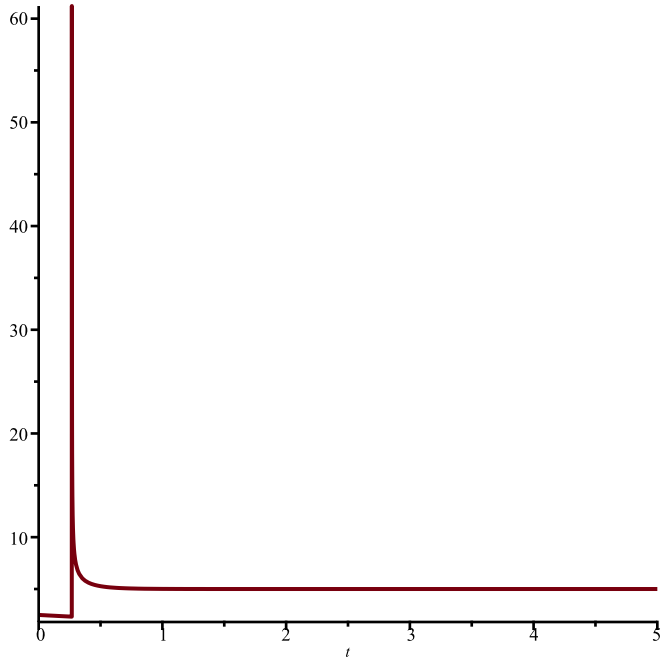
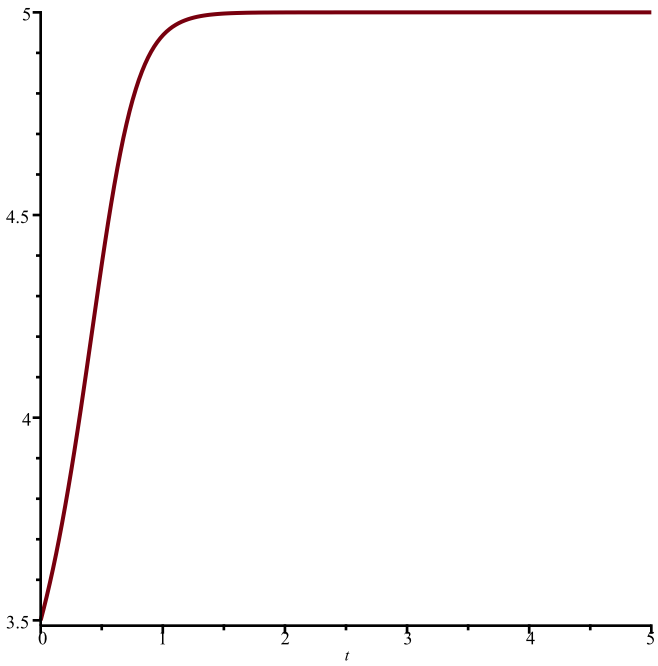
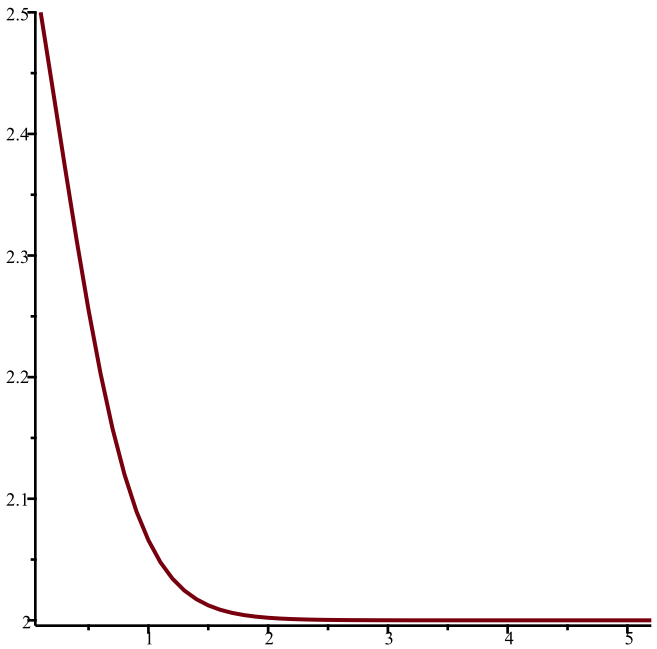
```
#Ok to post

#Anne Somalwar, hw15, 10.25.2021


read "C:/Users/aks238/OneDrive - Rutgers University/Documents/M15.txt"



#2

#(i)

dsolve( {D(x)(t) = (3 - x(t))·(2 - x(t))·(5 - x(t)), x(0) = (3 + 2)/2}, x(t) ) :

plot(rhs(%), t = 0..5)
```




```
dsolve( {D(x)(t) = (3 - x(t))·(2 - x(t))·(5 - x(t)), x(0) = (5 + 2)/2}, x(t) ) :
plot(rhs(%), t = 0..5)
```




```
>
>
>
>
>
>
>
>
> #(ii)
>
>
> Dis1((3 - x)·(2 - x)·(5 - x), x, (3 + 2)/2, 0.1, 5):
> plot(%)
```



