```
> #PROBLEM 1 HOMEWORK 16
> read `C:/Users/cgrie/Dynam Models Bio/Homeworks/HW15/M15.txt`
> print(Orbk);
```

proc$(k, z, f, INI, K1, K2)$ **(1)**

    local $L, i, newguy$;
    $L := INI$;
    if not $(type(k, integer)$ **and** $type(z, symbol)$ **and** $type(INI, list)$ **and** $nops(INI) = k$ **and**
    $type(K1, integer)$ **and** $type(K2, integer)$ **and** $0 < K1$ **and** $K1 < K2)$ **then**
        $print(`bad\ input`)$; $RETURN(FAIL)$
    end if;
    while $nops(L) < K2$ **do**
        $newguy := subs(\{seq(z[i] = L[-i], i = 1..k)\}, f)$; $L := [op(L), newguy]$
    end do;
    $[op(K1..K2, L)]$
end proc

```
> Digits:=10;

  orbit1 := evalf(Orbk(1,z, z[1]*((5/3)-z[1]),[0.5],1000,1020));

  print(`orbit 1 is stable`);
  print(`its stable fixed point is`);

  orbit2 := evalf(Orbk(1,z, z[1]*(2-z[1]),[0.5],1000,1020));

  print(`orbit 2 is also stable`)
```

$Digits := 10$

$orbit1 := [0.6666666670, 0.6666666670]$

orbit 1 is stable

its stable fixed point is

$orbit2 := [1.000000000, 1.000000000]$

orbit 2 is also stable **(2)**

```
> #We see that the point
Error, missing operator or `;`
```

Dynam Models Bio Homework 16

1) Decompose the following second order recurrences into systems of first order recurrences, find their respective fixed points, and determine the stability of the fixed points

(a) $x(n) = x(n-1) * (5/3 - x(n-2))$

Let $y(n) = x(n-1)$

Now we can create a first order system

$$\begin{cases} x(n) = x(n-1) * (5/3 - y(n-1)) \\ y(n) = x(n-1) \end{cases}$$

Where the values today ($x(n)$ and $y(n)$) solely depend on the values yesterday ($x(n-1)$ and $y(n-1)$)

After expansion results in

$$\begin{cases} x(n) = \frac{5}{3} * x(n-1) - x(n-1) * y(n-1) \\ y(n) = x(n-1) \end{cases}$$

~~It~~

Dynamic Models B10 HW 16

(11) to Find the equilibrium points, it is easier to look at the original 2nd order recurrence

$$x(n) = x(n-1) * \left(\frac{5}{3} - x(n-2)\right).$$

Because $x(n) = x(n-1) = x(n-2)$ is the definition of an equilibrium (the value of x never changes), we can substitute z for $x(n)$, $x(n-1)$ and $x(n-2)$ to get:

$$z = z\left(\frac{5}{3} - z\right)$$

which means $z = 0$ corresponds to one equilibrium and

$$\frac{z}{z} = 1 = \left(\frac{5}{3} - z\right) \Rightarrow z = \frac{2}{3}$$

corresponds to our other equilibrium

We see if the equilibrium points are stable by constructing a Jacobian Matrix

$$J = \begin{pmatrix} \frac{\partial F_1}{\partial x} & \frac{\partial F_1}{\partial y} \\ \frac{\partial F_2}{\partial x} & \frac{\partial F_2}{\partial y} \end{pmatrix}$$

Where $F_1 = \frac{5}{3}x(n-1) - x(n-1)y(n-1)$

$F_2 = x(n-1)$

Dynam Models Bio HW 16

Problem 1

Thus,

$$J = \begin{pmatrix} \frac{\partial}{\partial x}\left(\frac{5}{3}x - xy\right) & \frac{\partial}{\partial y}\left(\frac{5}{3}x - xy\right) \\ \frac{\partial}{\partial x}(x) & \frac{\partial}{\partial y}(x) \end{pmatrix}$$

$$= \begin{pmatrix} \frac{5}{3} - y & -x \\ 1 & 0 \end{pmatrix}$$

To Determine stability, Find the

When $(x_1, y_1) = (0, 0)$

Then

$$J_1 = \begin{pmatrix} \frac{5}{3} & 0 \\ 1 & 0 \end{pmatrix}$$, which has Characteristic equation

$$\lambda\left(\frac{5}{3} - \lambda\right) = 0$$

$\Rightarrow \lambda = 0$, and $\lambda = \frac{5}{3}$

since $\lambda = \frac{5}{3} > 1$, J_1 is unstable

When $(x_2, y_2) = \left(\frac{2}{3}, \frac{2}{3}\right)$

$$J_2 = \begin{pmatrix} 1 & -\frac{2}{3} \\ 1 & 0 \end{pmatrix}$$ Which has Characteristic Equation

