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> #Nikita John, Assignment 16
 #November 1st, 2021
> #M15.txt: Maple code for Lecture 15 of Dynamical Modesl in Biology, Fall 2021 (taught by Dr. Z.)

Help15 :=proc( ) : print( `HW3(u,v,w), HW2(u,v) , Dis1(F,y,y0,h,A), ToSys(k,z,f,INI) `) :end:



#ToSys(k,z,f,INI): converts the kth order difference equation x(n)=f(x[n-1],x[n-2],...x[n-k]) to a first-order system
#x1(n)=F(x1(n-1),x2(n-1), ...,xk(n-1))
#x2(n)=x1(n-1)
#...

#xk(n)=x[k-1](n-1). It gives the underlying transformation phrased in terms of z[1],...z[k], followed by the initial conditions. Try:
#ToSys:=proc(2,z,z[1]+z[2],[1,1])
ToSys :=proc(k, z, f, INI) local i :
[ f, seq(z[i-1], i=2 ..k) ], INI :
end:


#HW3(u,v,w): The Hardy-Weinberg unerlying transformation witu (u,v,w), Eqs. (53a,53b, 53c) in Edelestein-Keshet Ch. 3
HW3 :=proc(u, v, w) : [u^2 + u*v + (1/4)*v^2,  u*v + 2*u*w + 1/2*v^2 + v*w, 1/4 *v^2 + v*w + w^2] :end:


#HW2(u,v): The Hardy-Weinberg unerlying transformation witu (u,v,w), Eqs. (53a,53b,53c) in Edelestein-Keshet Ch. 3 using the fact that u+v+w=1
HW2 :=proc(u, v) : expand([u^2 + u*v + (1/4)*v^2 , u*v + 2*u*(1-u-v) + 1/2*v^2 + v*(1-u-v)]) :end:




#Dis1(F,y,y0,h,A): The approximate orbit of the Dynamical system approximating the 1D for the autonomous  continuous dynamical  process dy/dt=F(y(t)) , y(0)=y0 with mesh size h from t=0 to t=A
Dis1 :=proc(F, y, y0, h, A) local L, x, i :
L := Orb(x + h*subs(y=x, F), x, y0, 0, trunc(A/h)) :

L := [seq([i*h, L[i]], i=1 ..nops(L))] :
end:

##old stuff
```

```
#M13.txt: Maple code for Lecture 13 of Dynamical Modesl in Biology, Fall 2021 (taught by
Dr. Z.)
Help13 :=proc( ) :
 print( ` RT2(x,y,d,K), Orb2(F,x,y,pt0,K1,K2), FP2(F,x,y), SFP2(F,x,y), PlotOrb2(L), FP2drz
(F,x,y), SFP2drz(F,x,y) `) :end:

with(LinearAlgebra) :


#RT2(x,y,d,K): A random rational transformation of degree d from R^2 to R^2 with postiive
integer coefficients from 1 to K The inputs are variables x and y and
#the output is a pair of expressions  of (x,y) representing functions. It is for generating examples
#Try:
#RT2(x,y,2,10);
RT2 :=proc(x, y, d, K) local ra, i, j, f, g :
ra := rand(1 ..K) : #random integer from -K to K
f := add(add(ra( ) * x^i * y^j, j = 0 ..d - i), i = 0 ..d) / add(add(ra( ) * x^i * y^j, j = 0 ..d - i), i = 0
 ..d) :
g := add(add(ra( ) * x^i * y^j, j = 0 ..d - i), i = 0 ..d) / add(add(ra( ) * x^i * y^j, j = 0 ..d - i), i = 0
 ..d) :
[f, g] :
end:



#Orb2(F,x,y,pt,K1,K2): Inputs a mapping F=[f,g] from R^2  to R^2 where f and g describe
functions of x and y, an initial point pt0=[x0,y0]
#outputs the orbit starting at discrete time K1 and ending in discrete time K2. Try
#F:=RT2(x,y,2,10);
#Orb2(F,x,y,[1.1,1.2],1000,1010);
Orb2 :=proc(F, x, y, pt0, K1, K2)  local pt, L, i :
pt := pt0 :

for i from 1 to K1 - 1 do
pt := subs( {x = pt[1], y = pt[2]}, F) :
od:

L := [ ] :
for i from K1 to K2 do
L := [op(L), pt] :
pt := subs( {x = pt[1], y = pt[2]}, F) :

od:
L :
end:

