John Hermitt
Hw 16

1. a. i. $x(n) = x(n-1)\left(\frac{5}{3} - x(n-2)\right)$

$x_1(n) = x_1(n-1)\left(\frac{5}{3} - x_2(n-1)\right)$, $x_2(n) = x_1(n-1)$

ii. $z = z\left(\frac{5}{3} - z\right)$

$z = \frac{5}{3}z - z^2$

$0 = \frac{2}{3}z - z^2$

$0 = z\left(\frac{2}{3} - z\right)$

$z = 0, \frac{2}{3}$

$(0,0)$ $\left(\frac{2}{3}, \frac{2}{3}\right)$

iii. $(z_1, z_2) \to \left(z_1\left(\frac{5}{3} - z_2\right), z_1\right)$

$$J = \begin{pmatrix} \frac{5}{3} - z_2 & -z_1 \\ 1 & 0 \end{pmatrix}$$

$(0,0)$: $J = \begin{pmatrix} \frac{5}{3} & 0 \\ 1 & 0 \end{pmatrix}$

eigenvalues $= \frac{5}{3}, 0$ unstable

$\left(\frac{2}{3}, \frac{2}{3}\right)$: $J = \begin{pmatrix} 1 & -\frac{2}{3} \\ 1 & 0 \end{pmatrix}$