Dynam Models Bio Homework 16
Problem 2

2. Like problem 1,
to find equilibrium points of
$x(n) = x(n-1)*(a - x(n-2))$

Let $x(n) = x(n-1) = x(n-2) = z$

Thus,

$z = z(a - z) \Rightarrow$ $z = 0$ is an equilibrium.

Letting $a = z + 1$, we have

$z = z(z - z + 1)$ Answer

$\Updownarrow$

$z = z$

$\Rightarrow$ Therefore, our second equilibrium is $z = a - 1$

Therefore, we might actually have an infinite amount of equilibriums.

For example, if $z = 1$ then $a = 2$

then

$1 = 1(2 - 1)$

For example, if $z = 2$ then $a = 3$
then $2 = 2(3 - 2)$
$= 2(1)$

Dynam Models Bio HW 16

Problem 2

Save for later:

For the equilibrium associated with $Z=0$, we should look at:
The Jacobian matrix of the 2nd order recurrence
Decomposed system

$$\begin{cases} x(n) = x(n-1) * a - y(n-1) \\ y(n) = x(n-1) \end{cases}$$

Which is

$$\begin{pmatrix} \frac{\partial[ax-y]}{\partial x} & \frac{\partial[ax-y]}{\partial y} \\ \frac{\partial(x}{\partial x} & \frac{\partial}{\partial y} \end{pmatrix}$$

The underlying transformation, replacing $x(n-1)$ by z_1 and $y(n-1)$ by z_2 is:

$$(z_1, z_2) \longrightarrow (z_1(a - z_2), z_1)$$

Thus, $J_1 = \begin{pmatrix} \frac{\partial}{\partial z_1}[z_1(a-z_2)), & \frac{\partial}{\partial z_2}[z_1(a-z_2) \\ \frac{\partial}{\partial z_1}(z_1) & \frac{\partial}{\partial z_2}(z_1) \end{pmatrix}$

$$J_1 = \begin{pmatrix} a - z_2 & -z_1 \\ 1 & 0 \end{pmatrix}$$

Dynam Models Bio HW 16

To estimate the stability of the various equilibrium points, plug each into J

~~First~~ EQ: $(z_1, z_2) = (0; 0)$

$\Downarrow$

$$J_1 = \begin{pmatrix} a & 0 \\ 1 & 0 \end{pmatrix}$$

and we can find eigenvalues of J with $\det(J - \lambda I_2) = 0$

$$(a-\lambda)(-\lambda) - 0 = 0$$

$\Rightarrow$ $\lambda = 0$ and $\lambda = a$. Thus the values of a which dictate local stability for $(0,0) = EQ_1$ (same situation as $x=0$, Just worded differently) are $|a| < 1$

because the other eigenvalue is .

Now, If we have our other Equilibrium point being any (z_1, z_2), as long as $z_1 = z_2$ when a is STRICTLY $z_2 + 1$.

(Otherwise we dont have an equilibrium point),

We can look at the Jacobian

Dynam Models Bio

$(z_2, z_2) = (z_1, z_2) = $ EQ POINT 2 !

Therefore, our Jacobian Matrix is :

$$J_2 = \begin{pmatrix} a - z_1 & -z_2 \\ 1 & 0 \end{pmatrix}.$$

and with $a = z_2 + 1$ (or z_1, same thing)

And after

$$J_2 = \begin{pmatrix} 1 & -a+1 \\ 1 & 0 \end{pmatrix}$$

$$\Rightarrow (1-\lambda)(-\lambda) - (-a+1) = 0$$

which after expansion is

$$-\lambda^2 + \lambda + (a-1) = 0$$