```
> # Max Mekhanikov - HW 16 - Okay to post

> Orbk := proc(k, z, f, INI, K1, K2) local L, i, newguy :
L := INI : #We start out with the list of initial values

if not (type(k, integer) and type(z, symbol) and type(INI, list) and nops(INI) = k
    and type(K1, integer) and type(K2, integer) and K1 > 0 and K2 > K1) then
    #checking that the input is OK
 print(`bad input`) :
 RETURN(FAIL) :
fi:

while nops(L) < K2 do
 newguy := subs({seq(z[i] = L[-i], i = 1 ..k)}, f) :
    #Using what we know about the value yesterday, the day before yesterday, ... up to k days
    before yesterday we find the value of the sequence today
 L := [op(L), newguy] : #we append the new value to the running list of values of our sequence
od:

[op(K1 ..K2, L)] :

end:
```

> $Orbk\left(2, z, z[1]\cdot\left(\frac{5}{3} - z[2]\right), \left[\frac{2}{3}, \frac{2}{3}\right], 1000, 1010\right)$

$$\left[\frac{2}{3}, \frac{2}{3}, \frac{2}{3}, \frac{2}{3}, \frac{2}{3}, \frac{2}{3}, \frac{2}{3}, \frac{2}{3}, \frac{2}{3}, \frac{2}{3}, \frac{2}{3}\right] \qquad \textbf{(1)}$$

> $Orbk(2, z, z[1]\cdot(2 - z[2]), [1, 1], 1000, 1010)$

$$[1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1] \qquad \textbf{(2)}$$

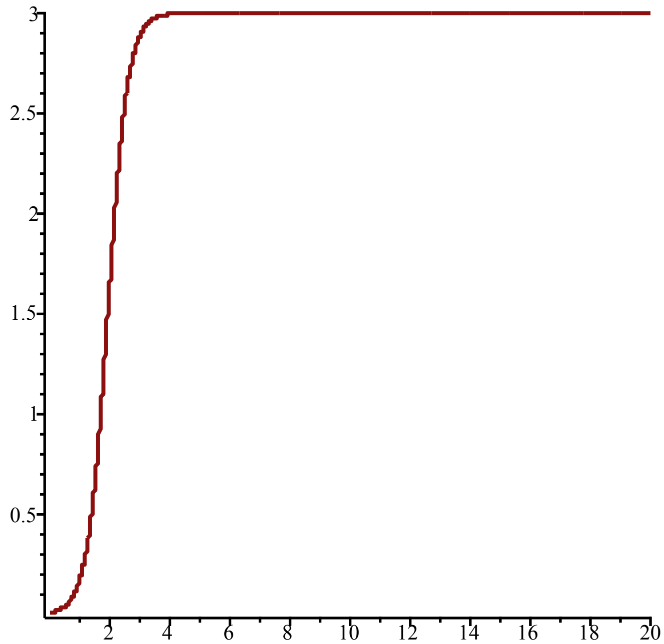
```
> Orb := proc(f, x, x0, K1, K2) local x1, i, L :
x1 := x0 :

for i from 1 to K1 do
 x1 := subs(x = x1, f) :
    #we don't record the first values of K1, since we are interested in the long-time behavior of the
    orbit
od:

L := [x1] :

for i from K1 to K2 do
 x1 := subs(x = x1, f) :   #we compute the next member of the orbit
 L := [op(L), x1] : #we append it to the list
od:

L : #that's the output
```

end:

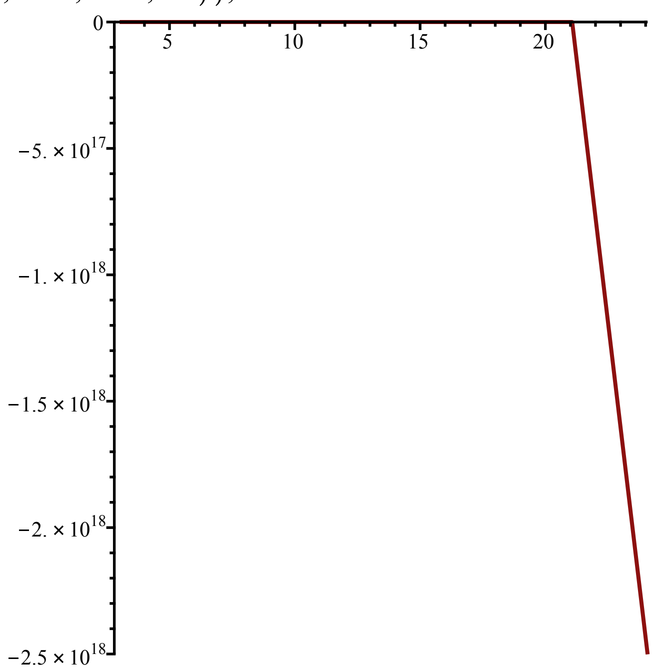
$Dis1 := \mathbf{proc}(F, y, y0, h, A)$ **local** L, x, i :
$L := Orb(x + h * subs(y = x, F), x, y0, 0, \mathrm{trunc}(A/h))$:

