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Question 1

a. $x(n) = x(n-1)\left(\frac{5}{3} - x(n-2)\right)$

i. $x_2(n) = x(n-1)$

$x_1(n) = x_1(n-1)\left(5/3 - x_2(n-1)\right)$

ii. $z = z(5/3 - z)$

$z = 5/3\, z - z^2$

$z^2 - 2/3\, z = 0$

$z(z - 2/3) = 0$

$z = 0, \quad z = 2/3$

Equilibrium points $(0,0)$ $(2/3, 2/3)$

iii $(z_1, z_2) \to (z_1(5/3 - z_2), z_1)$

$f(z_1, z_2) = z_1(5/3 - z_2)$ $g(z_1, z_2) = z_1$

$J = \begin{pmatrix} f_{z_1} & f_{z_2} \\ g_{z_1} & g_{z_2} \end{pmatrix}$ $\begin{pmatrix} 5/3 - z_2 & -z_1 \\ 1 & 0 \end{pmatrix}$

$(z_1, z_2) = (0, 0)$ $\begin{pmatrix} 5/3 & 0 \\ 1 & 0 \end{pmatrix}$ $\det(J - \lambda I) = (5/3 - \lambda)(-\lambda) - 1 = 0$

$\lambda^2 - 5/3\lambda - 1 = 0$

$\lambda = \frac{5/3 \pm \sqrt{(5/3)^2 + 4}}{2} = \frac{5/3 \pm \sqrt{\frac{25}{9} + \frac{36}{9}}}{2}$

$\lambda_{1,2} = \frac{5 \pm \sqrt{61}}{6}$

Since both eigenvalues are not less than 1 in absolute value $(0,0)$ is unstable

$(z_1, z_2) = \begin{pmatrix} 5/3 - z_2 & -z_1 \\ 1 & 0 \end{pmatrix}$

$(2/3, 2/3) = \begin{pmatrix} 1 & -2/3 \\ 1 & 0 \end{pmatrix}$

$\det(J - \lambda I) = (1-\lambda)-\lambda + 2/3 = 0$

$\lambda^2 - \lambda + 2/3$

$\lambda_{1,2} = \dfrac{1 \pm \sqrt{1-4(2/3)}}{2} = \dfrac{1 \pm \sqrt{3/3 - 8/3}}{2} = \dfrac{1 \pm i\sqrt{5/3}}{2}$

$|a+bi| = \sqrt{a^2+b^2} = \sqrt{1/4 + 5/12} = \sqrt{8/12} = \dfrac{2\sqrt{2}}{2\sqrt{3}}$

Both eigenvalues have absolute value < 1 so $(2/3, 2/3)$ is <u>stable</u>

b) $x(n) = x(n-1)(2 - x(n-2))$

i. $x_2(n) = x(n-1)$

$x_1(n) = x_1(n-1)(2 - x_2(n-1))$

ii $z = z(2-z)$

$z = 2z - z^2$

$z^2 - z = 0$

$z(z-1) = 0$

$z = 0, z = 1$

Equil. points $(0,0), (1,1)$

iii $(z_1, z_2) = (\overset{f(z_1,z_2)}{z_1(2-z_2)}, \overset{g(z_1,z_2)}{z_1})$

$J = \begin{pmatrix} f_{z_1} & f_{z_2} \\ g_{z_1} & g_{z_2} \end{pmatrix} = \begin{pmatrix} 2-z_2 & -z_1 \\ 1 & 0 \end{pmatrix}$

$J_{(0,0)} = \begin{pmatrix} 2 & 0 \\ 1 & 0 \end{pmatrix}$ $\det(J - \lambda I) = (2-\lambda)(-\lambda) - 1 = 0$

$= \lambda^2 - 2\lambda - 1 = 0$

$\lambda_{1,2} = \dfrac{2 \pm \sqrt{4+4}}{2} = \dfrac{2 \pm 2\sqrt{2}}{2} = 1+\sqrt{2}, 1-\sqrt{2}$

Since not both eigenvalues are <1 in absolute value, $(0,0)$ is unstable.

$$J_{(1,1)} = \begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix} \quad \det(J-\lambda I) = (1-\lambda)(-\lambda) + 1 = 0$$

$$\lambda^2 - \lambda + 1 = 0$$

$$\lambda_{1,2} = \frac{1 \pm \sqrt{1-4}}{2} = \frac{1 \pm \sqrt{-3}}{2} = \frac{1}{2} + \frac{i\sqrt{3}}{2},\ \frac{1}{2} - \frac{i\sqrt{3}}{2}$$

$$|a+bi| = \sqrt{a^2+b^2} = \sqrt{\left(\tfrac{1}{2}\right)^2 + \left(\tfrac{\sqrt{3}}{2}\right)^2} = \sqrt{\tfrac{1}{4} + \tfrac{3}{4}} = \sqrt{1} = 1$$

Since absolute value of eigenvalues $= 1$ we cannot conclude whether or not it is stable.

