Hrudai Bathini HW 16

1) a) $x(n) = x(n-1)\left(\frac{5}{3} - x(n-1)\right)$ $\quad x_1(n) = x(n)$ $\quad x_2(n) = x(n-1)$

i) $x_1(n) = x_1(n-1)\left(\frac{5}{3} - x_2(n-1)\right)$ $\quad z = x_1(n) = x(n-1) = x(n-2)$

ii) $z = z\left(\frac{5}{3} - z\right)$ $\quad z = 0$ $\quad z = \frac{2}{3}$

iii) $(t_1, t_2) = \left(t_1, \left(\frac{5}{3} - t_2\right), t_1\right)$ $\quad f(t_1, t_2) = t_1\left(\frac{5}{3} - t_2\right)$; $g(t_1, t_2) = t_1$

$$J = \begin{pmatrix} \frac{5}{3} - t_2 & -t_1 \\ 1 & 0 \end{pmatrix} \xrightarrow{(0,0)} \begin{pmatrix} \frac{5}{3} & 0 \\ 1 & 0 \end{pmatrix} \quad \left(\frac{5}{3} - \lambda\right)(-\lambda) = \lambda = 0, \frac{5}{3} \text{ unstable}$$

as both not less than 0

$\left(\frac{2}{3}, \frac{2}{3}\right)$ $\quad \begin{pmatrix} 1 & -\frac{2}{3} \\ 1 & 0 \end{pmatrix}$ $\quad (1-\lambda)(-\lambda) + \frac{2}{3} = \lambda^2 - \lambda + \frac{2}{3}$ $\quad |\lambda| = \left|\frac{1}{2} \pm \frac{\sqrt{7}}{2} i\right|$

$|\lambda| = .8164$ stable

Ans: The difference equation has two equilibrium points $x = 0$ & $x = \frac{2}{3}$ and $x = \frac{2}{3}$ is stable.

b) i) $x(n) = x(n-1)(2 - x(n-2))$ $\quad x_1 = x(n)$ $\quad x_2 = x(n-1)$

$x_1(n) = x_1(n)(2 - x_2(n-1))$

ii) $z = x_1(n) = x(n-1) = x(n-2)$

$z = z(2 - z)$ $\quad z = 0$ $\quad z = 1$

$f(z,t)$ $\quad g(z,t)$

iii) (t_1, t_2) $\quad (z_1(2 - t_2), z_1)$

$$J = \begin{pmatrix} 2 - 2z_2 & -z_1 \\ 1 & 0 \end{pmatrix} \xrightarrow{(0,0)} \begin{pmatrix} 2 & 0 \\ 1 & 0 \end{pmatrix} \quad (2 - \lambda)\lambda \quad \lambda = 2, 0 \text{ unstable}$$

$(1,1)$

$$J = \begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix} \quad \lambda = (1-\lambda)\lambda + 1 \quad \lambda^2 - \lambda + 1 \quad |\lambda| = 1, 1 \quad \frac{1 \pm \sqrt{1-4}}{2}$$

The difference eq. has two equilibrium pts. $x = 0, 1$, neither are stable.

$\dot{z} = z_1(a - z_2)$ $\quad z = 0$ $\quad 1 = a - z$ $\quad z = a - 1$ $\quad a > 0$

(z, z_2) $(z_1(a - z_2), z_1)$

$$J = \begin{pmatrix} a - z_2 & -z_1 \\ 1 & 0 \end{pmatrix} \quad (0,0) \quad \begin{pmatrix} a & 0 \\ 1 & 0 \end{pmatrix} \quad (a-\lambda)(\lambda) \quad \lambda^2 - a\lambda$$

$\lambda = 0, \lambda = a$

(0,0) is stable when $a < 1$.

$$J = \begin{pmatrix} a - a - 1 & -a + 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 1 - a \\ 1 & 0 \end{pmatrix} \quad (-1-\lambda)(-\lambda) - (1-a)$$

$\lambda^2 + \lambda + (a - 1)$ $\quad \frac{-1 \pm \sqrt{1 - 4(a-1)}}{2} = -\frac{1}{2} \pm \sqrt{1 - 4a + 4} \rightarrow \frac{\sqrt{5 - 4a}}{2}$

$-\frac{1}{2} \pm \frac{\sqrt{5-4a}}{2}$ $\quad (a-1, a-1)$ is stable when $a < 2$

a) $x'(t) = x(t)(3 - x(t))(5 - x(t))$

i) $F(x) \; x(3-x)(5-x)$ $\quad x = 0, 3, 5$

ii) $x' = (3x - x^2) \rightarrow (15x - 5x^2 - 3x^2 + x^3)' \rightarrow x' = 15 - 16x + 3x^2$