$L := [seq([i * h, L[i]], i = 1..nops(L))]$:
end:

> $plot(Dis1(y*(3-y), y, 0.01, 0.01, 20));$



> $plot(Dis1(y*(3-y), y, 0.01, 3.01, 20));$



>

>

> $eq4a := diff((x \cdot (3 - x) \cdot (5 - x)), x)$

(3)

$$eq4a := (3-x)\,(5-x) - x\,(5-x) - x\,(3-x) \tag{3}$$

> $subs(x = 0, eq4a)$

$$15 \tag{4}$$

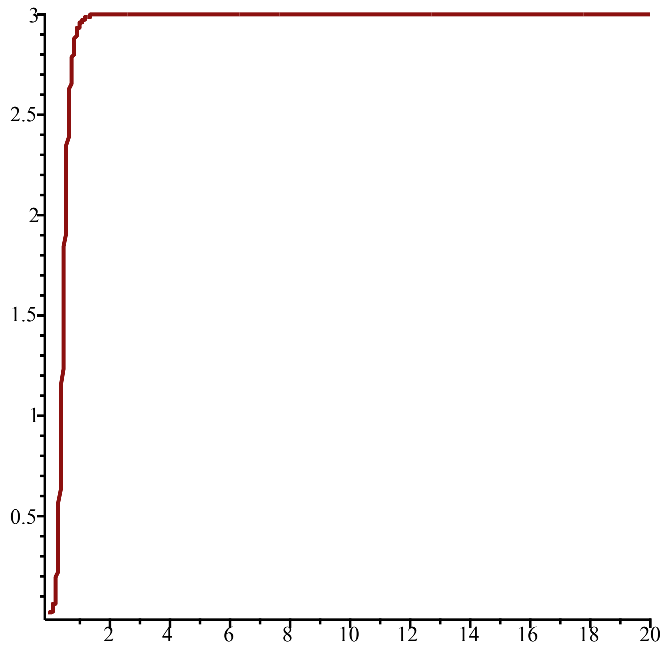
> $subs(x = 3, eq4a)$

$$-6 \tag{5}$$

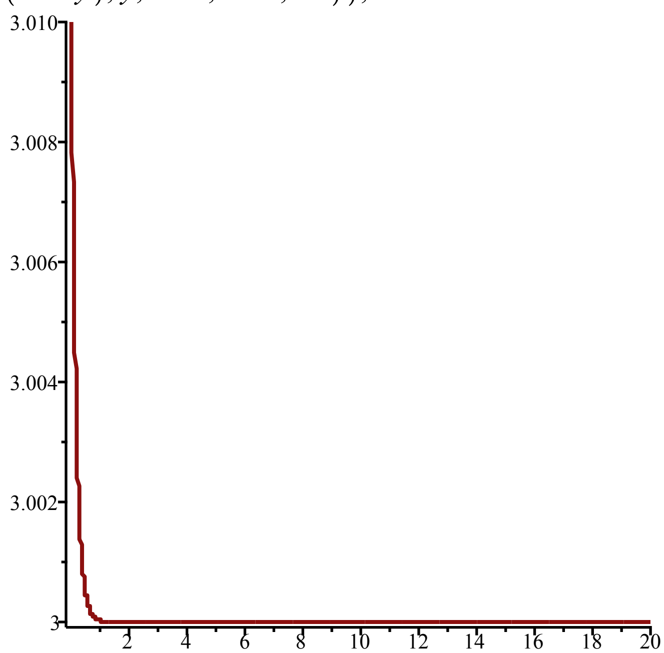
> $subs(x = 5, eq4a)$

$$10 \tag{6}$$

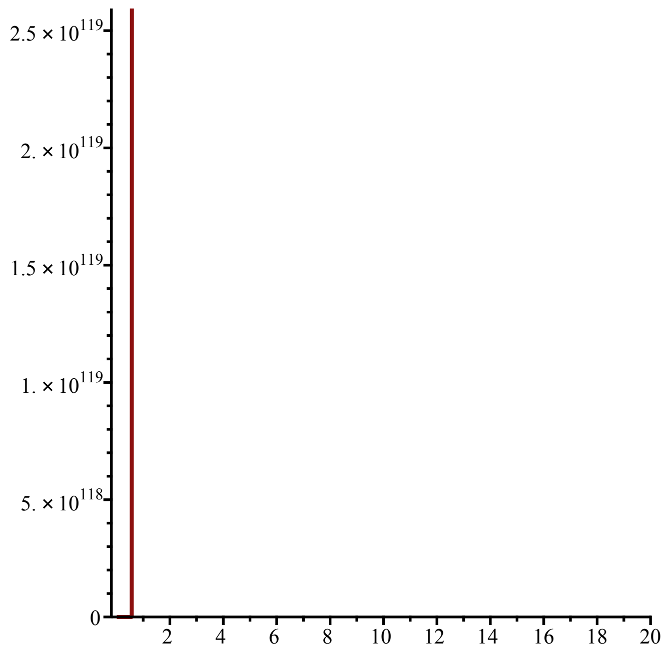
> $plot(Dis1(y\cdot(3-y)\cdot(5-y), y, 0.01, 0.01, 20));$



> $plot(Dis1(y\cdot(3-y)\cdot(5-y), y, 3.01, 0.01, 20));$



> $plot(Dis1(y\cdot(3-y)\cdot(5-y), y, 5.01, 0.01, 20));$



> # Question 4b

>

> $eq4b := diff\left(x^2 \cdot (3-x) \cdot (5-x) \cdot (7-x), x\right)$

$$eq4b := 2\,x\,(3-x)\,(5-x)\,(7-x) - x^2\,(5-x)\,(7-x) - x^2\,(3-x)\,(7-x) - x^2\,(3-x)\,(5-x) \tag{7}$$

