OK to post.

Anne Somalwar, hw16, 11/1/2021

1.

(a) $x(n) = x(n-1)\left(\frac{5}{3} - x(n-2)\right)$

SYSTEM

$$x(n) = x(n-1)\left(\frac{5}{3} - y(n-1)\right)$$

$$y(n) = x(n-1)$$

$$x = x(5/3 - x)$$

$$x = 0$$

$$1 = \frac{5}{3} - x$$

$$x = 2/3$$

FIXED PTS

$$x = 0, 2/3$$

$$J = \begin{pmatrix} \frac{5}{3} - y & -x \\ 1 & 0 \end{pmatrix}$$

$$J(0,0) = \begin{pmatrix} 5/3 & 0 \\ 1 & 0 \end{pmatrix}$$

$$(5/3 - \lambda)(-\lambda) = 0$$

$$\lambda^2 - \frac{5}{3}\lambda = 0$$

$$\lambda = 0, 5/3$$

$5/3 > 1$, so 0 is <u>not stable</u>.

$$J(2/3, 2/3) = \begin{pmatrix} 1 & -2/3 \\ 1 & 0 \end{pmatrix}$$

$$(1-\lambda)(-\lambda) + 2/3 = 0$$

$$\lambda^2 - \lambda + 2/3 = 0$$

$$\frac{1 \pm \sqrt{1 - \frac{8}{3}}}{2} = \frac{1}{2} \pm \frac{\sqrt{5/3}}{2} i$$

$$\left|\frac{1}{2} + \frac{\sqrt{5/3}}{2} i\right| = \left|\frac{1}{2} - \frac{\sqrt{5/3}}{2} i\right| = \sqrt{\frac{1}{4} + \frac{5}{12}} = \sqrt{\frac{8}{12}}$$

$\sqrt{\frac{8}{12}} < 1$, so $2/3$ is stable.

(b) $x(n) = x(n-1)\,(2 - x(n-2))$

SYSTEM

$$x(n) = x(n-1)\,(2 - y(n-1))$$

$$y(n) = x(n-1)$$

$$x = x(2-x)$$

$x = 0$

$1 = 2 - x$

$x = 1$

FIXED PTS

$x = 0, 1$

$$J = \begin{pmatrix} 2-y & -x \\ 1 & 0 \end{pmatrix}$$

$$J(0,0) = \begin{pmatrix} 2 & 0 \\ 1 & 0 \end{pmatrix}$$

$$(2-\lambda)(-\lambda) = 0$$

$$\lambda = 0, 2$$

$2 > 1$, so 0 is <u>not stable</u>.

$$J(1,1) = \begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix}$$

$$(1-\lambda)(-\lambda) + 1 = 0$$

$$\lambda^2 - \lambda + 1 = 0$$

$$\frac{1 \pm \sqrt{1-4}}{2}$$

$$= \frac{1 \pm \sqrt{3}\, i}{2}$$

$$\left| \frac{1+\sqrt{3}\, i}{2} \right| = \sqrt{1/4 + \tfrac{3}{4}} = 1$$

$1 \geq 1$, so 1 is <u>not stable</u>.

2) $x = x(a-x)$

$x = 0$

$1 = a - x$

$x = a - 1$

<u>FIXED PTS</u> $x = 0, a-1$

<u>SYSTEM</u>

$$x(n) = x(n-1)\,(a - y(n-1))$$

$$y(n) = x(n-1)$$

$$J = \begin{pmatrix} a-y & -x \\ 1 & 0 \end{pmatrix}$$

$$J(0,0) = \begin{pmatrix} a & 0 \\ 1 & 0 \end{pmatrix}$$

$$(a-\lambda)(-\lambda) = 0$$

$$\lambda = 0,\ a$$

0 <u>is stable when $|a| < 1$</u>.

$$J(a-1, a-1) = \begin{pmatrix} 1 & 1-a \\ 1 & 0 \end{pmatrix}$$

$$(1-\lambda)(-\lambda) - (1-a) = 0$$

$$\lambda^2 - \lambda + a - 1 = 0$$

$$\frac{1 \pm \sqrt{1-4(a-1)}}{2} = \frac{1 \pm \sqrt{5-4a}}{2}$$

$a-1$ is stable when

$\left| \frac{1 + \sqrt{5-4a}}{2} \right| < 1$ and

$$\left| \frac{1 - \sqrt{5-4a}}{2} \right| < 1 .$$

3) $x(n) = x(n-1)(a - x(n-2))(b - x(n-3))$

$$x = x(a-x)(b-x)$$

$$x = 0$$

$$1 = (a-x)(b-x)$$

$$1 = ab - ax - bx + x^2$$

$$x^2 - (a+b)x + ab - 1 = 0$$

Fixed pts

$$x = 0, \frac{a+b \pm \sqrt{(a+b)^2 - 4ab + 4}}{2}$$

This is pretty ugly, I think I won't continue.

4) (a) $x'(t) = x(t)(3-x(t))(5-x(t))$

$F(x) = x(3-x)(5-x)$

(i) $x(3-x)(5-x) = 0$

Eq. PTS

$x = 0, 3, 5$

(ii) $F(x) = (3x-x^2)(5-x) = 15x - 3x^2 - 5x^2 + x^3$

$= 15x - 8x^2 + x^3$

$F'(x) = 15 - 16x + 3x^2$

$F'(0) = 15 > 0$, so 0 is unstable

$F'(3) = 15 - 16(3) + 3(9)$

$= -6 < 0$ so 3 is stable.

$F'(5) = 15 - 16(5) + 3(25) = 10 > 0$

so 5 is unstable.

(b) $x'(t) = x(t)^2 (3 - x(t))(5 - x(t))(7 - x(t))$

$F(x) = x^2 (3-x)(5-x)(7-x)$

eq. PTS

$x = 0, 3, 5, 7$

see maple code for (ii) and (iii)