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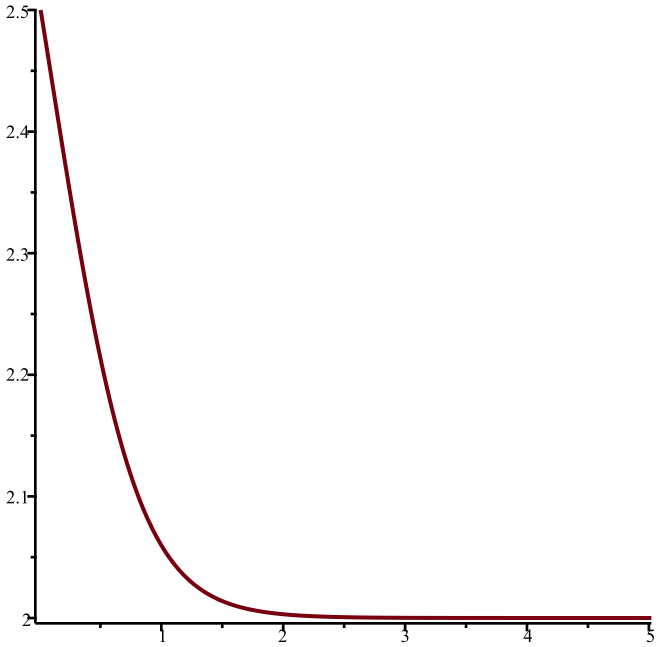
>

> $Dis1\left((3-x)\cdot(2-x)\cdot(5-x), x, \frac{(3+2)}{2}, 0.01, 5\right):$

> $plot(\%)$



>

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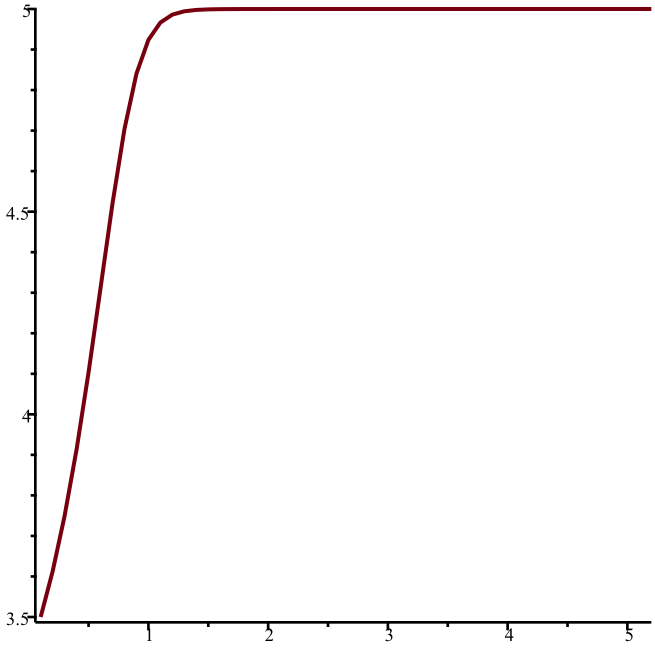
>