#FP2(F,x,y): The list of fixed points of the transformation [x,y]->F. Try
#FP2([x-y,x=y],x,y);
FP2 :=proc(F, x, y) local L, i :
```

```
L := [solve({F[1]=x, F[2]=y}, {x, y})]:

[seq(subs(L[i], [x, y]), i=1..nops(L))]:
end:


#SFP2(F,x,y): The list of Stable fixed points of the transformation [x,y]->F. Try
#SFP2([(1+x)/(1+y), (1+7*y)/(4+x)],x,y);
SFP2 :=proc(F, x, y) local L, J, S, J0, i, pt, EV:

L := evalf(FP2(F, x, y)):
    #F is the list of ALL fixed points of the transformation [x,y]->F using the previous procedure
    FP2(F,x,y), but since we are interested in numbers we take the floating point version using
    evalf

J := Matrix(normal([[diff(F[1], x), diff(F[1], y)], [diff(F[2], x), diff(F[2], y)]])):
    #J is the Jacobian matrix in general (in terms of the variables x and y). Note that J is a
    SYMBOLIC matrix featuring variables x and y

S := [ ]:  #S is the list of stable fixed points that starts out empty

for i from 1 to nops(L) do  #we examime it case by case
pt := L[i]: #pt is the current fixed point to be examined

J0 := subs({x=pt[1], y=pt[2]}, J):
    #J0 is the NUMERICAL matrix obtained by plugging-in the examined fixed pt

EV := Eigenvalues(J0):
    # We used Maple's command Eigenavalues to find the eigenvalues of this 2 by 2 matrix

if abs(EV[1]) < 1 and abs(EV[2]) < 1 then
 S := [op(S), pt]:
    #If both eigenvalues have absolute value less than 1 it means that they are stable, so we
    append the examined fixed point, pt, to the list of fixed points
fi:

od:
S: #the output is S
end:




###added Oct. 17, 20221
with(plots):

PlotOrb1 :=proc(L) local i, d:

d := textplot([L[1], 0, 0]):
```

```
for i from 2 to nops(L) do
d := d, textplot([L[i], 0, i-1]) :
od:
display(d) :
end:


PlotOrb2 :=proc(L) local i, d :

d := textplot([op(L[1]), 0]) :

for i from 2 to nops(L) do
d := d, textplot([op(L[i]), i-1]) :
od:
display(d) :
end:
###End added Oct. 17, 20221


###old stuff
#M11.txt: Maple code for Lecture 11 of Dynamical Models in Biology taught by Dr. Z.
Help11 :=proc() : print(` SFPe(f,x), Orbk(k,z,f,INI,K1,K2) `) :end:


    #SFPe(f,x): The set of fixed points of x->f(x) done exactly (and allowing symbolic
    parameters), followed by the condition of stability (if it is netween -1 and 1 it is stable)
#Try: FPe(k*x*(1-x),x);
#VERSION OF Oct. 12, 2021 (avoiding division by 0)
SFPe :=proc(f, x) local f1, L, i, M:
f1 := normal(diff(f, x)) :
L := [solve(numer(f-x), x)] :
M := [ ] :

for i from 1 to nops(L) do
 if subs(x=L[i], denom(f1) ) ≠ 0 then
  M := [op(M), [L[i], normal(subs(x=L[i],f1))]] :
 fi:
od:
M:

end:


#Added after class

    #Orbk(k,z,f,INI,K1,K2): Given a positive integer k, a letter (symbol), z, an expression f of z
    [1], ..., z[k] (representing a multi-variable function of the variables z[1],...,z[k]

    #a vector INI representing the initial values [x[1],..., x[k]], and (in applications) positive
```

```
integres K1 and K2, outputs the

#values of the sequence starting at n=K1 and ending at n=K2. of the sequence satisfying the difference equation
##x[n]=f(x[n-1],x[n-2],..., x[n-k+1]):

#This is a generalization to  higher-order difference equation of procedure Orb(f,x,x0,K1,K2). For example
#Orbk(1,z,5/2*z[1]*(1-z[1]),[0.5],1000,1010);  should be the same as
#Orb(5/2*z[1]*(1-z[1]),z[1],[0,5],1000,1010);
#Try:
#Orbk(2,z,(5/4)*z[1]-(3/8)*z[2],[1,2],1000,1010);
Orbk := proc(k, z, f, INI, K1, K2) local L, i, newguy :
L := INI :  #We start out with the list of initial values

if not (type(k, integer) and type(z, symbol) and type(INI, list) and nops(INI) = k and type(K1, integer) and type(K2, integer) and K1 > 0 and K2 > K1) then
#checking that the input is OK
 print(`bad input`) :
 RETURN(FAIL) :
fi:

while nops(L) < K2 do
 newguy := subs({seq(z[i]=L[-i], i=1 ..k)}, f) :
#Using what we know about the value yesterday, the day before yesterday, ... up to k days before yesterday we find the value of the sequence today
 L := [op(L), newguy] :  #we append the new value to the running list of values of our sequence
od:

