

Ok to post

Anne Somalwar, hw15, 10/23/2021

2.

(i)

$$x_1(n) = \frac{x_1(n-1) + 2x_2(n-1) + 3x_3(n-1) + 11x_4(n-1)}{x_1(n-1) + x_3(n-1)}$$

$$x_2(n) = x_1(n-1)$$

$$x_3(n) = x_2(n-1)$$

$$x_4(n) = x_3(n-1)$$

4.

		<u>Father</u>								
		<u>AA</u>			<u>Aa</u>			<u>aa</u>		
<u>Mother</u>	<u>AA</u>	$\frac{AA}{u^2}$	$\frac{Aa}{0}$	$\frac{aa}{0}$	$\frac{AA}{\frac{1}{2}uv}$	$\frac{Aa}{\frac{1}{2}uv}$	$\frac{aa}{0}$	$\frac{AA}{0}$	$\frac{Aa}{uv}$	$\frac{aa}{0}$
	<u>Aa</u>	$\frac{AA}{\frac{1}{2}uv}$	$\frac{Aa}{\frac{1}{2}uv}$	$\frac{aa}{0}$	$\frac{AA}{\frac{1}{4}v^2}$	$\frac{Aa}{\frac{1}{2}v^2}$	$\frac{aa}{\frac{1}{4}v^2}$	$\frac{AA}{0}$	$\frac{Aa}{\frac{1}{2}vw}$	$\frac{aa}{\frac{1}{2}vw}$
	<u>aa</u>	$\frac{AA}{0}$	$\frac{Aa}{uv}$	$\frac{aa}{0}$	$\frac{AA}{0}$	$\frac{Aa}{\frac{1}{2}vw}$	$\frac{aa}{\frac{1}{2}vw}$	$\frac{AA}{0}$	$\frac{Aa}{0}$	$\frac{aa}{w^2}$

AA total:

$$u^2 + \frac{1}{2}uv + \frac{1}{2}uv + \frac{v^2}{4} = u^2 + uv + \frac{v^2}{4}$$

Aa total:

$$\begin{aligned} & \frac{1}{2}uv + uv + \frac{1}{2}uv + \frac{v^2}{2} + \frac{1}{2}vw + uv + \frac{1}{2}vw \\ &= uv + 2uv + \frac{v^2}{2} + vw \end{aligned}$$

aa total:

$$v^2/4 + \frac{1}{2}v\omega + \frac{1}{2}v\omega + \omega^2 = v^2/4 + v\omega + \omega^2$$

5) (bonus)

$$M = \begin{array}{c} AA \\ Aa \\ aa \end{array} \begin{array}{ccc} AA & Aa & aa \\ \left[\begin{array}{ccc} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{array} \right] \end{array}$$

		<u>Father</u>								
		<u>AA</u>			<u>Aa</u>			<u>aa</u>		
<u>Mother</u>	<u>AA</u>	<u>AA</u> u^2 (multiply by M_{11})	<u>Aa</u> 0	<u>aa</u> 0	<u>AA</u> $\frac{1}{2}uv$ (Multiply by M_{12})	<u>Aa</u> $\frac{1}{2}uv$	<u>aa</u> 0	<u>AA</u> 0	<u>Aa</u> uv	<u>aa</u> 0 (Multiply by M_{13})
	<u>Aa</u>	<u>AA</u> $\frac{1}{2}uv$ (Multiply by M_{21})	<u>Aa</u> $\frac{1}{2}uv$	<u>aa</u> 0	<u>AA</u> $\frac{v^2}{4}$	<u>Aa</u> $\frac{v^2}{2}$	<u>aa</u> $\frac{v^2}{4}$	<u>AA</u> 0	<u>Aa</u> $\frac{1}{2}vw$	<u>aa</u> $\frac{1}{2}vw$ (Multiply by M_{23})
	<u>aa</u>	<u>AA</u> 0 (Multiply by M_{31})	<u>Aa</u> uv	<u>aa</u> 0	<u>AA</u> 0	<u>Aa</u> $\frac{1}{2}vw$	<u>aa</u> $\frac{1}{2}vw$	<u>AA</u> 0	<u>Aa</u> 0	<u>aa</u> w^2 (Multiply by M_{33})

So the transformation becomes:

$$u \rightarrow u^2 \cdot M_{11} + \frac{1}{2}uv \cdot M_{12} + \frac{1}{2}uv \cdot M_{21} + \frac{v^2}{4} \cdot M_{22}$$

$$\begin{aligned}v \rightarrow & \frac{1}{2}uv \cdot M_{12} + uv \cdot M_{13} + \frac{1}{2}uv \cdot M_{21} \\ & + \frac{v^2}{2} \cdot M_{22} + \frac{1}{2}vw \cdot M_{23} + uv \cdot M_{31} \\ & + \frac{1}{2}vw \cdot M_{32}\end{aligned}$$

$$\begin{aligned}w \rightarrow & \frac{v^2}{4} M_{22} + \frac{1}{2}vw \cdot M_{23} + \frac{1}{2}vw \cdot M_{32} \\ & + w^2 \cdot M_{33}\end{aligned}$$

(We normalize these by dividing each by the sum of them. I did this in the code.)