Question 2

$$x(n) = x(n-1)(a - x(n-2))$$

$$x_2(n) = x(n-1)$$

$$x_1(n) = x_1(n-1)(a - x_2(n-1))$$

$$z = z(a-z)$$

$$z = za - z^2$$

$$z^2 - (a-1)z = 0$$

$$z(z - (a-1)) = 0$$

$$z = 0, \quad z = a-1$$

$$f(z_1,z_2) \quad g(z_1,z_2)$$

$$(z_1,z_2) = (z_1(a-z_2),\ z_1)$$

$$J = \begin{pmatrix} a-z_2 & -z_1 \\ 1 & 0 \end{pmatrix}$$

$$J_{(0,0)} = \begin{pmatrix} a & 0 \\ 1 & 0 \end{pmatrix} \quad \det(J-\lambda I) = (a-\lambda)(-\lambda) - 1 = 0$$

$$\lambda^2 - a\lambda - 1 = 0$$

$$\lambda_{1,2} = \frac{a \pm \sqrt{a^2+4}}{2} \qquad \frac{a + \sqrt{a^2+4}}{2} < 1 \Rightarrow a + \sqrt{a^2+4} < 2$$

$$a^2+4 < 4 - 4a + a^2$$

$$a < 0$$

$$a - \sqrt{a^2+4} < 2$$

$$\sqrt{a^2+4} > -2 + a$$

$$a^2+4 > 4 - 4a + a^2, \quad a > 0$$

(0, 0) will not be stable

$J = \begin{pmatrix} a-2a & 2 \\ 1 & 0 \end{pmatrix}$ $J_{(a-1,a-1)} = \begin{pmatrix} 1 & a-1 \\ 1 & 0 \end{pmatrix}$

$(1-\lambda)-\lambda \quad -a+1 \quad = 0$

$\lambda^2-\lambda+1-a+1 \quad = 0$

$\lambda^2-\lambda-a+2 = 0$

$\lambda_{1,2} = \frac{1 \pm \sqrt{1-4(-a+2)}}{2} = \frac{1 \pm \sqrt{-7+4a}}{2}$

$\frac{1+\sqrt{-7+4a}}{2} < 1 \rightarrow 1+\sqrt{-7+4a} < 2$

$-7+4a < 1$

$a < 2$

$\frac{1-\sqrt{-7+4a}}{2} < 1 \rightarrow 1-\sqrt{-7+4a} < 2$

$\sqrt{-7+4a} > -1$

$a > 2$

```
> read "/Users/jeton/Desktop/Math 336/M15.txt"
> print(Orbk)
```

proc(k, z, f, INI, K1, K2) **(1)**

```
    local L, i, newguy;
    L := INI;
    if not (type(k, integer) and type(z, symbol) and type(INI, list) and nops(INI) = k and
    type(K1, integer) and type(K2, integer) and 0 < K1 and K1 < K2) then
        print(bad input); RETURN(FAIL)
    end if;
    while nops(L) < K2 do
        newguy := subs({seq(z[i] = L[-i], i = 1..k)}, f); L := [op(L), newguy]
    end do;
    [op(K1..K2, L)]
end proc
```

```
> f:=(z[1]*(5/3-z[2]))
```

$$f := z_1\left(\frac{5}{3} - z_2\right) \quad (2)$$

```
> Orbk(2,z,f,[0,0],1000,1020)
```

[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0] **(3)**

```
> Orbk(2,z,f,[2/3,2/3],1000,1020)
```

$$\left[\frac{2}{3}, \frac{2}{3}\right] \quad (4)$$

```
> Orbk(2,z,f,[.1111,.000001],1000,1020)
```

[0.6666666667, 0.6666666667, 0.6666666667, 0.6666666667, 0.6666666667, 0.6666666667, **(5)**
0.6666666667, 0.6666666667, 0.6666666667, 0.6666666667, 0.6666666667,
0.6666666667, 0.6666666667, 0.6666666667, 0.6666666667, 0.6666666667,
0.6666666667, 0.6666666667, 0.6666666667, 0.6666666667, 0.6666666667]

```
> #See that (2/3,2/3) is a stable point
> f:=(z[1]*(2-z[2]))
```

$$f := z_1\left(2 - z_2\right) \quad (6)$$

```
> Orbk(2,z,f,[0,0],1000,1020)
```

