HW10 - Alan Ho

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1) a) $x(n) = x(n-1)\left(\frac{5}{3} - x(n-2)\right)$

i) $x_1(n) = x(n) \qquad x_2(n) = x(n-1)$

$$\boxed{x_1(n) = x_1(n-1)\left(\frac{5}{3} - x_2(n-1)\right)}$$

ii) $z = z\left(\frac{5}{3} - z\right)$

$z = \frac{5}{3}z - z^2 \Rightarrow z^2 - \frac{2}{3}z = 0$

$z\left(z - \frac{2}{3}\right) = 0 \quad \Rightarrow \boxed{z = 0, \frac{2}{3}}$

iii) $(z_1, z_2) \to \left(z_1\left(\frac{5}{3} - z_2\right), z_1\right)$

$$J = \begin{bmatrix} \frac{5}{3} - z_2 & -z_1 \\ 1 & 0 \end{bmatrix}$$

$$J_{(0,0)} = \begin{bmatrix} \frac{5}{3} & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} \frac{5}{3} - \lambda & 0 \\ 1 & -\lambda \end{bmatrix} = -\frac{5\lambda}{3} + \lambda^2 = 0$$

$3\lambda^2 - 5\lambda = 0 \qquad \lambda(3\lambda - 5) \qquad \lambda = 0, \frac{5}{3}$ $\boxed{\therefore (0,0) \text{ not stable}}$

$$J_{\left(\frac{2}{3}, \frac{2}{3}\right)} = \begin{bmatrix} 1 & -\frac{2}{3} \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1-\lambda & -\frac{2}{3} \\ 1 & -\lambda \end{bmatrix} = -\lambda + \lambda^2 + \frac{2}{3}$$

$= \lambda^2 - \lambda + \frac{2}{3} = \frac{1 \pm \sqrt{1 - 8/3}}{2} = \frac{1}{2} - \frac{\sqrt{15}i}{6}, \ \frac{i(\sqrt{15} - 3i)}{6}$ $\boxed{\therefore \left(\frac{2}{3}, \frac{2}{3}\right) \text{ stable}}$

$0.5 - \frac{\sqrt{15}i}{6}$

b) $x(n) = x(n-1)(2 - x(n-2))$

i) $x_1(n) = x(n) \qquad x_2(n) = x(n-1)$

$$\boxed{x_1(n) = x_1(n-1)(2 - x_2(n-1))}$$

ii) $z = z(2 - z)$

$z = 2z - z^2 \Rightarrow z^2 - z = 0$

$z(z-1) = 0 \Rightarrow \boxed{z = 0, 1}$

iii) $(z_1, z_2) \to (z_1(2 - z_2), z_1)$

$$J = \begin{bmatrix} 2 - z_2 & -z_1 \\ 1 & 0 \end{bmatrix}$$

$$J_{(0,0)} = \begin{bmatrix} 2 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2-\lambda & 0 \\ 1 & -\lambda \end{bmatrix} = -2\lambda + \lambda^2 = 0$$

$\lambda = 0, 2$ $\boxed{\therefore (0,0) \text{ not stable}}$

$$J_{(1,1)} = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1-\lambda & -1 \\ 1 & -\lambda \end{bmatrix} = -\lambda + \lambda^2 + 1 = 0$$

$\lambda = \pm \frac{i(\sqrt{3} + i)}{2}$ $\boxed{\therefore (1,1) \text{ Stable}}$ NOT

$|1 + \frac{\sqrt{3}}{2}i| = 1.3229$

$\boxed{\text{Hw 16}}$ cont'd

2) $x(n) = x(n-1)\,(a - x(n-2))$

$x_1(n) = x_1(n-1)\,(a - x_2(n-2))$

$z = z(a - z) \Rightarrow z = az - z^2$

$z^2 + z - az = 0 \Rightarrow z(z+1-a) = 0$ $\quad \boxed{z = 0}, \ \boxed{z = a-1}$

$(z_1, z_2) \to (z_1(a - z_2),\ z_1)$

$J = \begin{bmatrix} a - z_2 & -z_1 \\ 1 & 0 \end{bmatrix}$ $\quad J_{(0,0)} = \begin{bmatrix} a & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} a-\lambda & 0 \\ 1 & -\lambda \end{bmatrix}$

$-a\lambda + \lambda^2 - 1 < \cancel{0} \Rightarrow \lambda^2 - a\lambda - 1 < |1|$

$\boxed{\lambda = \dfrac{a \pm \sqrt{a^2+4}}{2} < |1|}$

$J_{(a-1,\,a-1)} = \begin{bmatrix} 1 & -a+1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1-\lambda & -a+1 \\ 1 & -\lambda \end{bmatrix} = -\lambda + \lambda^2 + a - 1$

$\boxed{\lambda^2 - \lambda + a - 1 < |1|}$

4a) $x'(t) = x(t)\,(3 - x(t))\,(5 - x(t))$

i) $x(3-x)(5-x) = 0$

$(3x - x^2)(5-x) = 0$

$15x - 5x^2 - 3x^2 + x^3 = 0$

$x^3 - 8x^2 + 15x = 0$

$x(x^2 - 8x + 15) = 0$

$x(x-5)(x-3)$ $\quad \boxed{x = 0, 5, 3}$

ii) $F'(x) =$

$3x^2 - 16x + 15$

$\boxed{\begin{aligned} F'(0) &= 15, \text{ unstable} \\ F'(5) &= 10, \text{ unstable} \\ F'(3) &= -6, \text{ stable} \end{aligned}}$

4b) $x'(t) = x(t)^2\,(3 - x(t))\,(5 - x(t))\,(7 - x(t)))$

i) $x^2(3-x)(5-x)(7-x)$

$= x^2(15-8x+x^2)(7-x) = (105 - \cancel{56x} + \cancel{7x^2} - \cancel{15x} + \cancel{8x^2} - \cancel{x^3})$

$= -x^5 + \cancel{8x^4}\ 15x^4 - 71x^3 + 105x^2$

$= x^2(-x^3 + 15x^2 - 71x + 105)$

$\boxed{x = 0, 3, 5, 7}$

ii) $F'(x) = -5x^4 + 60x^3 - 213x^2 + 210x$

$F'(0) = 0$ unstable
$F'(3) = -72$ stable
$F'(5) = 100$ ~~stab~~ unstable
$F'(7) = -392$ stable