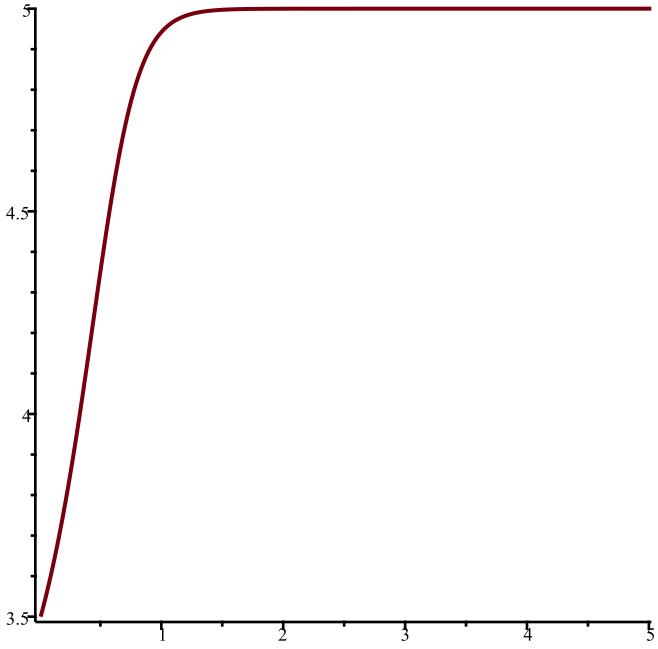
> $Dis1\left((3-x)\cdot(2-x)\cdot(5-x), x, \frac{(5+2)}{2}, 0.1, 5\right):$

> $plot(\%)$



> $Dis1\left((3-x)\cdot(2-x)\cdot(5-x), x, \frac{(5+2)}{2}, 0.01, 5\right):$

> $plot(\%)$



>

>

>

>

>

```
#2

#(i) See written work

#(ii)

ToSys(4, x, (x[1] + 2·x[2] + 3·x[3] + 11·x[4])/(x[1] + x[3]), [1, 5, 5, 2])
```

$$\left[\frac{x_1 + 2\,x_2 + 3\,x_3 + 11\,x_4}{x_1 + x_3}, x_1, x_2, x_3\right] \quad (1)$$

```
#3

Orbk(2, x, (1 − x[1])·(1 − x[2]), [2.5, 2.7], 1000, 1010);
```

$$[0.3819660113, 0.3819660113, 0.3819660112, 0.3819660113, 0.3819660113, 0.3819660112, 0.3819660113, 0.3819660113, 0.3819660112, 0.3819660113, 0.3819660113] \quad (2)$$

```
ToSys(2, x, (1 − x[1])·(1 − x[2]), [2.5, 2.7])
```

$$[(1 - x_1)\,(1 - x_2), x_1] \quad (3)$$

```
SFP2(%, x[1], x[2])
```

$$[[0.3819660113, 0.3819660113]] \quad (4)$$

```
# 0.38 is a stable fixed point.
```

```
> #4
> #See written work
> #5 (bonus)
> #See written work for explanation
> HW3g := proc(u, v, w, M) :
  with(LinearAlgebra) :
  (1/(u^2·M[1][1] + (1/2)·u*v·M[1][2] + (1/2)·u·v·M[2][1] + (1/4)*v^2
     ·M[2][2] + (1/2)·u*v·M[1][2] + u*w·M[1][3] + (1/2)·u·v·M[2][1] + (1/2)*v
     ^2·M[2][2] + (1/2)·v·w·M[2][3] + u*w·M[3][1] + (1/2)·v·w·M[3][2] + (1/4)
     *v^2·M[2][2] + (1/2)·v*w·M[2][3] + (1/2)·v·w·M[3][2] + w^2·M[3][3]))·[u
     ^2·M[1][1] + (1/2)·u*v·M[1][2] + (1/2)·u·v·M[2][1] + (1/4)*v^2·M[2][2],
       (1/2)·u*v·M[1][2] + u*w·M[1][3] + (1/2)·u·v·M[2][1] + (1/2)*v^2·M[2][2]
     + (1/2)·v·w·M[2][3] + u*w·M[3][1] + (1/2)·v·w·M[3][2], (1/4)*v^2·M[2][2]
     + (1/2)·v*w·M[2][3] + (1/2)·v·w·M[3][2] + w^2·M[3][3]]:
end:
> HW3g(u, v, w, [[0.2, 0.2, 0.2], [0.2, 0.2, 0.2], [0.2, 0.2, 0.2]])
```