> $subs(x=0, eq4b)$

$$0 \tag{8}$$

> $subs(x=3, eq4b)$

$$-72 \tag{9}$$

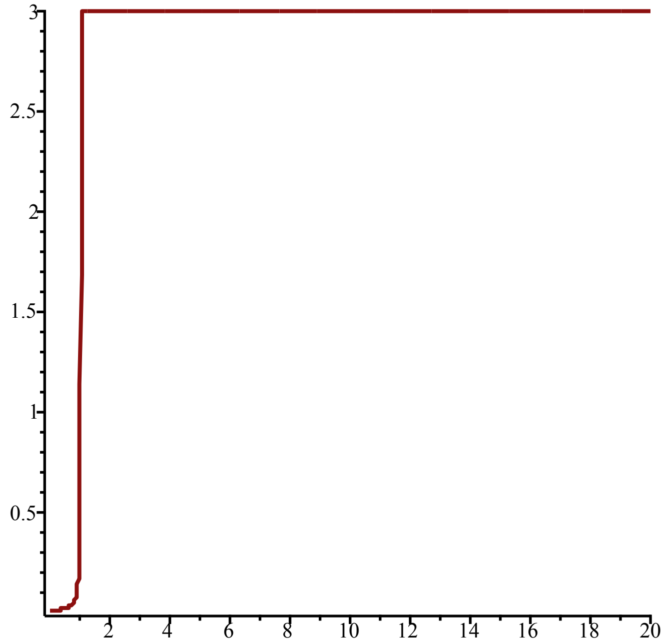
> $subs(x=5, eq4b)$

$$100 \tag{10}$$

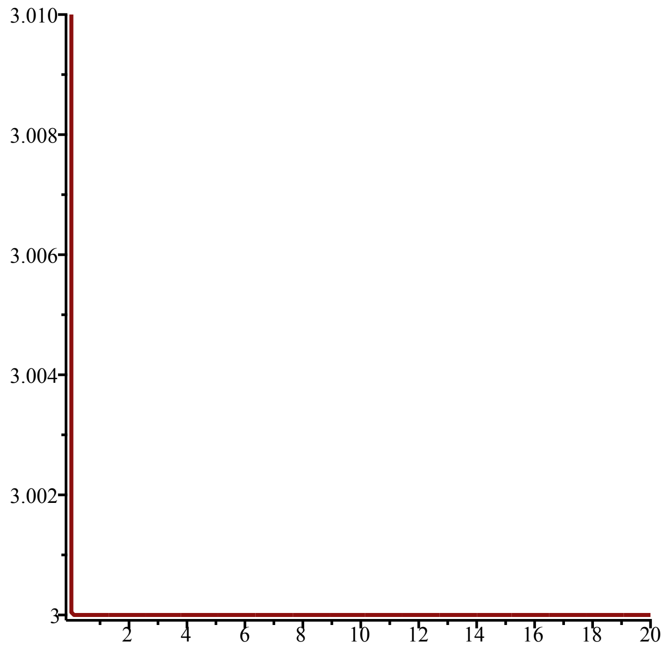
> $subs(x=7, eq4b)$

$$-392 \tag{11}$$

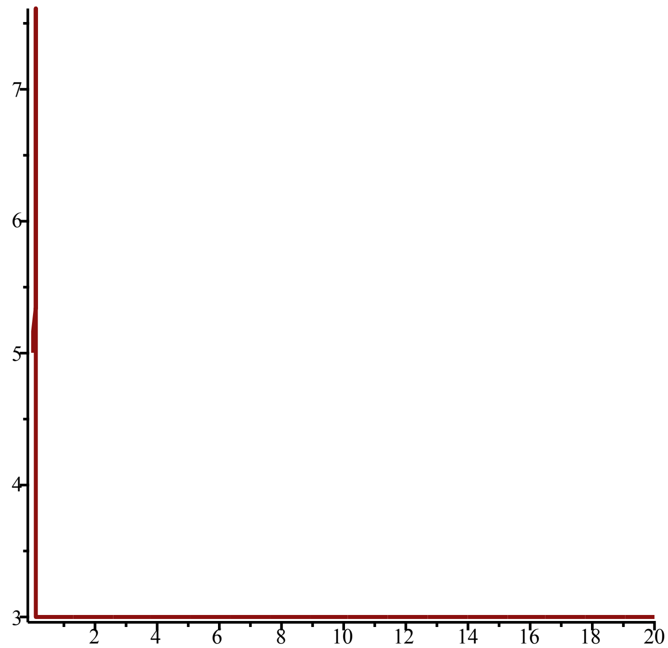
> $plot\left(Dis1\left(y^2 * (3-y)\cdot(5-y)\cdot(7-y), y, 0.01, 0.01, 20\right)\right);$



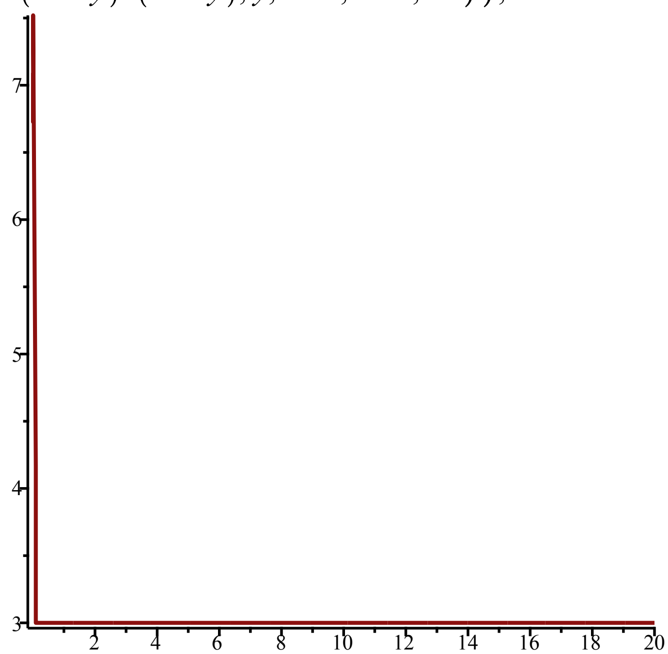
> $plot\left(Dis1\left(y^2 * (3-y)\cdot(5-y)\cdot(7-y), y, 3.01, 0.01, 20\right)\right);$



> $plot\left(Dis1\left(y^2 * (3-y)\cdot(5-y)\cdot(7-y), y, 5.01, 0.01, 20\right)\right);$



> $plot\big(Dis1\big(y^2 * (3-y)\cdot(5-y)\cdot(7-y), y, 7.01, 0.01, 20\big)\big);$



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Max M. - HW 16

1. a) $x(n-1)(5/3 - x(n-2))$

$$X_2(n) = x(n-1)$$

$$X_1(n) = X_1(n-1)(5/3 - X_2(n-1))$$

$$x(n) = z$$

$$z = z(5/3 - z)$$

$$z = 0, \quad z = \frac{2}{3}$$

eq. points are $(0,0)$ & $(2/3, 2/3)$

$$(z_1, z_2) \rightarrow (z_1(5/3 - z_2), z_1)$$

$$f(z_1, z_2) = z_1(5/3 - z_2)$$

$$g(z_1, z_2) = z_1$$

$$J = \begin{pmatrix} 5/3 - z_2 & -z_1 \\ 1 & 0 \end{pmatrix}$$

$(z_1, z_2) = (0,0)$

$$J = \begin{pmatrix} 5/3 & 0 \\ 1 & 0 \end{pmatrix}, \quad \lambda = 0, 5/3$$

unstable (>1)

