Dynamical Modeling HW15 - Okay to Post

2) (i) $$x(n)=\frac{x(n-1)+2x(n-2)+3x(n-3)+11x(n-4)}{x(n-1)+x(n-3)}$$

$$x_1(n)=\frac{x_1(n-1)+2x_1(n-2)+3x_1(n-3)+11x_1(n-4)}{x_1(n-1)+x_1(n-3)}$$

$x_2(n)=x_1(n-1)$

$x_3(n)=x_1(n-2)$

$x_4(n)=x_1(n-3)$

$$\boxed{\begin{aligned}
x_1(n) &= \frac{x_1(n-1)+2x_2(n-1)+3x_3(n-1)+11x_4(n-1)}{x_1(n-1)+x_3(n-1)}\\
x_2(n) &= x_1(n-1)\\
x_3(n) &= x_1(n-2)\\
x_4(n) &= x_1(n-3)
\end{aligned}}$$

(ii) $$f=\frac{z[1]+2\cdot z[2]+3\cdot z[3]+11\cdot z[4]}{z[1]+z[3]}$$

$k=4$

INI = [1, 5, 5, 2]

output provided in Maple

4)

	Parent Genotype	Frequency	Offspring Genotype Frequencies		
			AA	Aa	aa
	Mom = AA, Dad = AA	u^2	u^2	0	0
same ⇒ don't duplicate	Mom : AA, Dad = Aa	$2uv$	uv	uv	0
	Mom = Aa, Dad = AA	$2uv$	uv	uv	0
same ⇒ don't duplicat	Mom = AA, Dad = aa	$2uw$	0	$2uw$	0
	Mom = aa, Dad = AA	$2uw$	0	$2uw$	0
	Mom = Aa, Dad = Aa	v^2	$v^2/4$	$v^2/2$	$v^2/4$
same ⇒ don't duplicate	Mom = Aa, Dad = aa	$2vw$	0	vw	vw
	Mom = aa, Dad = Aa	$2yw$	0	vw	vw
	Mom = aa, Dad = aa	w^2	0	0	w^2
		Total	$u^2 + uv + \frac{v^2}{4}$	$uv + 2uw + vw + \frac{v^2}{2}$	$\frac{v^2}{4} + vw + w^2$