[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0] **(7)**

```
> Orbk(2,z,f,[.1111,.111],1000,1020)
```

[1.047967048, 1.057894352, 1.007150283, 0.9488419700, 0.9420574814, 0.9902512863, **(8)**
1.047628940, 1.057841975, 1.007458083, 0.9491847177, 0.9421056193, 0.9899789820,
1.047293202, 1.057788146, 1.007761958, 0.9495252628, 0.9421550876, 0.9897101179,
1.046959813, 1.057732906, 1.008061967]

```
> Orbk(2,z,f,[.25,.25],1000,1020)
```

[1.053361492, 0.9968080683, 0.9436169025, 0.9466288635, 1.000002731, 1.053374013, **(9)**
1.053371136, 0.9971484913, 0.9439295436, 0.9466211672, 0.9996986477, 1.053061395,

1.053378737, 0.9974849918, 0.9442405028, 0.9466152752, 0.9993980668, 1.052750658, 1.053384343, 0.9978176258, 0.9445497874]

```
> #Question 4
> #a x'(t)=x(t)*(3-x(t))*(5-x(t))
> solve(x*(3-x)*(5-x),x)
```

$$0, 3, 5 \qquad (10)$$

```
> expand(x*(3-x)*(5-x))
```

$$x^3 - 8x^2 + 15x \qquad (11)$$

```
> f:=diff(expand(x*(3-x)*(5-x)),x)
```

$$f := 3x^2 - 16x + 15 \qquad (12)$$

```
> subs(x=0,f)
```

$$15 \qquad (13)$$

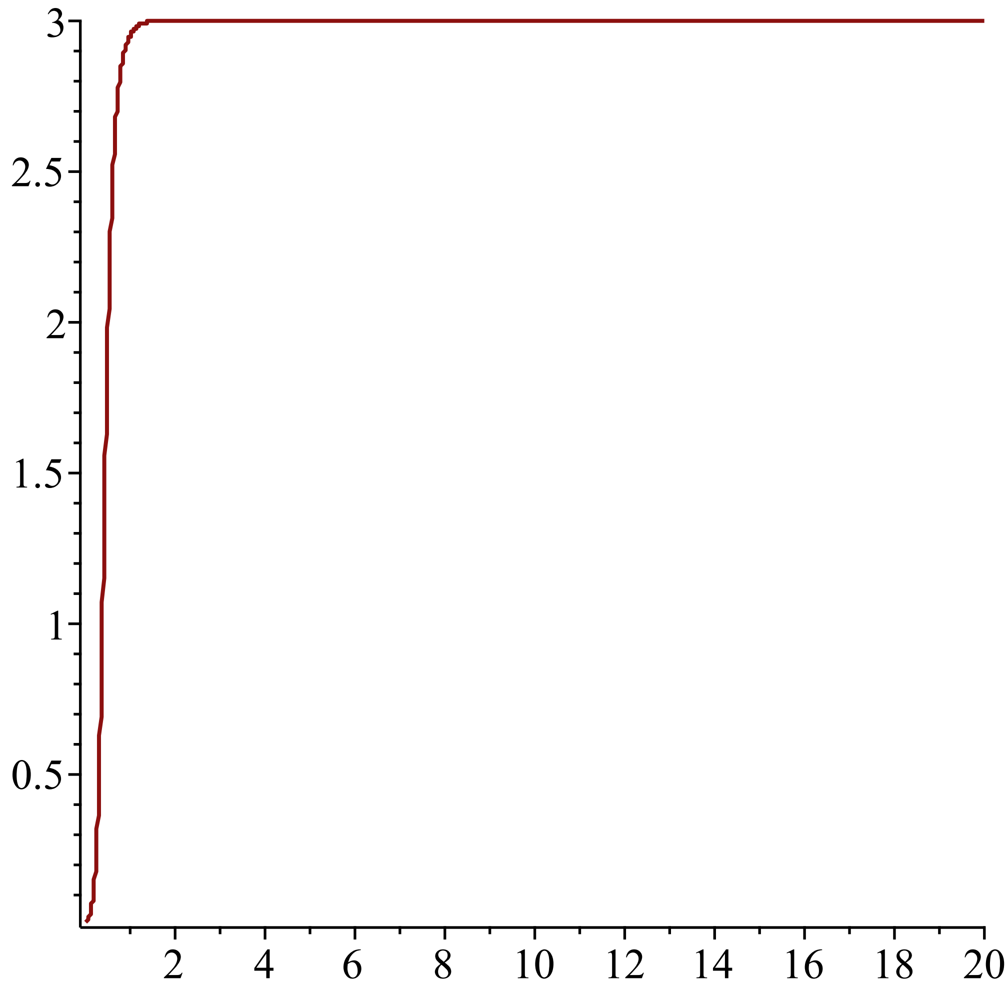
```
> subs(x=3,f)
```

$$-6 \qquad (14)$$

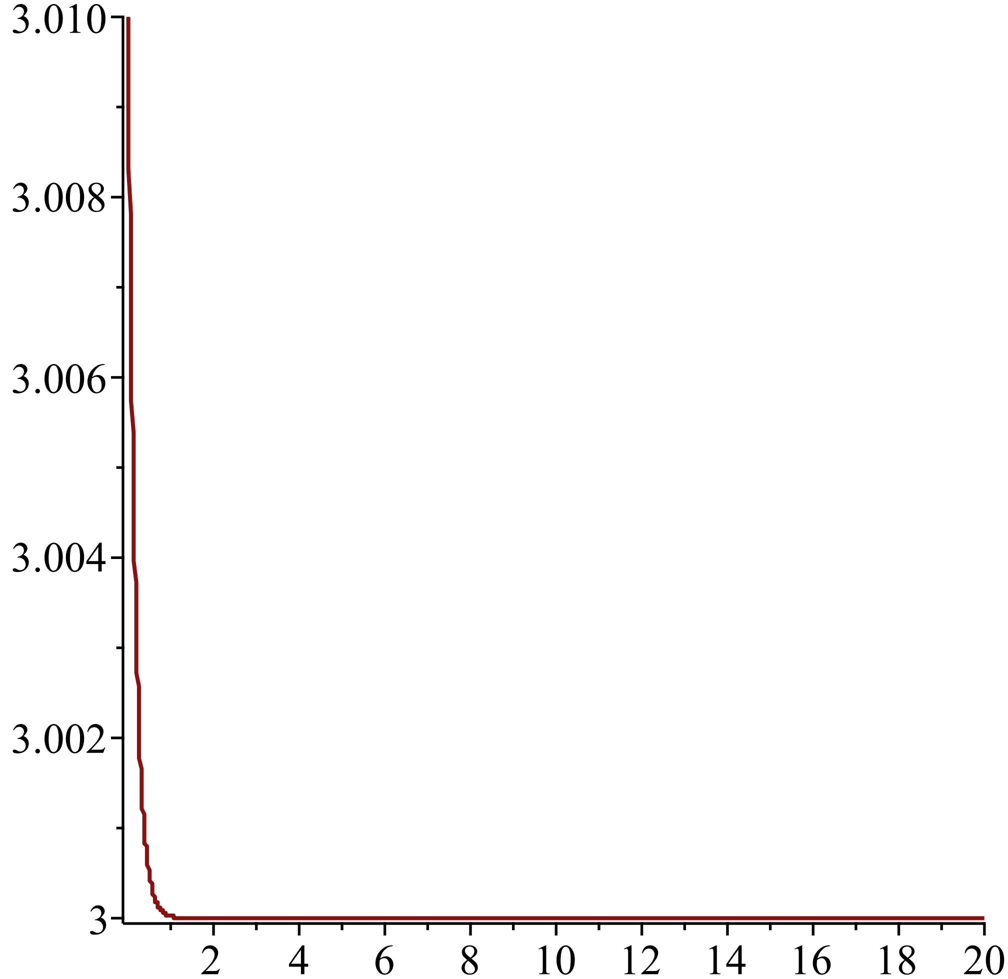
```
> subs(x=5,f)
```