[op(K1 ..K2, L)] :

end:


####STAFT FROM M9.txt
#M9.txt: Maple Code for "Dynamical models in Biology" (Math 336) taught by Dr. Z., Lecture 9

Help9 := proc( ) :
  print(` Orb(f,x,x0,K1,K2), Orb2D(f,x,x0,K) , FP(f,x) , SFP(f,x) , Comp(f,x)  `) :end:


#Orb(f,x,x0,K1,K2): Inputs an expression f in x (desccribing) a function of x, an initial point, x0, and a positive integer K, outputs
#the values of x[n] from n=K1 to n=K2. Try: where x[n]=f(x[n-1]), . Try:
#Orb(2*x*(1-x),x,0.4,1000,2000);
Orb := proc(f, x, x0, K1, K2) local x1, i, L :
x1 := x0 :

for i from 1 to K1 do
```

```
 x1 := subs(x = x1, f) :
     #we don't record the first values of K1, since we are interested in the long-time behavior of
     the orbit
od:

L := [x1] :

for i from K1 to K2 do
 x1 := subs(x = x1, f) :   #we compute the next member of the orbit
 L := [op(L), x1] : #we append it to the list
od:

L : #that's the output

end:

#Orb2D(f,x,x0,K): 2D version of Orb(f,x,x0,0,K), just for illustration
Orb2D :=proc(f, x, x0, K) local L, L1, i :
L := Orb(f, x, x0, 0, K) :
L1 := [[L[1], 0], [L[1], L[2]], [L[2], L[2]] ] :
for i from 3 to nops(L) do
 L1 := [op(L1), [L[i-1], L[i]], [L[i], L[i]]] :
od:
L1 :
end:


#FP(f,x): The list of fixed points of the map x->f where f is an expression in x. Try:
#FP(2*x*(1-x),x);
FP :=proc(f, x)
evalf([solve(f=x, x)]) :
end:


#SFP(f,x): The list of stable fixed points of the map x->f where f is an expression in x. Try:
#SFP(2*x*(1-x),x);
SFP :=proc(f, x) local L, i, f1, pt, Ls :
L := FP(f, x) : #The list of fixed points (including complex ones)

Ls := [ ] :      #Ls is the list of stable fixed points, that starts out as the empty list

f1 := diff(f, x) :  #The derivative of the function f w.r.t. x

for i from 1 to nops(L) do
pt := L[i] :

if abs(subs(x = pt, f1)) < 1 then

 Ls := [op(Ls), pt] :  # if pt,  is stable we add it to the list of stable points
```

```
fi:

od:

Ls :   #The last line is the output

end:

#Comp(f,x): f(f(x))
Comp :=proc( f, x) : normal(subs(x=f, f ) ) :end:




##added Oct. 17, 2021
#FP2drz(F,x,y): The list of fixed points of the transformation [x,y]->F. Dr. Z.'s way
#FP2([x-y,x+y],x,y);
FP2drz :=proc(F, x, y) local eq, i, L, S1 :
eq := [numer(F[1]-x), numer(F[2]-y)] :

L := Groebner[Basis](eq, plex(x, y)) :

S1 := evalf([solve(L[1], y)]) :
[seq([solve(subs(y=S1[i], L[2]), x), S1[i]], i=1..nops(S1))] :
end:


#SFP2drz(F,x,y): The list of Stable fixed points of the transformation [x,y]->F. Try
#SFP2drz([(1+x)/(1+y), (1+7*y)/(4+x)],x,y);
SFP2drz :=proc(F, x, y) local L, J, S, J0, i, pt, EV :

L := FP2drz(F, x, y) :
    #F is the list of ALL fixed points of the transformation [x,y]->F using the previous procedure
    FP2(F,x,y), but since we are interested in numbers we take the floating point version using
    evalf

J := Matrix(normal([[diff(F[1], x), diff(F[2], x)], [diff(F[1], y), diff(F[2], y)]])) :
    #J is the Jacobian matrix in general (in terms of the variables x and y). Note that J is a
    SYMBOLIC matrix featuring variables x and y