$$\left[0.2\,u^2 + 0.2000000000\,v\,u + 0.05000000000\,v^2,\ 0.2000000000\,v\,u + 0.4\,u\,w + 0.1000000000\,v^2 + 0.2000000000\,v\,w,\ 0.05000000000\,v^2 + 0.2000000000\,v\,w + 0.2\,w^2\right] \Big/ \left(0.2\,u^2 + 0.4\,v\,u + 0.2\,v^2 + 0.4\,u\,w + 0.4\,v\,w + 0.2\,w^2\right) \quad \textbf{(5)}$$

```
>
```

```
>
> #Same as HW3g but replaces w by 1-u-v
> HW2g := proc(u, v, M) :
  with(LinearAlgebra) :
  (1/(u^2·M[1][1] + (1/2)·u*v·M[1][2] + (1/2)·u·v·M[2][1] + (1/4)*v^2
     ·M[2][2] + (1/2)·u*v·M[1][2] + u*(1-u-v)·M[1][3] + (1/2)·u·v·M[2][1]
     + (1/2)*v^2·M[2][2] + (1/2)·v·(1-u-v)·M[2][3] + u*(1-u-v)·M[3][1]
     + (1/2)·v·(1-u-v)·M[3][2] + (1/4)*v^2·M[2][2] + (1/2)·v*(1-v-u)
     ·M[2][3] + (1/2)·v·(1-v-u)·M[3][2] + (1-v-u)^2·M[3][3]))·[u^2
     ·M[1][1] + (1/2)·u*v·M[1][2] + (1/2)·u·v·M[2][1] + (1/4)*v^2·M[2][2],
     (1/2)·u*v·M[1][2] + u*(1-v-u)·M[1][3] + (1/2)·u·v·M[2][1] + (1/2)*v
     ^2·M[2][2] + (1/2)·v·(1-v-u)·M[2][3] + u*(1-v-u)·M[3][1] + (1/2)·v
     ·(1-v-u)·M[3][2]]:
  end:
> #First, we test the case where the survival rate is the same for everyone.
> HW2g(u, v, [[0.2, 0.2, 0.2], [0.2, 0.2, 0.2], [0.2, 0.2, 0.2]])
```

$$\left[0.2\,u^2 + 0.2000000000\,v\,u + 0.05000000000\,v^2,\ 0.2000000000\,v\,u + 0.4\,u\,(1-u-v) + 0.1000000000\,v^2 + 0.2000000000\,v\,(1-u-v)\right] \Big/ \left(0.2\,u^2 + 0.4\,v\,u + 0.2\,v^2 + 0.4\,u\,(1-u-v) + 0.4\,v\,(1-u-v) + 0.2\,(1-u-v)^2\right) \tag{6}$$

```
>
>
> Orb2(%, u, v, [0.3, 0.4], 1000, 1010)
```

$$[[0.2500000000, 0.5000000000], [0.2500000000, 0.5000000000], [0.2500000000, 0.5000000000], [0.2500000000, 0.5000000000], [0.2500000000, 0.5000000000], [0.2500000000, 0.5000000000], [0.2500000000, 0.5000000000], [0.2500000000, 0.5000000000], [0.2500000000, 0.5000000000], [0.2500000000, 0.5000000000]] \tag{7}$$

```
> #The  frequencies are stable, as Hardy-Weinberg suggests.
>
>
>
> #Now, let's try the case where not all survival rates are equal.
>  HW2g(u, v, [[0.2, 0.2, 0.3], [0.2, 0.4, 0.2], [0.2, 0.8, 0.2]])
```