$x'(0) = 15$ not negative → unstable ; $x'(3) = -6$ negative → stable ; $x'(5) = 10$ not negative unstable

iii → maple

b) i) = $F(x) = x^2(3-x)(5-x)(7-x)$ $\quad x = 0, 3, 5, 7$

ii) maple

iii) maple

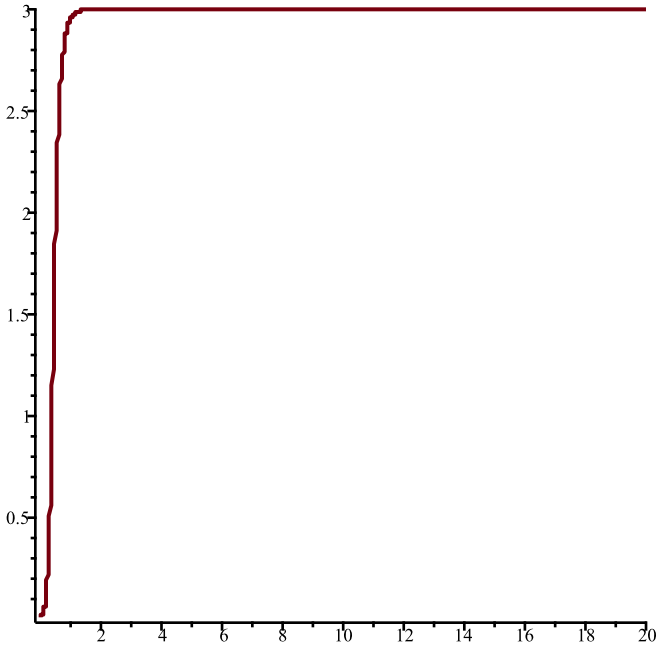
```
> #Hrudai Battini hw 16
  read "/Users/hb334/Documents/M15.txt";
> #1
  #a
  Orbk(2,z,evalf((z[1]*((5/3)-z[2]))), [0.5,0.5], 1000,1020);
  #b
  Orbk(2,z,evalf((z[1]*((2)-z[2]))), [0.5,0.5], 1000,1020);
```

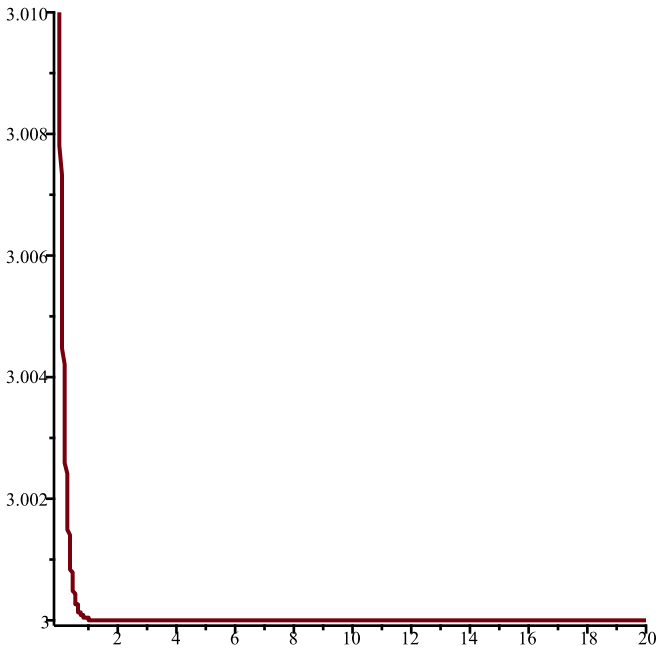
[0.6666666666, 0.6666666666, 0.6666666666, 0.6666666666, 0.6666666666, 0.6666666666,
0.6666666666, 0.6666666666, 0.6666666666, 0.6666666666, 0.6666666666,
0.6666666666, 0.6666666666, 0.6666666666, 0.6666666666, 0.6666666666,
0.6666666666, 0.6666666666, 0.6666666666, 0.6666666666, 0.6666666666]