$$10 \qquad (15)$$

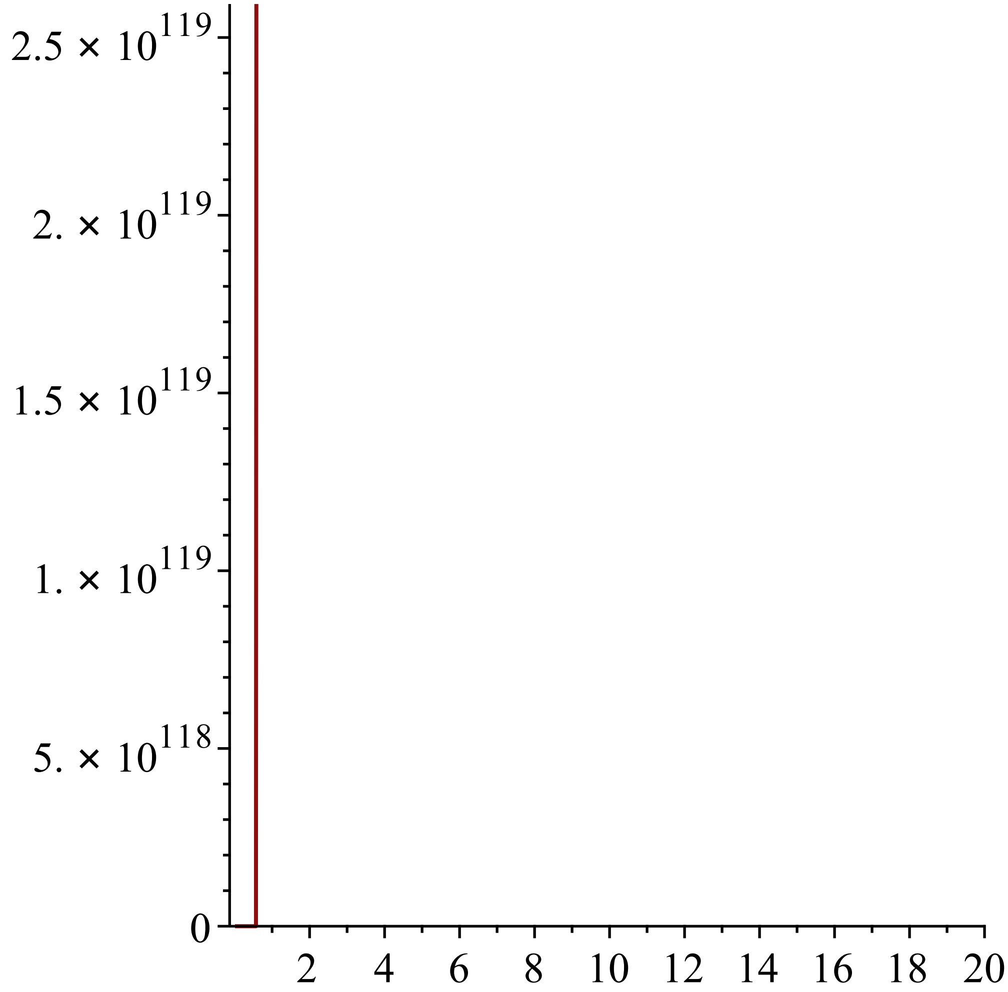
```
> #x=0, is unstable, because 15 is not < 0, x=3 is stable, since -6 <
  0, x=5 is unstable since 10 is not < 0
> plot(Dis1(x*(3-x)*(5-x),x,0.01,0.01,20));
```




```
> plot(Dis1(x*(3-x)*(5-x),x,3.01,.01,20));
```




```
> plot(Dis1(x*(3-x)*(5-x),x,5.01,.01,20));
```




```
> #b x'(t)=(x'(t))^2*(3-x(t))*(5-x(t))*(7-x(t))
> solve(x^2*(3-x)*(5-x)*(7-x))
```

$$3, 5, 7, 0, 0 \quad (16)$$

```
> expand(x^2*(3-x)*(5-x)*(7-x))
```

$$-x^5 + 15\,x^4 - 71\,x^3 + 105\,x^2 \quad (17)$$

```
> f:=diff(expand(x^2*(3-x)*(5-x)*(7-x)),x)
```

