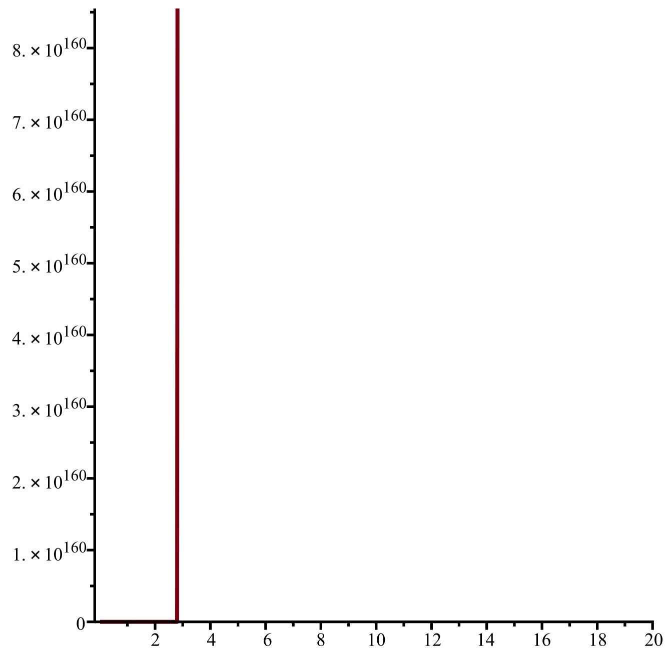
> *#Problem 4a*

> *#Around x=3*

> $plot\left(Dis1\left(y^2 - 8\cdot y + 15, y, 3.01, 0.01, 20\right)\right)$

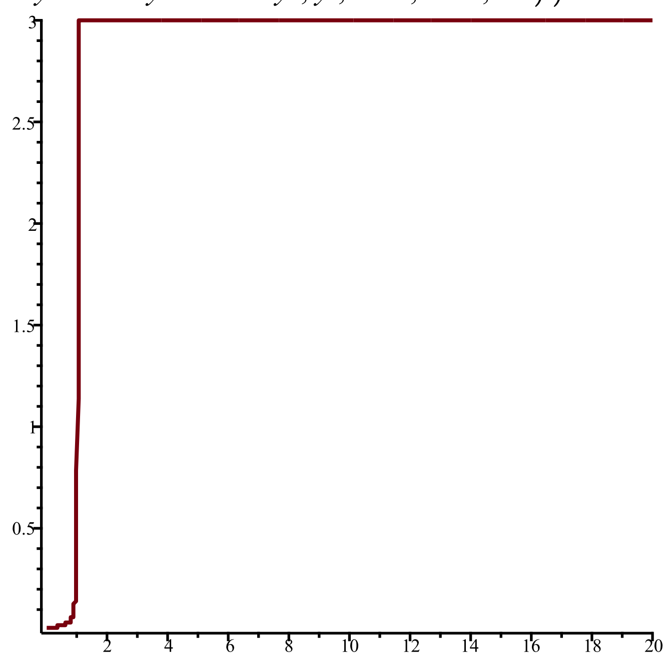


> $plot\left(Dis1\left(y^2 - 8\cdot y + 15, y, 5.01, 0.01, 20\right)\right)$

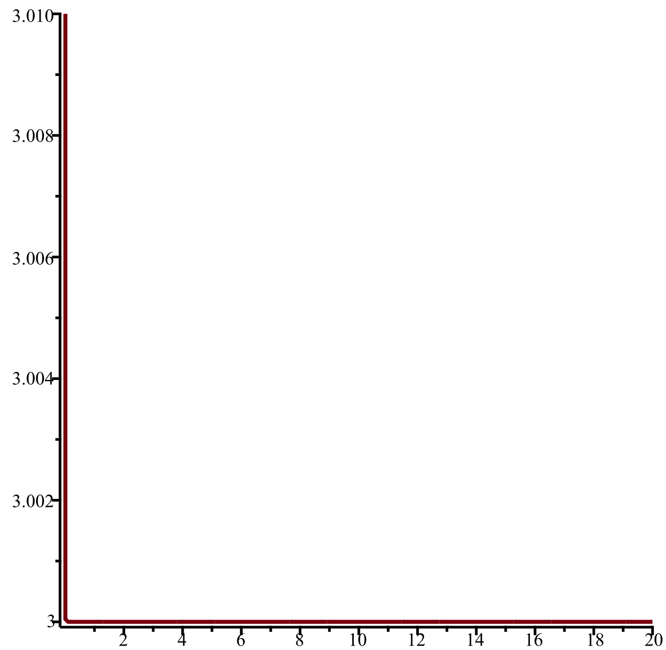


> *#Problem 4b*

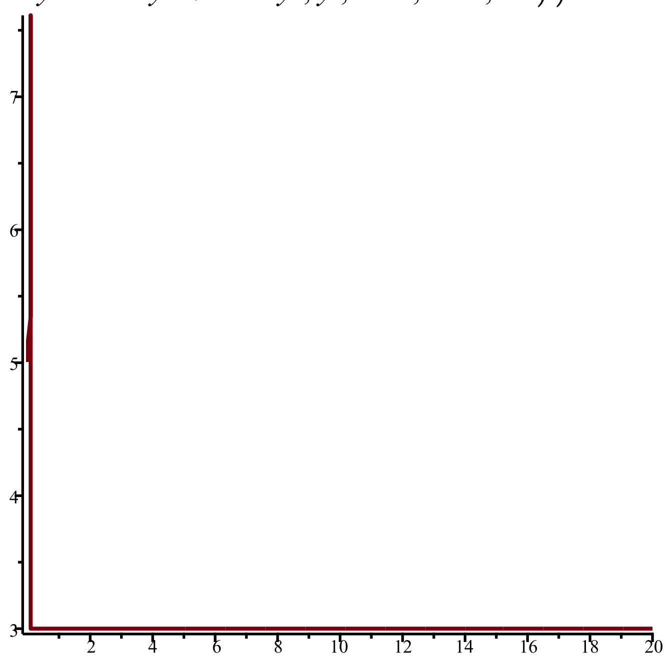
> $plot\left(Dis1\left(-y^5 + 15 \cdot y^4 - 71 \cdot y^3 + 105 \cdot y^2, y, 0.01, 0.01, 20\right)\right)$