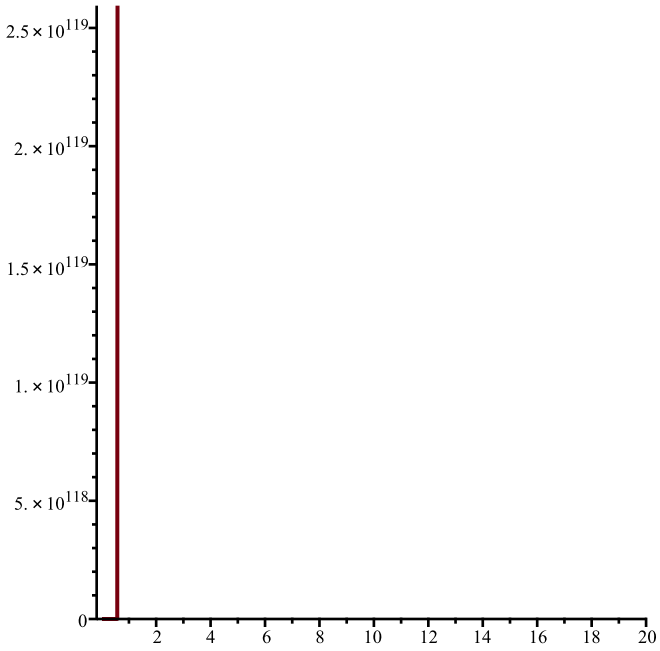
[0.9883413838, 0.9405167599, 0.9514818836, 1.008079109, 1.056989208, 1.048449677, **(1)**
0.9886993603, 0.9407971956, 0.9514288060, 1.007756059, 1.056703974, 1.048508116,
0.9890535391, 0.9410764153, 0.9513778716, 1.007436466, 1.056420171, 1.048564138,
0.9894039700, 0.9413544191, 0.9513290388]

```
> #4
  #iii a

  plot(Dis1(x*(3-x)*(5-x),x,0.01,0.01,20));
  plot(Dis1(x*(3-x)*(5-x),x,3.01,0.01,20)); #Stable
  plot(Dis1(x*(3-x)*(5-x),x,5.01,0.01,20));
```








```
> #4 b
  #ii
  F:= expand(diff(x^2*(3-x)*(5-x)*(7-x),x));
  eval(F,x=0); #Not negative so Unstable
  eval(F,x=3); #Negative so Stable
  eval(F,x=5); #Postive so Unstable
  eval(F,x=7); #Negative so Stable

  plot(Dis1(x^2*(3-x)*(5-x)*(7-x),x,0.01,0.01,20));
  plot(Dis1(x^2*(3-x)*(5-x)*(7-x),x,3.01,0.01,20));
  plot(Dis1(x^2*(3-x)*(5-x)*(7-x),x,5.01,0.01,20));
  plot(Dis1((x^2)*(3-x)*(5-x)*(7-x),x,7.01,0.01,20)); #Negative
  however the Asymptote is at 3.
```

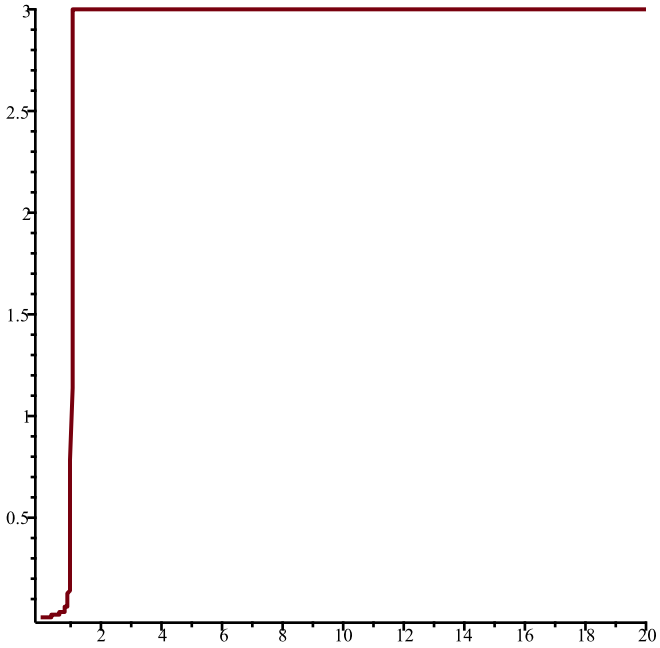
$$F := -5\,x^4 + 60\,x^3 - 213\,x^2 + 210\,x$$