$$f := -5\,x^4 + 60\,x^3 - 213\,x^2 + 210\,x \quad (18)$$

```
> subs(x=0,f)
```

$$0 \quad (19)$$

```
> subs(x=3,f)
```

$$-72 \quad (20)$$

```
> subs(x=5,f)
```

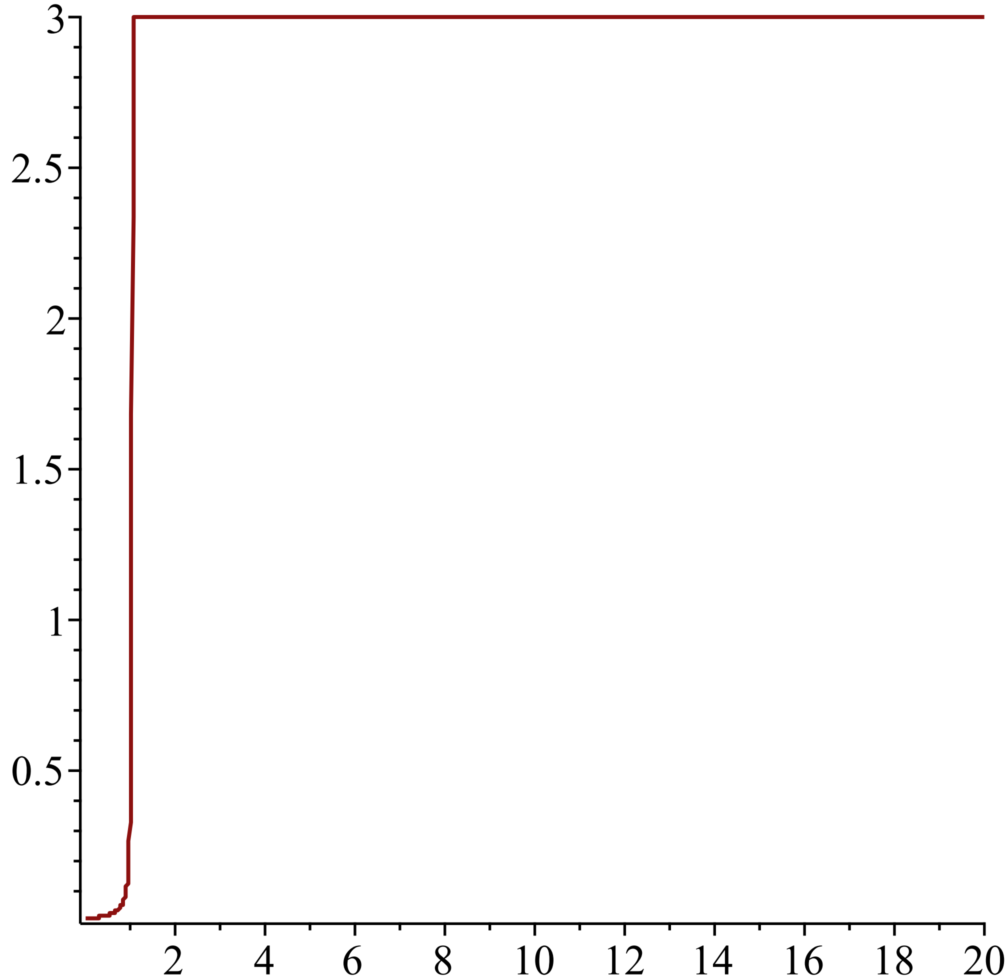
$$100 \quad (21)$$

```
> subs(x=7,f)
```