↑ 53(a) ↑ 53(b) ↑ 53(c)

```
> # Nikita John, Assignment 15 (Okay to Post)
  #October 25th, 2021
> #M15.txt: Maple code for Lecture 15 of Dynamical Modesl in Biology, Fall 2021 (taught by Dr.
     Z.)

Help15 :=proc( ) : print( ` HW3(u,v,w), HW2(u,v) , Dis1(F,y,y0,h,A), ToSys(k,z,f,INI) `) :end:



     #ToSys(k,z,f,INI): converts the kth order difference equation x(n)=f(x[n-1],x[n-2],...x[n-k]) to
     a first-order system
#x1(n)=F(x1(n-1),x2(n-1), ...,xk(n-1))
#x2(n)=x1(n-1)
#...

     #xk(n)=x[k-1](n-1). It gives the underlying transformation phrased in terms of z[1],...z[k],
     followed by the initial conditions. Try:
#ToSys:=proc(2,z,z[1]+z[2],[1,1])
ToSys :=proc(k, z, f, INI) local i :
[ f, seq(z[i-1], i=2 ..k) ], INI :
end:


     #HW3(u,v,w): The Hardy-Weinberg unerlying transformation witu (u,v,w), Eqs. (53a,53b,
     53c) in Edelestein-Keshet Ch. 3
HW3 :=proc(u, v, w) : [u^2 + u * v + (1/4) * v^2,  u * v + 2 * u * w + 1/2 * v^2 + v * w, 1/4
      * v^2 + v * w + w^2 ] :end:


     #HW2(u,v): The Hardy-Weinberg unerlying transformation witu (u,v,w), Eqs. (53a,53b,53c)
     in Edelestein-Keshet Ch. 3 using the fact that u+v+w=1
HW2 :=proc(u, v) : expand( [u^2 + u * v + (1/4) * v^2 , u * v + 2 * u * (1-u-v) + 1/2 * v^2
      + v * (1-u-v) ]) :end:





     #Dis1(F,y,y0,h,A): The approximate orbit of the Dynamical system approximating the 1D for
     the autonomous  continuous dynamical  process dy/dt=F(y(t)) , y(0)=y0 with mesh size h
     from t=0 to t=A
Dis1 :=proc(F, y, y0, h, A) local L, x, i :
L := Orb(x + h * subs(y=x, F), x, y0, 0, trunc(A/h) ) :

L := [seq( [i * h, L[i]], i=1 ..nops(L) ) ] :
end:

##old stuff
```

```
    #M13.txt: Maple code for Lecture 13 of Dynamical Modesl in Biology, Fall 2021 (taught by
    Dr. Z.)
Help13 :=proc( ) :
     print( ` RT2(x,y,d,K), Orb2(F,x,y,pt0,K1,K2), FP2(F,x,y), SFP2(F,x,y), PlotOrb2(L), FP2drz
    (F,x,y), SFP2drz(F,x,y) `) :end:

with(LinearAlgebra) :


    #RT2(x,y,d,K): A random rational transformation of degree d from R^2 to R^2 with postiive
    integer coefficients from 1 to K The inputs are variables x and y and
#the output is a pair of expressions  of (x,y) representing functions. It is for generating examples
#Try:
#RT2(x,y,2,10);
RT2 :=proc(x, y, d, K) local ra, i, j, f, g :
ra := rand(1 ..K) : #random integer from -K to K
f := add(add(ra( ) * x^i * y^j, j = 0 ..d - i), i = 0 ..d) / add(add(ra( ) * x^i * y^j, j = 0 ..d - i), i = 0
     ..d) :
g := add(add(ra( ) * x^i * y^j, j = 0 ..d - i), i = 0 ..d) / add(add(ra( ) * x^i * y^j, j = 0 ..d - i), i = 0
     ..d) :
[f, g] :
end:



    #Orb2(F,x,y,pt,K1,K2): Inputs a mapping F=[f,g] from R^2  to R^2 where f and g describe
    functions of x and y, an initial point pt0=[x0,y0]
#outputs the orbit starting at discrete time K1 and ending in discrete time K2. Try
#F:=RT2(x,y,2,10);
#Orb2(F,x,y,[1.1,1.2],1000,1010);
Orb2 :=proc(F, x, y, pt0, K1, K2)  local pt, L, i :
pt := pt0 :

for i from 1 to K1 - 1 do
pt := subs( {x = pt[1], y = pt[2]}, F) :
od:

L := [ ] :
for i from K1 to K2 do
L := [op(L), pt] :
pt := subs( {x = pt[1], y = pt[2]}, F) :

od:
L :
end:

#FP2(F,x,y): The list of fixed points of the transformation [x,y]->F. Try
#FP2([x-y,x=y],x,y);
FP2 :=proc(F, x, y) local L, i :
```

```
L ≔ [solve({F[1]=x, F[2]=y}, {x, y})]:

[seq(subs(L[i], [x, y]), i=1 ..nops(L))]:
end:


#SFP2(F,x,y): The list of Stable fixed points of the transformation [x,y]->F. Try
#SFP2([(1+x)/(1+y), (1+7*y)/(4+x)],x,y);
SFP2 ≔proc(F, x, y) local L, J, S, J0, i, pt, EV:

L ≔ evalf(FP2(F, x, y)) :
     #F is the list of ALL fixed points of the transformation [x,y]->F using the previous procedure
     FP2(F,x,y), but since we are interested in numbers we take the floating point version using
     evalf

J ≔ Matrix(normal([[diff(F[1], x), diff(F[1], y)], [diff(F[2], x), diff(F[2], y)]])) :
     #J is the Jacobian matrix in general (in terms of the variables x and y). Note that J is a
     SYMBOLIC matrix featuring variables x and y

S ≔ [ ] :  #S is the list of stable fixed points that starts out empty

for i from 1 to nops(L) do  #we examime it case by case
pt ≔ L[i] : #pt is the current fixed point to be examined

J0 ≔ subs({x=pt[1], y=pt[2]}, J) :
     #J0 is the NUMERICAL matrix obtained by plugging-in the examined fixed pt

EV ≔ Eigenvalues(J0) :
     # We used Maple's command Eigenavalues to find the eigenvalues of this 2 by 2 matrix

if abs(EV[1]) < 1 and abs(EV[2]) < 1  then
 S ≔ [op(S), pt] :
     #If both eigenvalues have absolute value less than 1 it means that they are stable, so we
     append the examined fixed point, pt, to the list of fixed points
fi:

od:
S : #the output is S
end:




###added Oct. 17, 20221
with(plots) :

PlotOrb1 ≔proc(L) local i, d :

d ≔ textplot([L[1], 0, 0]) :
```

```
for i from 2 to nops(L) do
d := d, textplot( [L[i], 0, i-1 ]) :
od:
display(d) :
end:


PlotOrb2 :=proc(L) local i, d :

d := textplot( [ op(L[1 ]), 0 ]) :

for i from 2 to nops(L) do
d := d, textplot( [ op(L[i]), i-1 ]) :
od:
display(d) :
end:
###End added Oct. 17, 20221


###old stuff
#M11.txt: Maple code for Lecture 11 of Dynamical Models in Biology taught by Dr. Z.
Help11 :=proc( ) : print( ` SFPe(f,x), Orbk(k,z,f,INI,K1,K2) `) :end:


    #SFPe(f,x): The set of fixed points of x->f(x) done exactly (and allowing symbolic
