1) a. $x(n) = x(n-1)\left(\frac{5}{3} - x(n-2)\right)$

i) $x_2(n) = x(n-1)$

$x_1(n) = x_1(n-1)\left(\frac{5}{3} - x_2(n-1)\right), \quad x_2(n) = x_1(n-1)$

ii) $z = z\left(\frac{5}{3} - z\right)$

$z = \frac{5}{3}z - z^2$

$z^2 = \frac{2}{3}z$

$z = \frac{2}{3}, \; z = 0$

iii) $(z_1, z_2) \to \left(z_1\left(\frac{5}{3} - z_2\right), z_1\right)$

$$J = \begin{pmatrix} \frac{5}{3} - z_2 & -z_1 \\ 1 & 0 \end{pmatrix}$$

$(z_1, z_2) = (0, 0)$

$\begin{pmatrix} \frac{5}{3} & 0 \\ 1 & 0 \end{pmatrix}, \quad \lambda = 0, \frac{5}{3}$ Unstable

$(z_1, z_2) = \left(\frac{2}{3}, \frac{2}{3}\right)$

$\begin{pmatrix} 1 & -\frac{2}{3} \\ 1 & 0 \end{pmatrix} \quad \lambda = \left|\frac{1}{2} \pm \frac{\sqrt{15}}{6} i\right| = \sqrt{\frac{2}{3}}$ Stable

6. $x(n) = x(n-1)(2 - x(n-2))$

i) $x_1(n) = x_1(n-1)(2 - x_2(n-1))$, $x_2(n) = x_1(n-1)$

ii) $z = z(2-z)$

$z = 0,\ z = 1$

iii) $(z_1, z_2) \Rightarrow (z_1(2 - z_2), z_1)$

$$J = \begin{pmatrix} 2 - z_2 & -z_1 \\ 1 & 0 \end{pmatrix}$$

$(z_1, z_2) = (0\ 0)$

$$\begin{pmatrix} 2 & 0 \\ 1 & 0 \end{pmatrix},\ \lambda = 0, 2 \quad \text{unstable}$$

$(z_1, z_2) = (1, 1)$

$$\begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix},\ \lambda = \left|\frac{1}{2} \pm \frac{\sqrt{3}}{2} i\right| = 1 \quad \text{unstable}$$

2) $x(n) = x(n-1)(a - x(n-2))$

$z = z(a-z)$

$1 = a - z$

$z = a-1,\ z = 0$, equillibrium Points

$$J = \begin{pmatrix} a - z_2 & -z_1 \\ 1 & 0 \end{pmatrix}$$

$(z_1, z_2) = (0,0)$

$$\begin{pmatrix} a & 0 \\ 1 & 0 \end{pmatrix} \det \begin{pmatrix} a-\lambda & 0 \\ 1 & -\lambda \end{pmatrix}$$

$(a-\lambda)(-\lambda) = 0$

$-\lambda a + \lambda^2 = 0,\ \lambda^2 = \lambda a,\ \lambda = a$

$\lambda = 0, a$

For the equillibrium point $(0,0)$, the system is stable for all $|a| < 1$, otherwise it is unstable.

$$(z_1, z_2) = (a-1, a-1)$$

$$\begin{pmatrix} a-(a-1) & -(a-1) \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1-a \\ 1 & 0 \end{pmatrix}$$

$$\det \begin{pmatrix} 1-\lambda & 1-a \\ 1 & -\lambda \end{pmatrix}$$

$$(1-\lambda)(-\lambda) - (1-a) = 0$$

$$-\lambda + \lambda^2 - 1 + a = 0$$

$$\lambda = \frac{1 \pm \sqrt{1-4(a-1)}}{2}$$

$$\lambda = \frac{1 \pm \sqrt{-4a+5}}{2}$$

$$1 > \frac{1+\sqrt{-4a+5}}{2}$$

$$1 > \sqrt{-4a+5}$$

$$1 > 5-4a$$

$$-4 > -4a$$

$$1 < a$$

$$-1 < \frac{1+\sqrt{-4a+5}}{2}$$

$$-3 < \sqrt{-4a+5}$$

$$-3 < -4a+5$$

$$-8 < -4a$$

$$-2 > a$$

?

4) a. $x'(t) = x(t)(3-x(t))(5-x(t))$

i) $F(x) = x(3-x)(5-x)$

$x(3-x)(5-x) = 0$

$x = 0, 3, 5$

ii) $F(x) = 15x - 8x^2 + x^3$

$F'(x) = 3x^2 - 16x + 15$

$x=0$, $F'(0) = 15$, unstable

$x=3$, $F'(0) = -6$, stable

$x=5$, $F'(0) = 10$, unstable

b. $x'(t) = x(t)^2(3-x(t))(5-x(t))(7-x(t))$

$F(x) = x^2(3-x)(5-x)(7-x)$

$x = 0, 3, 5, 7$

ii) $F'(x) = -5x^4 + 60x^3 - 213x + 210x$

$x=0$, $F'(0) = 0$, undetermined

$x=3$, $F'(0) = -72$, stable

$x=5$, $F'(0) = 100$, unstable

$x=7$, $F'(0) = -392$, stable