S := [ ] :   #S is the list of stable fixed points that starts out empty

for i from 1 to nops(L) do   #we examime it case by case
pt := L[i] :  #pt is the current fixed point to be examined

J0 := subs({x=pt[1], y=pt[2]}, J) :
    #J0 is the NUMERICAL matrix obtained by plugging-in the examined fixed pt

EV := Eigenvalues(J0) :
    # We used Maple's command Eigenavalues to find the eigenvalues of this 2 by 2 matrix
```

```
if abs(EV[1]) < 1 and abs(EV[2]) < 1 then
 S := [op(S), pt] :
    #If both eigenvalues have absolute value less than 1 it means that they are stable, so we
    append the examined fixed point, pt, to the list of fixed points
fi:

od:
S : #the output is S
end:
```

```
> #1 (a)
  Orbk(2, z, z[1]·(5/3 − z[2]), [0.5, 0.5], 1000, 1020);
```

[0.6666666666, 0.6666666666] **(1)**

```
> #1 (b)
  Orbk(2, z, z[1]·(2 − z[2]), [0.5, 0.5], 1000, 1020);
```

[0.9883413838, 0.9405167599, 0.9514818836, 1.008079109, 1.056989208, 1.048449677, 0.9886993603, 0.9407971956, 0.9514288060, 1.007756059, 1.056703974, 1.048508116, 0.9890535391, 0.9410764153, 0.9513778716, 1.007436466, 1.056420171, 1.048564138, 0.9894039700, 0.9413544191, 0.9513290388] **(2)**