eigenvalues: $\frac{1}{2} \pm i\frac{\sqrt{15}}{6}$ unstable

b. i. $x(n) = x(n-1)(2 - x(n-2))$

$x_1(n) = x_1(n-1)(2 - x_2(n-1))$ $\quad x_2(n) = x_1(n-1)$

ii. $z = z(2-z)$

$z = 2z - z^2$

$0 = z - z^2$

$0 = z(1-z)$

$z = 0, 1$

$(0,0)$ $(1,1)$

iii. $(z_1, z_2) \to (z_1(2-z_2), z_1)$

$J = \begin{pmatrix} 2-z_2 & -z_1 \\ 1 & 0 \end{pmatrix}$

$(0,0)$: $J = \begin{pmatrix} 2 & 0 \\ 1 & 0 \end{pmatrix}$

eigenvalues: $2, 0$ unstable

$(1,1)$: $J = \begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix}$

eigenvalues: $\frac{1}{2} \pm i\frac{\sqrt{3}}{2}$ unstable

2. $x(n) = x(n-1)(a - x(n-2))$

$x_1(n) = x_1(n-1)(a - x_2(n-1))$, $x_2(n) = x_1(n-1)$

$z = z(a-z)$

$z = az - z^2$

$0 = az - z - z^2$

$0 = z(a-1-z)$

$z = 0, a-1$

$(0,0)$ $(a-1, a-1)$

$J = \begin{pmatrix} a - z_2 & -z_1 \\ 1 & 0 \end{pmatrix}$

$(0,0)$; $J = \begin{pmatrix} a & 0 \\ 1 & 0 \end{pmatrix}$

eigenvalues : $a, 0$ to be stable, $-1 < a < 1$

$(a-1, a-1)$; $J = \begin{pmatrix} a-(a-1) & -(a-1) \\ 1 & 0 \end{pmatrix}$

$J = \begin{pmatrix} 1 & -a+1 \\ 1 & 0 \end{pmatrix}$

$0 = (1-\lambda)(-\lambda) - (-a+1)$

$0 = \lambda^2 - \lambda + a - 1$

$\lambda = \frac{1 \pm \sqrt{1 - 4(a-1)}}{2}$

$\lambda = \frac{1 \pm \sqrt{-4a+5}}{2}$

to be stable, $-1 < a < 1$

$\therefore -1 < \frac{1 \pm \sqrt{-4a+5}}{2} < 1$

$-2 < 1 \pm \sqrt{-4a+5} < 2$

$-3 < \pm\sqrt{-4a+5} < 1$

$9 < -4a + 5 < 1$

$4 < -4a < -4$

$-1 > a > 1$

$-3 < -\sqrt{-4a+5} < 1$

$3 > \sqrt{-4a+5} > 1$

$9 > -4a+5 > 1$

$4 > -4a > -4$

$-1 < a < 1$

4.a. $x'(t) = x(t)(3-x(t))(5-x(t))$

$F(x) = x(3-x)(5-x)$

$0 = x(3-x)(5-x)$

$x = 0, 3, 5$

$F'(x) = 3x^2 - 16x + 15$

$F'(0) = 15$ unstable

$F'(3) = -6$ stable

$F'(5) = 10$ unstable

b. $x'(t) = x(t)^2(3-x(t))(5-x(t))(7-x(t))$

$F(x) = x^2(3-x)(5-x)(7-x)$

$0 = x^2(3-x)(5-x)(7-x)$

$x = 0, 3, 5, 7$

$F'(x) = -5x^4 + 60x^3 - 213x^2 + 210x$

$F'(0) = 0$ unstable

$F'(3) = -72$ stable

$F'(5) = 100$ unstable

$F'(7) = -392$ stable

```
> read "/John/Rutgers/Senior Fall/Dynamic Models/M15.txt";
  Help15( );
```

HW3(u,v,w), HW2(u,v) , Dis1(F,y,y0,h,A), ToSys(k,z,f,INI) **(1)**

```
> #1 a
  Orbk(2, z, z[1]·(5/3 − z[2]), [0, 0], 1000, 1020);
```

[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0] **(2)**

```
> Orbk(2, z, z[1]·(5/3 − z[2]), [2/3, 2/3], 1000, 1020);
```

$\left[\frac{2}{3}, \frac{2}{3}\right]$ **(3)**

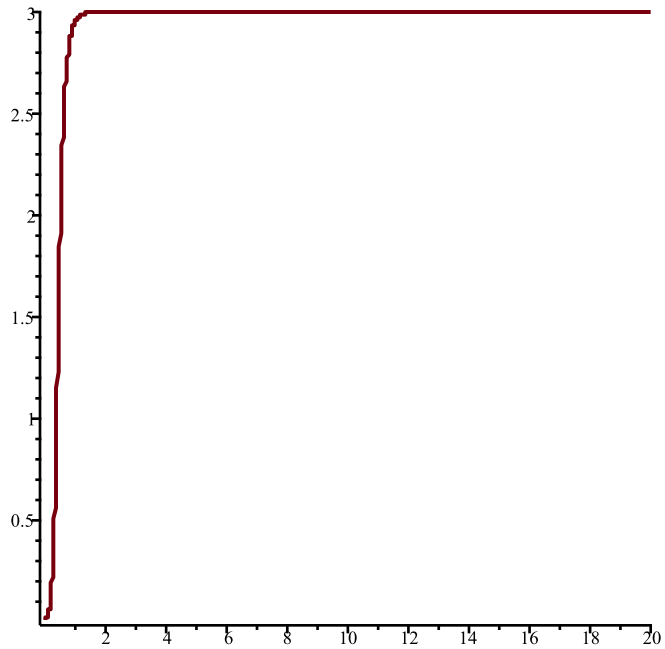
```
> #b
  Orbk(2, z, z[1]·(2 − z[2]), [0, 0], 1000, 1020);
```

[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0] **(4)**

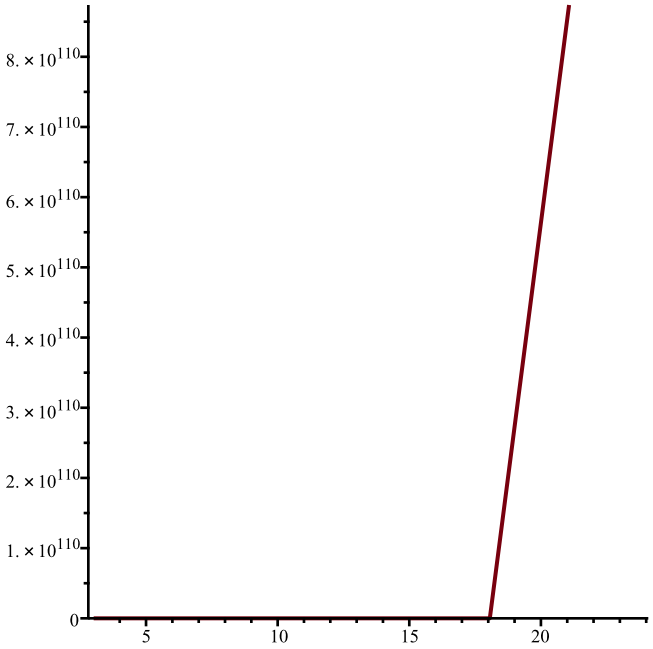
```
> Orbk(2, z, z[1]·(2 − z[2]), [1, 1], 1000, 1020)
```