    parameters), followed by the condition of stability (if it is netween -1 and 1 it is stable)
#Try: FPe(k*x*(1-x),x);
#VERSION OF Oct. 12, 2021 (avoiding division by 0)
SFPe :=proc(f, x) local f1, L, i, M:
f1 := normal(diff (f, x) ) :
L := [solve(numer(f-x), x) ] :
M := [ ] :

for i from 1 to nops(L) do
 if subs(x=L[i], denom(f1) ) ≠ 0 then
  M := [op(M), [L[i], normal(subs(x=L[i],f1) ) ]] :
 fi:
od:
M:

end:


#Added after class

    #Orbk(k,z,f,INI,K1,K2): Given a positive integer k, a letter (symbol), z, an expression f of z
    [1], ..., z[k] (representing a multi-variable function of the variables z[1],...,z[k]

    #a vector INI representing the initial values [x[1],..., x[k]], and (in applications) positive
```

```
    integres K1 and K2, outputs the

    #values of the sequence starting at n=K1 and ending at n=K2. of the sequence satisfying the
    difference equation
##x[n]=f(x[n-1],x[n-2],..., x[n-k+1]):

    #This is a generalization to  higher-order difference equation of procedure Orb(f,x,x0,K1,K2)
    . For example
#Orbk(1,z,5/2*z[1]*(1-z[1]),[0.5],1000,1010);  should be the same as
#Orb(5/2*z[1]*(1-z[1]),z[1],[0,5],1000,1010);
#Try:
#Orbk(2,z,(5/4)*z[1]-(3/8)*z[2],[1,2],1000,1010);
Orbk ≔proc(k, z, f, INI, K1, K2) local L, i, newguy :
L ≔ INI :  #We start out with the list of initial values

if not (type(k, integer) and type(z, symbol) and type(INI, list) and nops(INI) = k and type(K1,
    integer) and type(K2, integer) and K1 > 0 and K2 > K1) then
    #checking that the input is OK
 print(`bad input`) :
 RETURN(FAIL) :
fi:

while nops(L) < K2 do
 newguy ≔ subs( {seq(z[i] = L[ -i], i = 1 ..k) }, f ) :
    #Using what we know about the value yesterday, the day before yesterday, ... up to k days
    before yesterday we find the value of the sequence today
 L ≔ [op(L), newguy] :  #we append the new value to the running list of values of our sequence
od:

[op(K1 ..K2, L) ] :

end:


####STAFT FROM M9.txt
#M9.txt: Maple Code for "Dynamical models in Biology" (Math 336) taught by Dr. Z., Lecture 9

Help9 ≔proc( ) :
     print( ` Orb(f,x,x0,K1,K2), Orb2D(f,x,x0,K) , FP(f,x) , SFP(f,x) , Comp(f,x)   `) :end:


    #Orb(f,x,x0,K1,K2): Inputs an expression f in x (desccribing) a function of x, an initial point,
    x0, and a positive integer K, outputs
#the values of x[n] from n=K1 to n=K2. Try: where x[n]=f(x[n-1]), . Try:
#Orb(2*x*(1-x),x,0.4,1000,2000);
Orb ≔proc(f, x, x0, K1, K2) local x1, i, L :
x1 ≔ x0 :

for i from 1 to K1 do
```

```
 x1 ≔ subs(x=x1, f) :
     #we don't record the first values of K1, since we are interested in the long-time behavior of
     the orbit
od:

L ≔ [x1] :

for i from K1 to K2 do
 x1 ≔ subs(x=x1, f) :   #we compute the next member of the orbit
 L ≔ [op(L), x1] : #we append it to the list
od:

L : #that's the output

end:

#Orb2D(f,x,x0,K): 2D version of Orb(f,x,x0,0,K), just for illustration
Orb2D ≔proc(f, x, x0, K) local L, L1, i :
L ≔ Orb(f, x, x0, 0, K) :
L1 ≔ [[L[1], 0], [L[1], L[2]], [L[2], L[2]] ] :
for i from 3 to nops(L) do
 L1 ≔ [op(L1), [L[i-1], L[i]], [L[i], L[i]]] :
od:
L1 :
end:


#FP(f,x): The list of fixed points of the map x->f where f is an expression in x. Try:
#FP(2*x*(1-x),x);
FP ≔proc(f, x)
evalf([solve(f=x, x)]) :
end:


#SFP(f,x): The list of stable fixed points of the map x->f where f is an expression in x. Try:
#SFP(2*x*(1-x),x);
SFP ≔proc(f, x) local L, i, f1, pt, Ls :
L ≔ FP(f, x) :  #The list of fixed points (including complex ones)

Ls ≔ [ ] :        #Ls is the list of stable fixed points, that starts out as the empty list

f1 ≔ diff(f, x) :   #The derivative of the function f w.r.t. x

for i from 1 to nops(L) do
pt ≔ L[i] :

if abs(subs(x=pt, f1)) < 1 then

 Ls ≔ [op(Ls), pt] :   # if pt,  is stable we add it to the list of stable points
```

```
fi:

od:

Ls :   #The last line is the output

end:

#Comp(f,x): f(f(x))
Comp ≔proc(f, x) : normal(subs(x=f,f) ) :end:




##added Oct. 17, 2021
#FP2drz(F,x,y): The list of fixed points of the transformation [x,y]->F. Dr. Z.'s way
#FP2([x-y,x+y],x,y);
FP2drz ≔proc(F, x, y) local eq, i, L, S1 :
eq ≔ [numer(F[1]-x), numer(F[2]-y)] :

L ≔ Groebner[Basis](eq, plex(x, y)) :

S1 ≔ evalf([solve(L[1], y)]) :
[seq([solve(subs(y=S1[i], L[2]), x), S1[i]], i=1 ..nops(S1))] :
end:


#SFP2drz(F,x,y): The list of Stable fixed points of the transformation [x,y]->F. Try
#SFP2drz([(1+x)/(1+y), (1+7*y)/(4+x)],x,y);
SFP2drz ≔proc(F, x, y) local L, J, S, J0, i, pt, EV :

L ≔ FP2drz(F, x, y) :
    #F is the list of ALL fixed points of the transformation [x,y]->F using the previous procedure
    FP2(F,x,y), but since we are interested in numbers we take the floating point version using
    evalf

J ≔ Matrix(normal([[diff(F[1], x), diff(F[2], x)], [diff(F[1], y), diff(F[2], y)]])) :
    #J is the Jacobian matrix in general (in terms of the variables x and y). Note that J is a
    SYMBOLIC matrix featuring variables x and y

S ≔ [ ] :  #S is the list of stable fixed points that starts out empty

for i from 1 to nops(L) do  #we examime it case by case
pt ≔ L[i] : #pt is the current fixed point to be examined

J0 ≔ subs({x=pt[1], y=pt[2]}, J) :
    #J0 is the NUMERICAL matrix obtained by plugging-in the examined fixed pt

EV ≔ Eigenvalues(J0) :
    # We used Maple's command Eigenavalues to find the eigenvalues of this 2 by 2 matrix
```