(8)

$$\left[0.2\,u^2 + 0.2000000000\,v\,u + 0.1000000000\,v^2,\ 0.2000000000\,v\,u + 0.5\,u\,(1-u-v) + 0.2000000000\,v^2 + 0.5000000000\,v\,(1-u-v)\right] \Big/ \left(0.2\,u^2 + 0.4\,v\,u + 0.4\,v^2 + 0.5\,u\,(1-u-v) + 1.0\,v\,(1-u-v) + 0.2\,(1-u-v)^2\right) \tag{8}$$

```
> Orb2(%, u, v, [0.3, 0.4], 1000, 1010)
```

$$[[0.08400002707, 0.4699326384], [0.08400002707, 0.4699326384], [0.08400002707, 0.4699326384], [0.08400002707, 0.4699326384], [0.08400002707, 0.4699326384], [0.08400002707, 0.4699326384], [0.08400002707, 0.4699326384], [0.08400002707, 0.4699326384], [0.08400002707, 0.4699326384], [0.08400002707, 0.4699326384]] \tag{9}$$

```
> #Looks like the frequencies stabilize anyway!
>
>
> #Some more tests:
>
> HW2g(u, v, [[0.2, 0.1, 0.3], [0.6, 0.4, 0.2], [0.5, 0.8, 0.2]])
```

$$\left[0.2\,u^2 + 0.3500000000\,v\,u + 0.1000000000\,v^2,\ 0.3500000000\,v\,u + 0.8\,u\,(1-u-v) + 0.2000000000\,v^2 + 0.5000000000\,v\,(1-u-v)\right] \Big/ \left(0.2\,u^2 + 0.7\,v\,u + 0.4\,v^2 + 0.8\,u\,(1-u-v) + 1.0\,v\,(1-u-v) + 0.2\,(1-u-v)^2\right) \tag{10}$$

```
> Orb2(%, u, v, [0.2, 0.7], 1000, 1010)
```

$$[[0.1345596521, 0.5117727345], [0.1345596521, 0.5117727345], [0.1345596521, 0.5117727345], [0.1345596521, 0.5117727345], [0.1345596521, 0.5117727345], [0.1345596521, 0.5117727345], [0.1345596521, 0.5117727345], [0.1345596521, 0.5117727345], [0.1345596521, 0.5117727345], [0.1345596521, 0.5117727345]] \tag{11}$$

```
>
>
>
> HW2g(u, v, [[0.1, 0.1, 0.6], [0.3, 0.7, 0.8], [0.2, 0.8, 0.1]])
```

$$\left[0.1\,u^2 + 0.2000000000\,v\,u + 0.1750000000\,v^2,\ 0.2000000000\,v\,u + 0.8\,u\,(1-u-v) + 0.3500000000\,v^2 + 0.8000000000\,v\,(1-u-v)\right] \Big/ \left(0.1\,u^2 + 0.4\,v\,u + 0.7\,v^2 + 0.8\,u\,(1-u-v) + 1.6\,v\,(1-u-v) + 0.1\,(1-u-v)^2\right) \tag{12}$$

```
> Orb2(%, u, v, [0.2, 0.7], 1000, 1010)
```

$$[[0.1011294756, 0.5134483837], [0.1011294757, 0.5134483837], [0.1011294757, 0.5134483835], [0.1011294756, 0.5134483837], [0.1011294757, 0.5134483837], [0.1011294757, 0.5134483835], [0.1011294756, 0.5134483837], [0.1011294757, 0.5134483837], [0.1011294757, 0.5134483835], [0.1011294756, 0.5134483837], [0.1011294757, 0.5134483837]] \tag{13}$$

```
>
```