[1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1] **(5)**

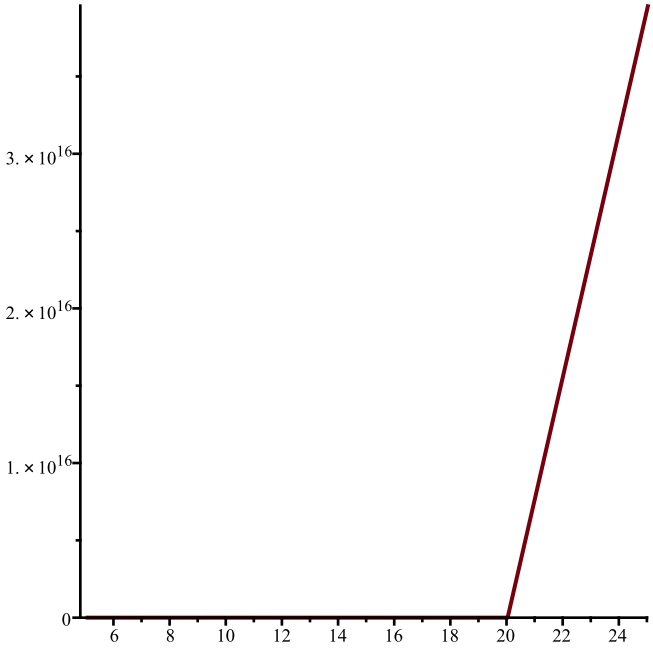
```
> #4 a
  plot(Dis1(y·(3 − y)·(5 − y), y, 0.01, 0.01, 20));
```



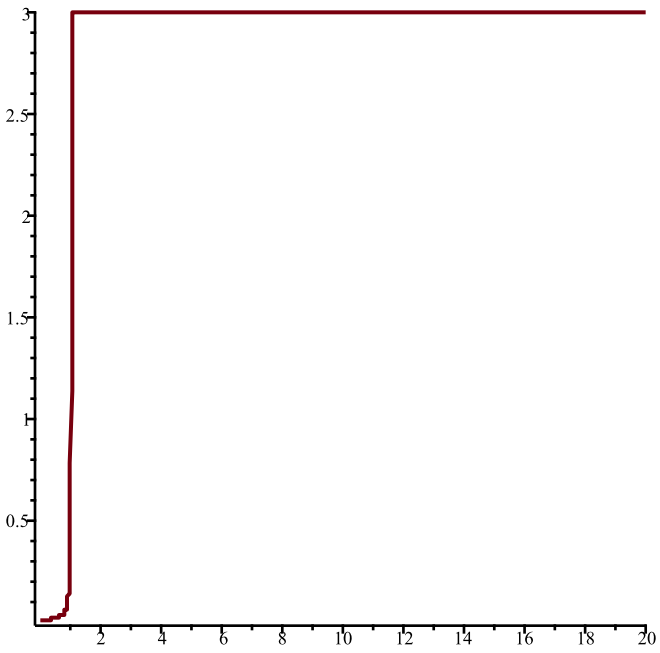

```
> plot(Dis1(y·(3 − y)·(5 − y), y, 0.01, 3.01, 20));
```



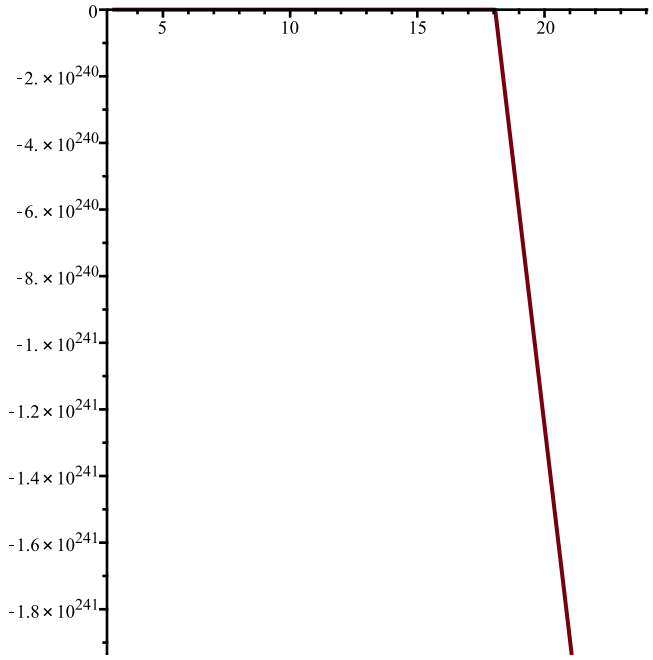
> $plot(Dis1(y\cdot(3-y)\cdot(5-y),y,0.01,5.01,20));$