$(z_1, z_2) = (2/3, 2/3)$

$$J = \begin{pmatrix} 1 & -2/3 \\ 1 & 0 \end{pmatrix}$$

$$\lambda = \frac{2}{3} \pm \frac{2}{3} i \quad (<1) \rightarrow \begin{matrix} (2/3, 2/3) \\ \text{stable} \end{matrix}$$

$X = 0$ unstable, $X = 2/3$ stable.

b) $x(n-1)(2 - x(n-2))$

$$X_2(n) = x(n-1)$$

$$X_1(n) = X_1(n-1)(2 - X_2(n-1))$$

$$X(n) = z$$

$$z = z(2 - z)$$

$$z = 0, \quad z = 1$$

eq. points: $(0,0)$, $(1,1)$

$$f(z_1, z_2) = z_1(2 - z_2)$$
$$g(z_1, z_2) = z_1$$

$$J = \begin{pmatrix} 2 - z_2 & -z_1 \\ 1 & 0 \end{pmatrix}$$

$(z_1, z_2) = (0,0)$

$$J = \begin{pmatrix} 2 & 0 \\ 1 & 0 \end{pmatrix}$$

$\lambda = 0, 2 \longrightarrow$ unstable (>1)

$(z_1, z_2) = (1,1)$

$$J = \begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix}$$

$\lambda = 1/2 \pm \frac{\sqrt{3}}{2} i$ abs val $= 1$

$\hookrightarrow$ unstable

Both eq. $x=0$ and $x=1$ are unstable.

2) $x(n) = x(n-1)(a - x(n-2))$

$x_2(n) = x(n-1)$

$x_1(n) = x_1(n-1)(a - x_2(n-1))$

$x(n) = a$

$z = z(a-z)$

$z = 0, z = a-1$

$z = az - z^2$

$1 = a - z$

$z + 1 = a$

eq. points $(0,0), (a-1, a-1)$

$$f(z_1, z_2) = z_1(a - z_2)$$
$$g(z_1, z_2) = z_1$$
$$J = \begin{pmatrix} a - z_2 & -z_1 \\ 1 & 0 \end{pmatrix}$$

$(z_1, z_2) = (0,0)$

$$J = \begin{pmatrix} a & 0 \\ 1 & 0 \end{pmatrix}$$

$$\lambda = 0, a$$

↳ stable if $a < 1$

$(z_1, z_2) = (a-1, a-1)$

$$J = \begin{pmatrix} 1 & 1-a \\ 1 & 0 \end{pmatrix} \qquad \det(J - \lambda I) = 0$$

$$\begin{vmatrix} 1-\lambda & 1-a \\ 1 & -\lambda \end{vmatrix} = 0$$

$$-\lambda + \lambda^2 - 1 + a = 0$$
$$\lambda^2 - \lambda - 1 + a = 0$$

$X = 0$ is stable if $a < 1$,
$x = a - 1$ is stable if ______.

4. $x'(t) = x(t)(3 - x(t))$

$F(x) = x(3 - x)$

↳ solve $F(x) = 0$, $x = 0$, $x = 3$

$F'(x) = 3 - 2x$

$F'(0) = 3 \rightarrow$ unstable b/c positive

$F'(3) = -3 \rightarrow$ stable b/c negative

a) $x'(t) = x(t)(3 - x(t))(5 - x(t))$

$F(x) = x(3 - x)(5 - x)$

↳ $x = 0$, $x = 3$, $x = 5$

$F'(x) = (3 - x)(5 - x) - x(5 - x) - x(3 - x)$

$F'(0) = 15 \rightarrow$ unstable

$F'(3) = -6 \rightarrow$ stable

$F'(5) = 10 \rightarrow$ unstable

b) $x'(t) = x(t)^2(3-x(t))(5-x(t))(7-x(t))$

$F(x) = x^2(3-x)(5-x)(7-x)$

$\hookrightarrow x=0,\ x=3,\ x=5,\ x=7$

$$F'(x) = 2x(3-x)(5-x)(7-x) - x^2(3-x)(5-x) - x^2(5-x)(7-x) - x^2(3-x)(7-x)$$

$F'(0) = 0$

$F'(3) = -72 \longrightarrow$ stable

$F'(5) = 100$

$F'(7) = -392 \longrightarrow$ stable