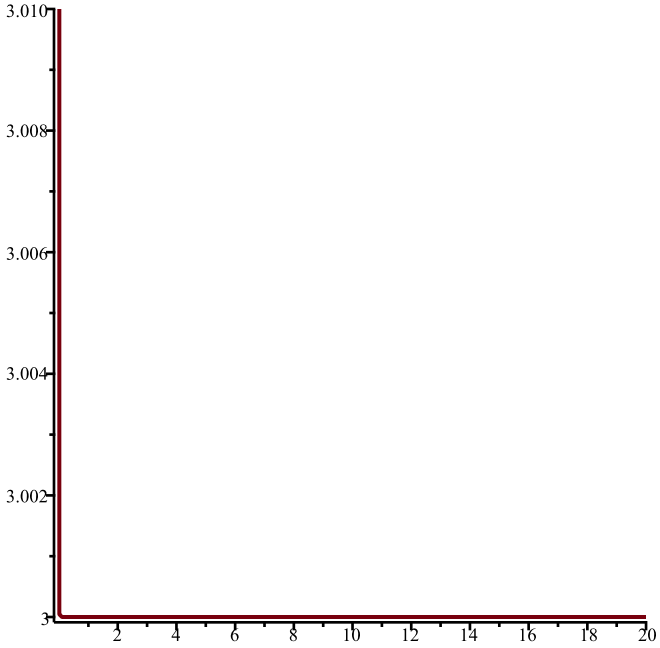
0

-72

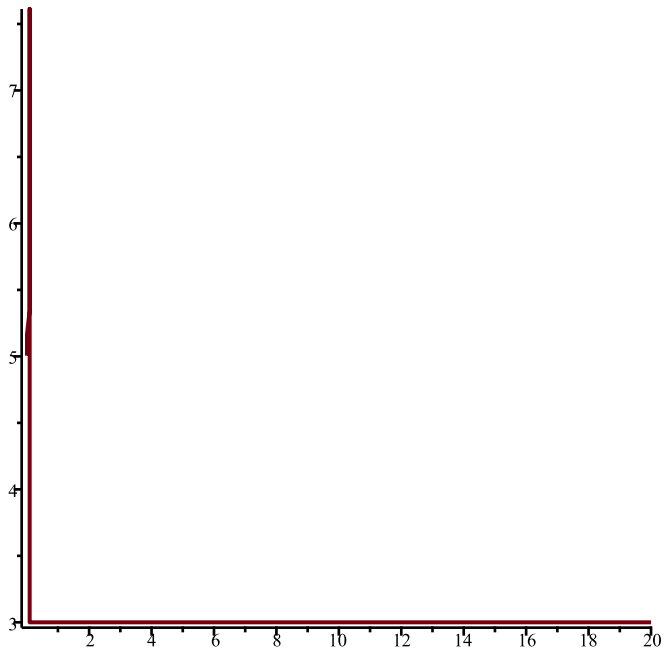
100

-392


3
2.5
2
1.5
1
0.5
2
4
6
8
10
12
14
16
18
20


3.010
3.008
3.006
3.004
3.002
3
2
4
6
8
10
12
14
16
18
20

```
>
```


7
6
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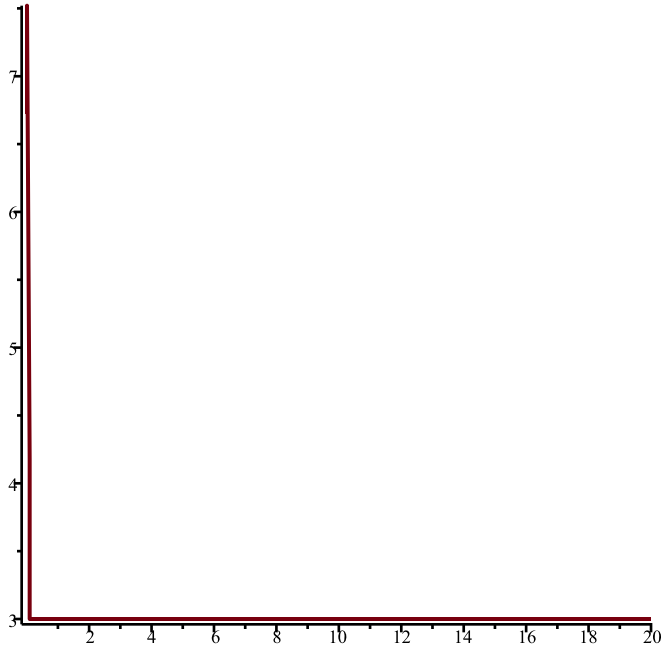
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