```
> #4 (a)
  F := diff(x·(3 − x)·(5 − x), x);
  subs(x = 0, F);
  subs(x = 3, F);
  subs(x = 5, F);
```

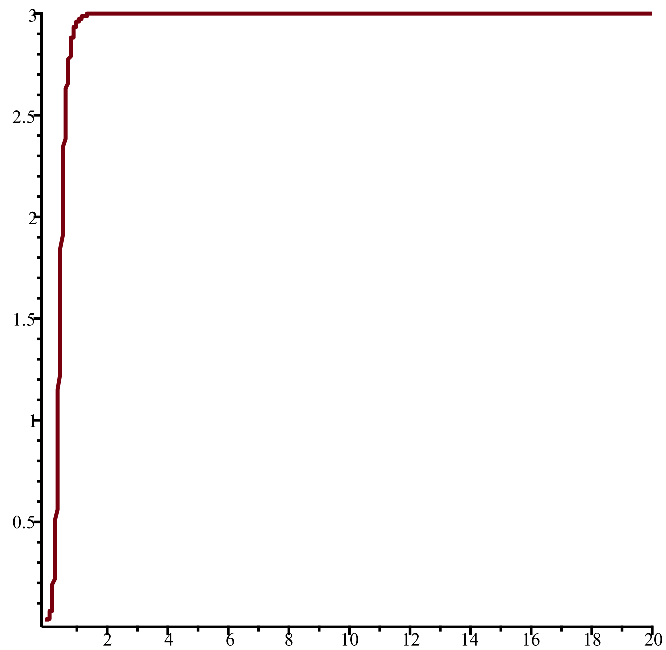
$$F := (3 - x)\,(5 - x) - x\,(5 - x) - x\,(3 - x)$$

$$15$$

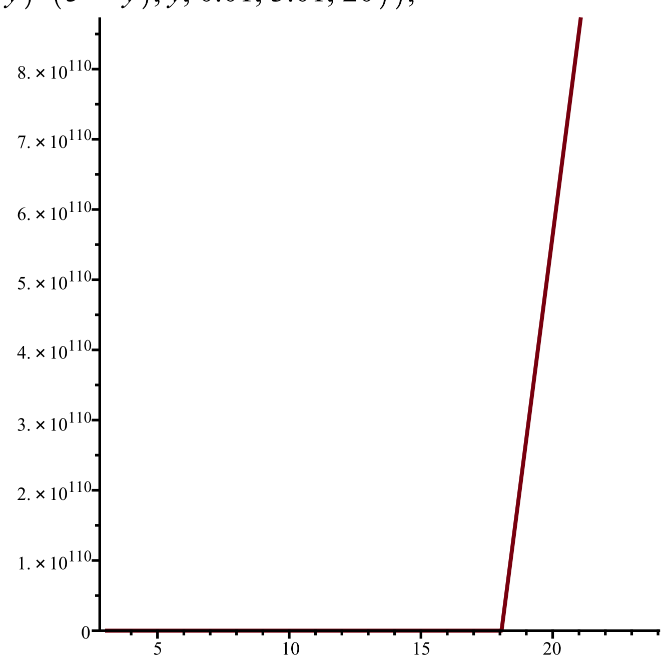
$$-6$$

$$10 \quad \textbf{(3)}$$

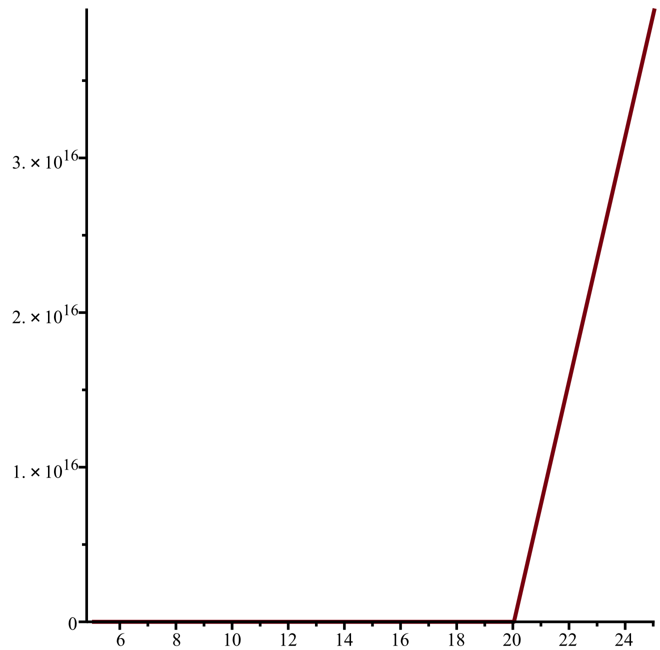
```
>  plot(Dis1(y·(3 − y)·(5 − y), y, 0.01, 0.01, 20));
```



> $plot(Dis1(y \cdot (3 - y) \cdot (5 - y), y, 0.01, 3.01, 20));$



> $plot(Dis1(y \cdot (3 - y) \cdot (5 - y), y, 0.01, 5.01, 20));$



> *#4 (b)*

$F2 := diff\left(x^2 \cdot (3-x) \cdot (5-x) \cdot (7-x), x\right);$

$subs(x=0, F2);$

$subs(x=3, F2);$

$subs(x=5, F2);$

$subs(x=7., F2);$

$$F2 := 2\,x\,(3-x)\,(5-x)\,(7-x) - x^2\,(5-x)\,(7-x) - x^2\,(3-x)\,(7-x) - x^2\,(3-x)\,(5-x)$$

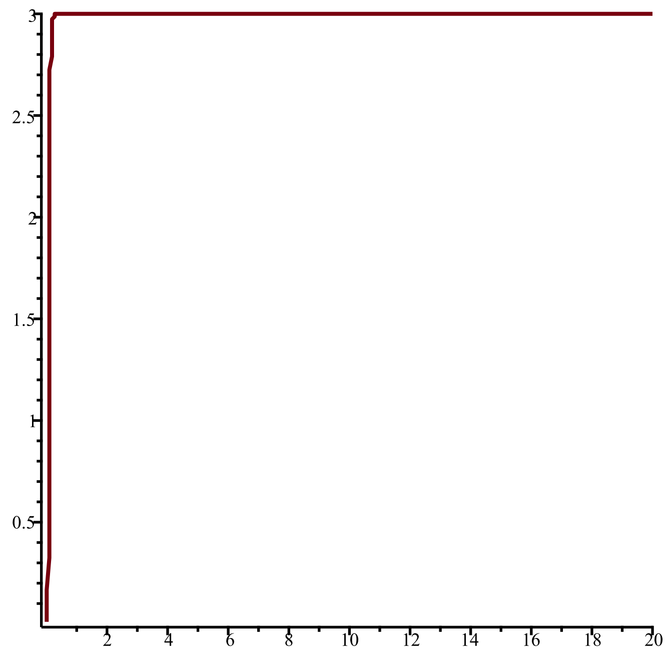
$$0$$

$$-72$$

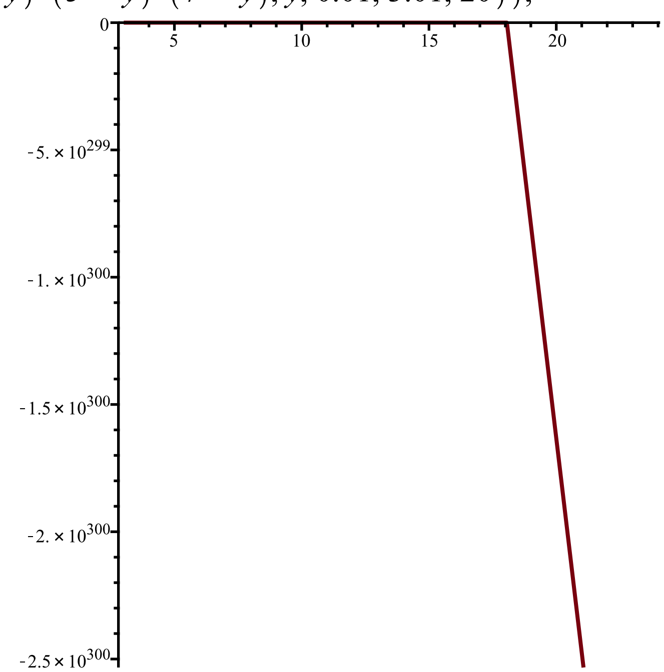
$$100$$

$$-392. \qquad \textbf{(4)}$$

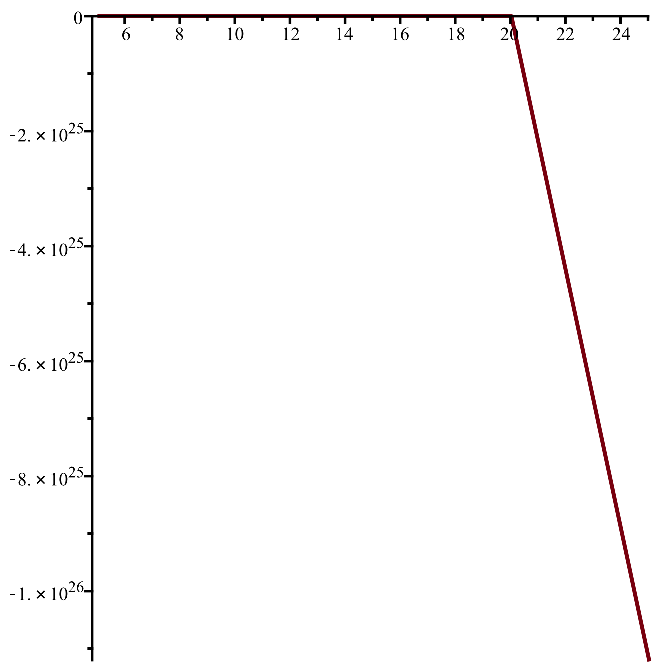
> $plot(Dis1(y \cdot (3-y) \cdot (5-y) \cdot (7-y), y, 0.01, 0.01, 20));$



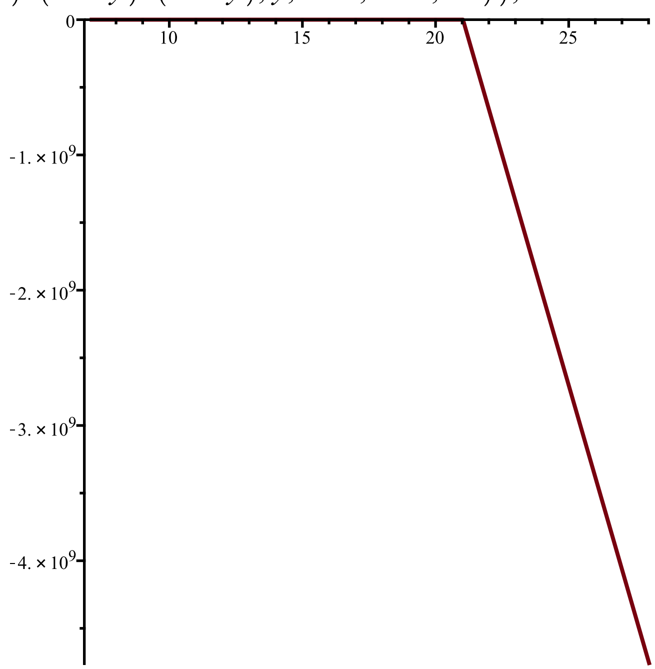
> $plot(Dis1(y \cdot (3 - y) \cdot (5 - y) \cdot (7 - y), y, 0.01, 3.01, 20));$



> $plot(Dis1(y \cdot (3 - y) \cdot (5 - y) \cdot (7 - y), y, 0.01, 5.01, 20));$



> $plot(Dis1(y \cdot (3-y) \cdot (5-y) \cdot (7-y), y, 0.01, 7.01, 20));$



>

Dynamic Modeling HW 16 - Okay to Post

1) A $x(n) = x(n-1)\left(\frac{5}{3} - x(n-2)\right)$

$x_1(n) = x_1(n-1)\left(\frac{5}{3} - x(n-2)\right)$

$x_2(n) = x(n-1)$

(i) $x_1(n) = x_1(n-1)\left(\frac{5}{3} x_2(n-1)\right)$

$x_2(n) = x_1(n-1)$

