

1.

HW16

a.

$$\begin{aligned} x(x) &= x(n-1)(\frac{5}{3} - x(n-2)) \\ x_1(n) &= x_1(n-1)(5/3 - x_2(n-2)), \quad x_2(n) = x_1(n-1) \\ z &= z(\frac{5}{3} - z) \\ z &= 0, \frac{5}{3} \end{aligned}$$

Eq. points: $(0,0)$, $(\frac{5}{3}, \frac{5}{3})$

$$\begin{aligned} (z_1, z_2) &\rightarrow (z_1(\frac{5}{3} - z_2), z_1) \\ f(z_1, z_2) &= (z_1(\frac{5}{3} - z_2)), g(z_1, z_2) = z_1 \end{aligned}$$

$$J = \begin{bmatrix} \frac{5}{3} - z_2 & -z_1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} f_{z_1} & f_{z_2} \\ g_{z_1} & g_{z_2} \end{bmatrix}$$

$$(z_1, z_2) = (0,0) \rightarrow \begin{bmatrix} \frac{5}{3} & 0 \\ 1 & 0 \end{bmatrix}$$

eigenvalues = $\frac{5}{3}, 0$ so unstable

$$(z_1, z_2) = (\frac{5}{3}, \frac{5}{3}) \rightarrow \begin{bmatrix} 1 & -\frac{5}{3} \\ 1 & 0 \end{bmatrix}$$

eigenvalues = $1 \pm i\sqrt{\frac{16}{9}}$ so unstable

$$\begin{aligned} b. \quad x(n) &= x(n-1)(2-x(n-2)), \\ x_1(n) &= x_1(n-1)(2-x_2(n-1)), \quad x_2(n) = x_2(n-1) \\ z &= z(2-z) \\ z^2 - z &= 0 \\ z &= 0, 1 \end{aligned}$$

$$f(z_1, z_2) = (z_1(z - z_2), z_1)$$

(-1) > (-5)

$$S = \begin{bmatrix} 2 & 0 \\ 1 & 0 \end{bmatrix}$$

eigenvalues = 2 > 0 so unstable

$$(z_1, z_2) = (t, 1)$$

$$\mathcal{T} = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$$

$$\text{eigenvalues} = \frac{1}{2} \pm i \frac{\sqrt{3}}{2} \quad \text{so unstable}$$

```
;
> NULL;
> Orbk(2, z, z[1]*(2 - z[2]), [1, 1], 1000, 1020);
[1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]
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```
;
> NULL;
> Orbk(2, z, z[1]*(2 - z[2]), [0, 0], 1000, 1020);
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
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2.

2. $x(n) = x_{1}(n-1)(a - x_{1}(n-2))^{(2-\lambda)}(1)x_{2}(n) = (1)^{n}$

$x_{1}(n) = x_{1}(n-1)(a - x_{2}(n-1)), x_{2}(n) = x_{2}(n-1)$

$\lambda = \alpha - \omega$

$\lambda = a - \omega^2$

$\lambda = 0, a-1$

$(0, 0), (\alpha-1, a-1)$

$\Sigma = \begin{bmatrix} a-\omega_2 & -\omega_1 \\ 1 & 0 \end{bmatrix}^{(2-\lambda)}(1)x = (1)^n$

(Q6.) $\Sigma = \begin{bmatrix} a & 0 \\ 1 & 0 \end{bmatrix}$

eigenvalues = $a, 0$

$(a-1, a-1) \quad \Sigma = \begin{bmatrix} 1 & -a+1 \\ 1 & 0 \end{bmatrix}$

$0 = (1-\lambda)(-\lambda) - (-a+1)$

$\lambda = \frac{1 \pm \sqrt{4a+5}}{2}$

4.

4.3a $x'(t) = x(t)(3-x(t))(5-x(t))$

 $F(x) = x(3-x)(5-x)$
 $x = 0, 3, 5$
 $f'(x) = 3x^2 - 16x + 15$
 $f'(0) = 15 \text{ Unstable}$
 $f'(3) = -6 \text{ Stable}$
 $f'(5) = 0 \text{ Unstable}$

b $x'(t) = x(t)^2(3-x(t))(5-x(t))(7-x(t))$

 $F(x) = x^2(3-x)(5-x)(7-x)$
 $x = 0, 3, 5, 7$
 $F'(x) = -5x^4 + 60x^3 - 213x^2 + 210x$
 $F'(0) = 0 \text{ Unstable}$
 $F'(3) = -72 \text{ Stable}$
 $F'(5) = 100 \text{ Unstable}$
 $F'(7) = -392 \text{ Stable}$

$$(1-x)(-1)(x-1) = 0$$

$$\frac{c_3 + c_4}{5} = 0$$