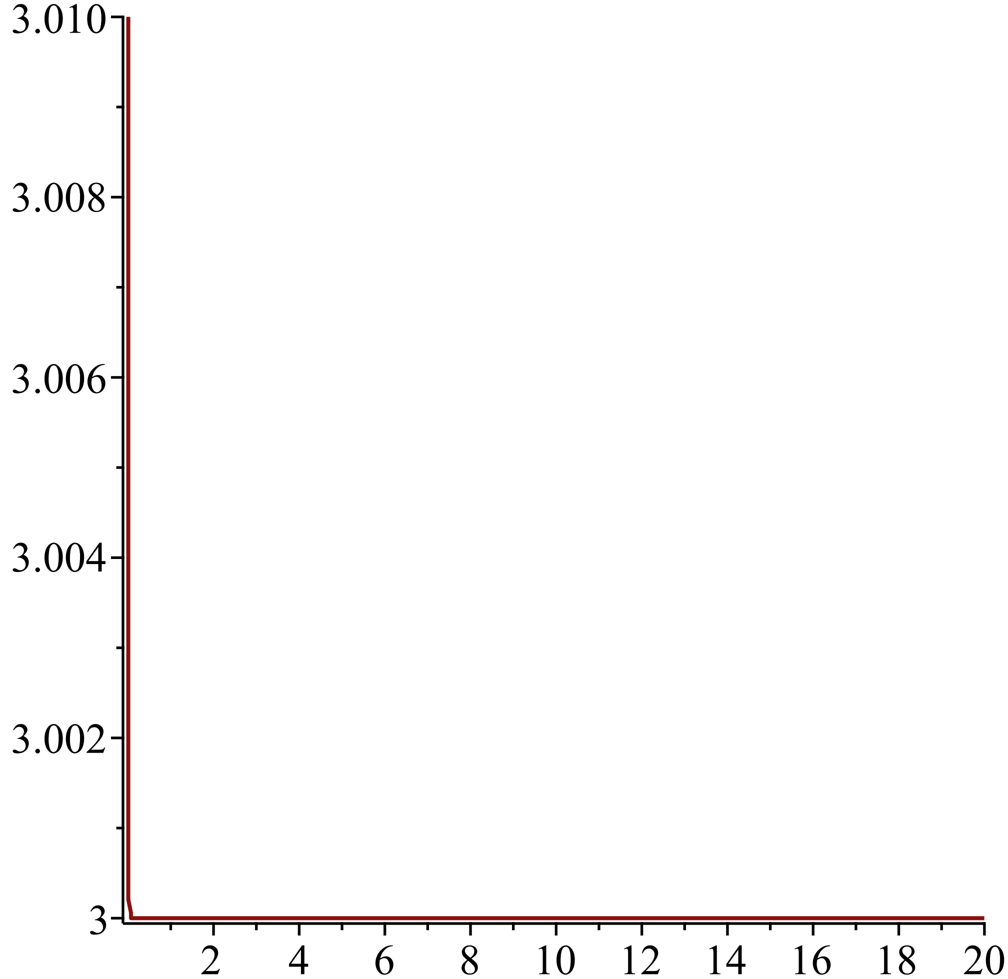
$$-392 \quad (22)$$

```
> #x=0 is unstable since f'(0) is not less than 0, x=3 is stable
  since f'(3) < 0, x=5 is unstable since f'(5) is not less than 0, x=
  7 is stable since f'(7) < 0.
```

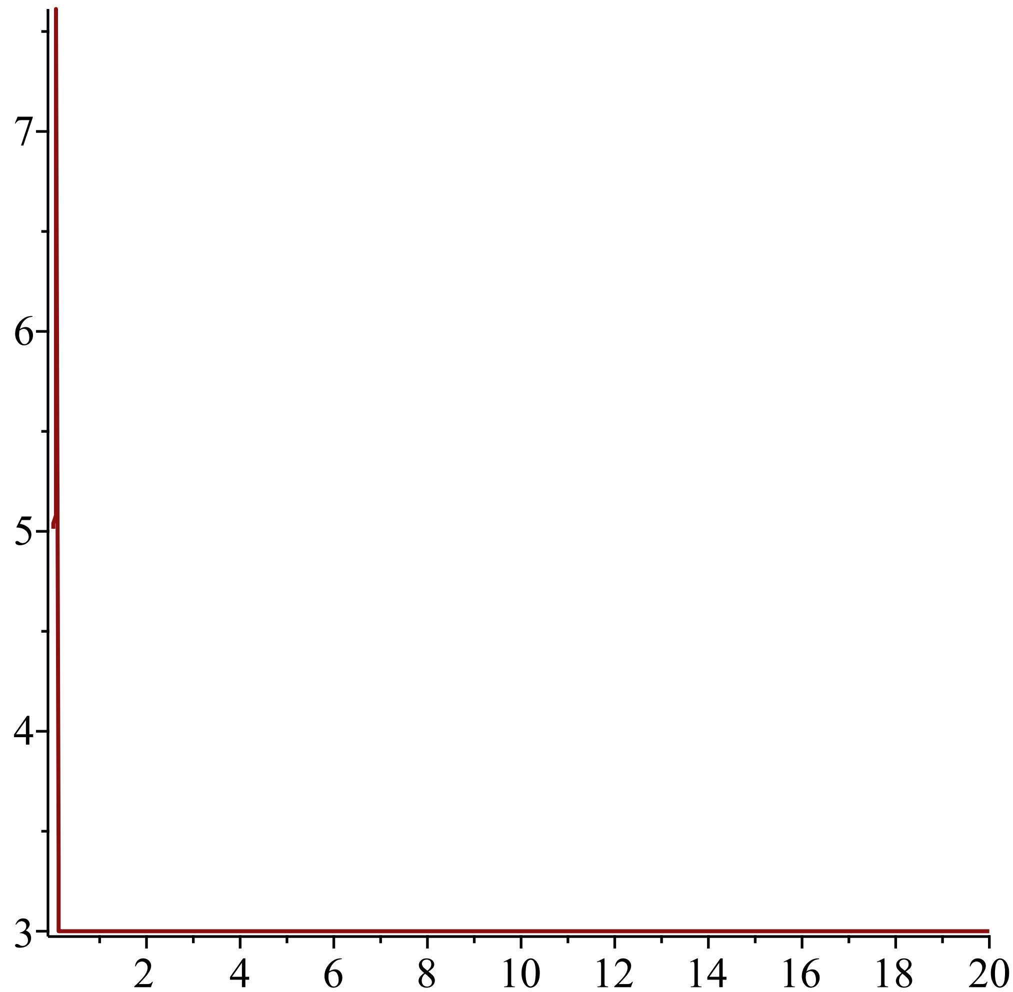
```
> plot(Dis1(x^2*(3-x)*(5-x)*(7-x),x,0.01,.01,20))
```