(ii) $f(x_1,x_2) = x_1\left(\frac{5}{3} - x_2\right)$

$g(x_1,x_2) = x_1$

Equilibrium points: $(0,0), \left(\frac{2}{3}, \frac{2}{3}\right)$

$x_2 = x_1$

$x_2 = x_2\left(\frac{5}{3} - x_2\right)$

$x_2 = \frac{5}{3}x_2 - x_2^2$

$x_2^2 - \frac{2}{3}x_2 = 0$

$x_2\left(x_2 - \frac{2}{3}\right) = 0$

$x_1 = x_2 = 0, \frac{2}{3}$

(iii) Jacobian: $\begin{bmatrix} f_{x_1}(x_1^\circ, x_2^\circ) & f_{x_2}(x_1^\circ, x_2^\circ) \\ g_{x_1}(x_1^\circ, x_2^\circ) & g_{x_2}(x_1^\circ, x_2^\circ) \end{bmatrix}$

$\left.\begin{matrix} f_{x_1} = \frac{5}{3} - x_2 & g_{x_1} = 1 \\ f_{x_2} = x_1 & g_{x_2} = 0 \end{matrix}\right\} \Rightarrow \begin{bmatrix} \frac{5}{3} - x_2 & x_1 \\ 1 & 0 \end{bmatrix}$

* $(0,0)$

$\begin{bmatrix} \frac{5}{3} & 0 \\ 1 & 0 \end{bmatrix} \Rightarrow \det\begin{bmatrix} \frac{5}{3} - \lambda & 0 \\ 1 & -\lambda \end{bmatrix} = -\lambda\left(\frac{5}{3} - \lambda\right) = 0$

$\lambda = 0; \frac{5}{3}$

Eigenvalues are not both less than 1; (0,0) is unstable

* $\left(\frac{2}{3}, \frac{2}{3}\right)$

$\begin{bmatrix} 1 & \frac{2}{3} \\ 1 & 0 \end{bmatrix} \Rightarrow \det\begin{bmatrix} 1-\lambda & \frac{2}{3} \\ 1 & -\lambda \end{bmatrix} = -\lambda(1-\lambda) - \frac{2}{3} = 0$

$3\left(\lambda^2 - \lambda - \frac{2}{3} = 0\right)$

$3\lambda^2 - 3\lambda - 2 = 0$

$\lambda = \frac{3 \pm \sqrt{9 - 4(3)(-2)}}{2(3)} = \frac{3 \pm \sqrt{9+24}}{6} = \frac{3 \pm \sqrt{33}}{6}$

$\lambda = \frac{3+\sqrt{33}}{6} = 1.46, \ \lambda = \frac{3-\sqrt{33}}{6} = -0.46$

Eigenvalues are not both less than 1, $\left(\frac{2}{3}, \frac{2}{3}\right)$ is unstable