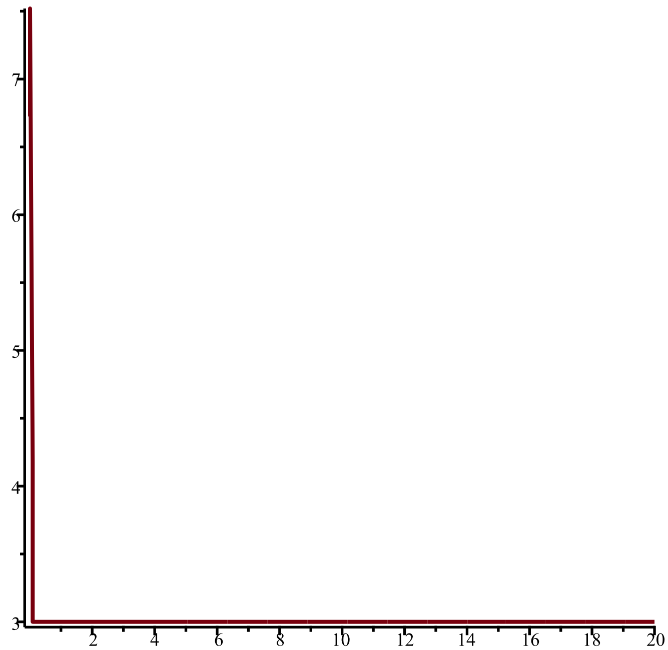
> $plot\left(Dis1\left(-y^5 + 15 \cdot y^4 - 71 \cdot y^3 + 105 \cdot y^2, y, 3.01, 0.01, 20\right)\right)$



> $plot\left(Dis1\left(-y^5 + 15 \cdot y^4 - 71 \cdot y^3 + 105 \cdot y^2, y, 5.01, 0.01, 20\right)\right)$



> $plot\left(Dis1\left(-y^5 + 15 \cdot y^4 - 71 \cdot y^3 + 105 \cdot y^2, y, 7.01, 0.01, 20\right)\right)$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