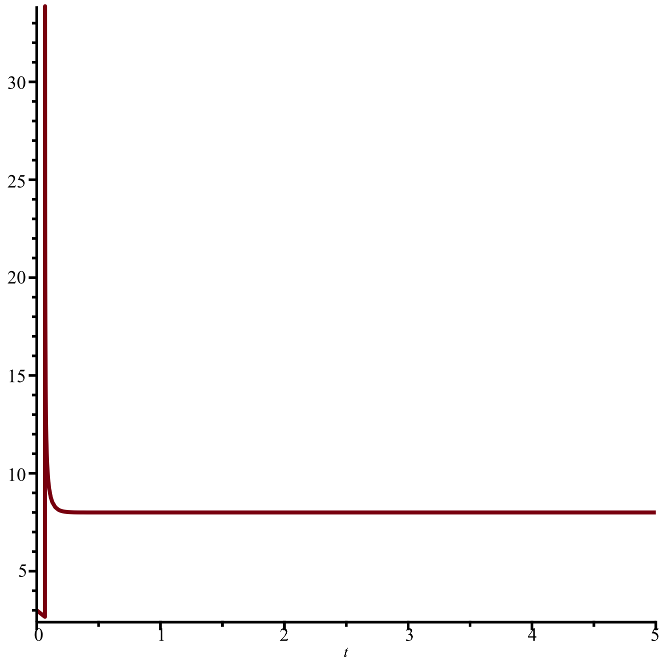
```
if abs(EV[1]) < 1 and abs(EV[2]) < 1  then
 S := [op(S), pt] :
     #If both eigenvalues have absolute value less than 1 it means that they are stable, so we
     append the examined fixed point, pt, to the list of fixed points
fi:

od:
S : #the output is S
end:
```

```
> #1 (i)
  # first eq: a3 = 4, a4 = 2, a7 = 8
```