B $x(n) = x(n-1)(2 - x(n-2))$

(i) $x_1(n) = x_1(n-1)(2 - x_2(n-1))$

$x_2(n) = x_1(n-1)$

(ii) $f(x_1, x_2) = x_1(2-x_2)$

$g(x_1, x_2) = x_1$

Equilibrium points: (0,0), (1,1)

$x_2 = x_1$

$x_2 = x_2(2-x_2)$

$x_2 = 2x_2 - x_2^2$

$x_2^2 - x_2 = 0$

$x_2(x_2-1) = 0$

$x_1 = x_2 = 0, 1$

(iii) Jacobian: $\begin{bmatrix} f_{x_1}(x_1,x_2) & f_{x_2}(x_1,x_2) \\ g_{x_1}(x_1,x_2) & g_{x_2}(x_1,x_2) \end{bmatrix}$

$\left.\begin{matrix} f_{x_1} = 2-x_2 & g_{x_1} = 1 \\ f_{x_2} = x_1 & g_{x_2} = 0 \end{matrix}\right\} = \begin{bmatrix} 2-x_2 & x_1 \\ 1 & 0 \end{bmatrix}$

* (0,0)

$\begin{bmatrix} 2 & 0 \\ 1 & 0 \end{bmatrix} \Rightarrow \det \begin{bmatrix} 2-\lambda & 0 \\ 1 & -\lambda \end{bmatrix} = -\lambda(2-\lambda) = 0$

$\lambda = 2, 0$

Equilibrium points are not both less than 1; (0,0) is unstable

* (1,1)

$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \Rightarrow \det \begin{bmatrix} 1-\lambda & 1 \\ 1 & -\lambda \end{bmatrix} = -\lambda(1-\lambda) - 1 = 0$

$\lambda^2 - \lambda - 1 = 0$

$\lambda = \frac{1 \pm \sqrt{1-4(1)(-1)}}{2} = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$

$\lambda = \frac{1+\sqrt{5}}{2} = 1.62$

$\lambda = \frac{1-\sqrt{5}}{2} = -0.62$

Equilibrium points are not both less than 1; (1,1) is unstable

2) $x(n) = x(n-1)(a - x(n-2))$

$x_1(n) = x_1(n-1)(a - x_2(n-1))$

$x_2(n) = x_1(n-1)$

$f(x_1,x_2) = x_1(a-x_2)$

$g(x_1,x_2) = x_1$

$x_2 = x_1$

$x_2 = x_2(a-x_2)$

$x_2 = ax_2 - x_2^2$

$x_2^2 + x_2 - ax_2 = 0$

$\rightarrow x_2^2 + x_2(1-a) = 0$