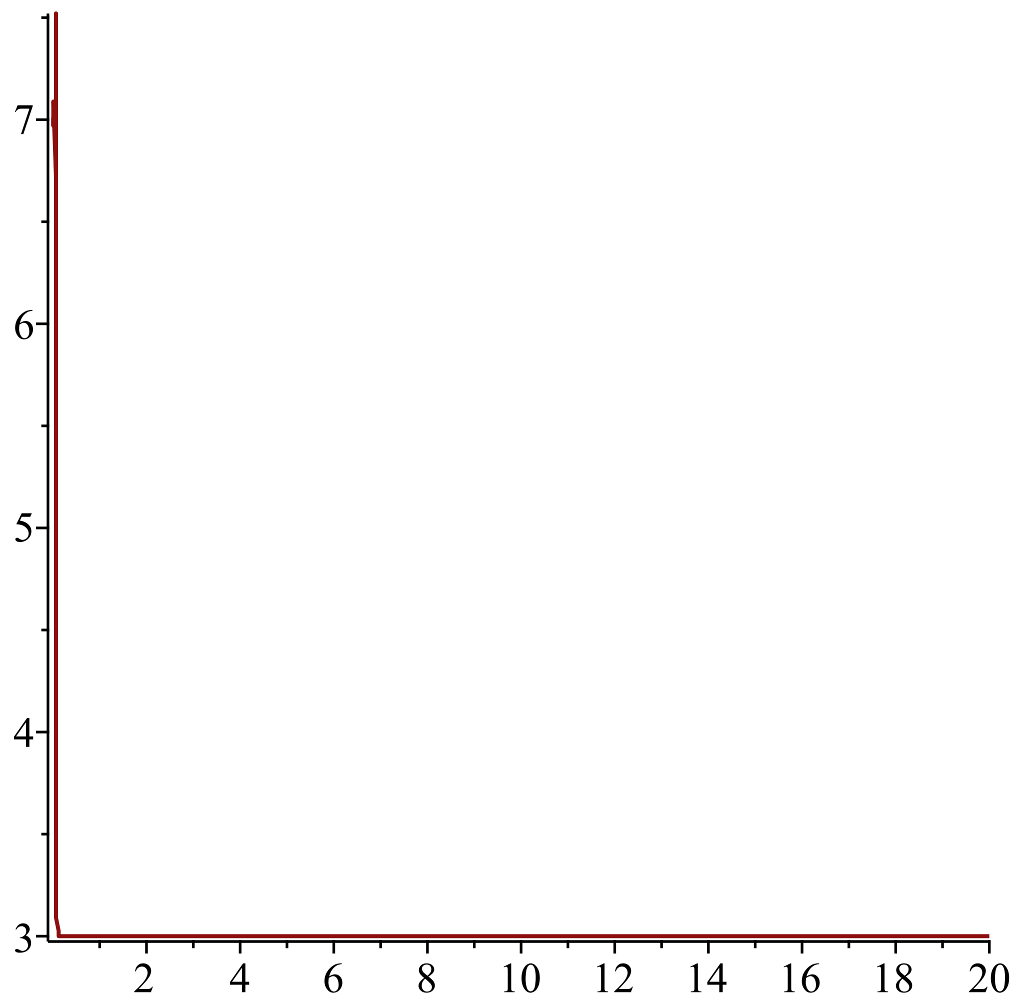

```
> plot(Dis1(x^2*(3-x)*(5-x)*(7-x),x,3.01,.01,20))
```




```
> plot(Dis1(x^2*(3-x)*(5-x)*(7-x),x,5.01,.01,20))
```




```
> plot(Dis1(x^2*(3-x)*(5-x)*(7-x),x,7.01,.01,20))
```



7
6
5
4
3
2
4
6
8
10
12
14
16
18
20