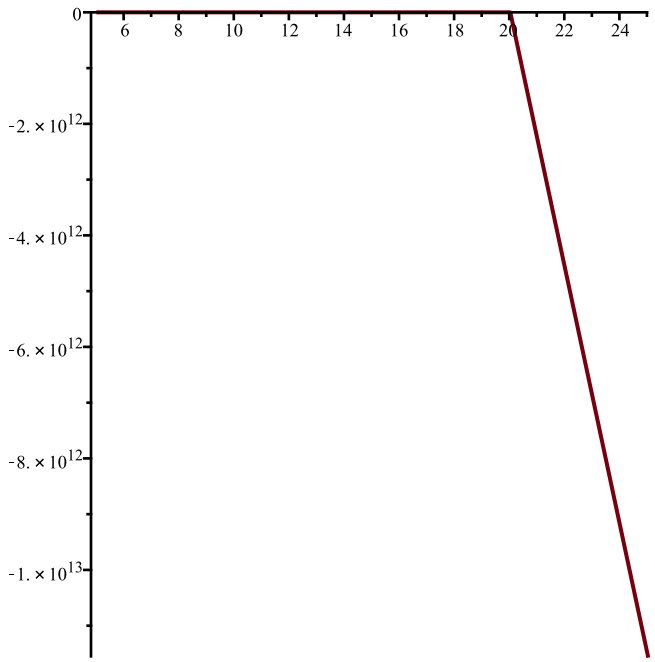
> #b
$plot\left(Dis1\left(y^2\cdot(3-y)\cdot(5-y)\cdot(7-y),y,0.01,0.01,20\right)\right)$



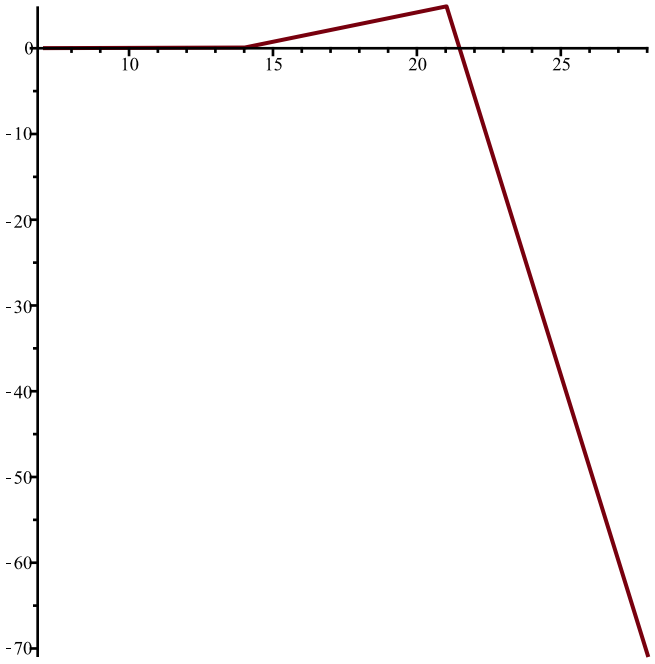
> $plot\left(Dis1\left(y^2\cdot(3-y)\cdot(5-y)\cdot(7-y), y, 0.01, 3.01, 20\right)\right)$



> $plot\left(Dis1\left(y^2\cdot(3-y)\cdot(5-y)\cdot(7-y), y, 0.01, 5.01, 20\right)\right)$



> $plot\left(Dis1\left(y^2 \cdot (3-y) \cdot (5-y) \cdot (7-y), y, 0.01, 7.01, 20\right)\right)$



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