$x_2(x_2+1-a) = 0$

$x_2 = 0, a-1$

Equilibrium points: $(0,0), (a-1, a-1)$

Jacobian: $\begin{bmatrix} f_{x_1}(x_1^0, x_2^0) & f_{x_2}(x_1^0, x_2^0) \\ g_{x_1}(x_1^0, x_2^0) & g_{x_2}(x_1^0, x_2^0) \end{bmatrix}$

$\left.\begin{matrix} f_{x_1} = a - x_2 & g_{x_1} = 1 \\ f_{x_2} = x_1 & g_{x_2} = 0 \end{matrix}\right\} \Rightarrow \begin{bmatrix} a-x_2 & x_1 \\ 1 & 0 \end{bmatrix}$

* $(0,0)$

$\begin{bmatrix} a & 0 \\ 1 & 0 \end{bmatrix} \Rightarrow \det \begin{bmatrix} a-\lambda & 0 \\ 1 & -\lambda \end{bmatrix} = -\lambda(a-\lambda) = 0$

$\lambda = 0, a$

(0,0) will be stable if $|a| < 1$

* $(a-1, a-1)$

$\begin{bmatrix} 1 & a-1 \\ 1 & 0 \end{bmatrix} \Rightarrow \det \begin{bmatrix} 1-\lambda & a-1 \\ 1 & -\lambda \end{bmatrix} = -\lambda(1-\lambda) - a + 1 = 0$

$\lambda^2 - \lambda - a + 1 = 0$

$\lambda = \frac{1 \pm \sqrt{1-4(1)(-a+1)}}{2} = \frac{1 \pm \sqrt{1+4a+4}}{2} = \frac{1 \pm \sqrt{5+4a}}{2}$

(a-1, a-1) will be stable if [ ]

4) [A] $x'(t) = x(t)(3-x(t))(5-x(t))$

$F(x) = x(3-x)(5-x)$

$0 = x(3-x)(5-x)$

$x = 0, 3, 5$

$F'(x) = (3-x)(5-x) - x(5-x) - x(3-x)$

$F'(0) = (3)(5) - 0(5) - 0(3) = 15$

$F'(3) = (0)(2) - 3(2) - 3(0) = -6$

$F'(5) = (-2)(0) - 5(0) - 5(-2) = 10$

$F'(0) > 0$: unstable

$F'(3) < 0$: stable

$F'(5) > 0$: unstable

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(B) $x'(t) = x(t)^2(3-x(t))(5-x(t))(7-x(t))$

$F(x) = x^2(3-x)(5-x)(7-x)$

$0 = x^2(3-x)(5-x)(7-x)$

$x = 0, 3, 5, 7$

$F'(0) = 0 = 0$; unstable

$F'(3) = -72 < 0$; stable

$F'(5) = 100 > 0$; unstable

$F'(7) = -392 < 0$; stable