$$F := dsolve\left(\left\{diff(x(t),t) = (4-x(t))\cdot(2-x(t))\cdot(8-x(t)),\, x(0) = \frac{(4+2)}{2}\right\},\, x(t)\right);$$

$$F := x(t) = \frac{2\,5^{2/3}\left(\left(e^{t}\right)^{24}\left(\sqrt{-\frac{1}{\frac{\left(e^{t}\right)^{24}}{5}-1}}+1\right)\left(\frac{\left(e^{t}\right)^{24}}{5}-1\right)^{2}\right)^{1/3}}{5\left(\frac{\left(e^{t}\right)^{24}}{5}-1\right)} + \frac{2\left(e^{t}\right)^{24}\,5^{1/3}}{5\left(\left(e^{t}\right)^{24}\left(\sqrt{-\frac{1}{\frac{\left(e^{t}\right)^{24}}{5}-1}}+1\right)\left(\frac{\left(e^{t}\right)^{24}}{5}-1\right)^{2}\right)^{1/3}} + 4 \tag{1}$$

$$> plot\left(\frac{2\,5^{2/3}\left(\left(e^{t}\right)^{24}\left(\sqrt{-\frac{1}{\frac{\left(e^{t}\right)^{24}}{5}-1}}+1\right)\left(\frac{\left(e^{t}\right)^{24}}{5}-1\right)^{2}\right)^{1/3}}{5\left(\frac{\left(e^{t}\right)^{24}}{5}-1\right)} + \frac{2\left(e^{t}\right)^{24}\,5^{1/3}}{5\left(\left(e^{t}\right)^{24}\left(\sqrt{-\frac{1}{\frac{\left(e^{t}\right)^{24}}{5}-1}}+1\right)\left(\frac{\left(e^{t}\right)^{24}}{5}-1\right)^{2}\right)^{1/3}} + 4,\, t=0..5\right);$$



> *#second eq*

$$G := dsolve\left(\left\{diff(x(t), t) = (4 - x(t))\cdot(2 - x(t))\cdot(8 - x(t)), x(0) = \frac{(8+2)}{2}\right\}, x(t)\right);$$

$$G := x(t) = -\frac{\left(-\frac{\left(e^{t}\right)^{24}\left(\sqrt{-\frac{1}{-\frac{\left(e^{t}\right)^{24}}{27}-1}}+1\right)\left(-\frac{\left(e^{t}\right)^{24}}{27}-1\right)^{2}}{27}\right)^{1/3}}{-\frac{\left(e^{t}\right)^{24}}{27}-1} + \frac{\left(e^{t}\right)^{24}}{27\left(-\frac{\left(e^{t}\right)^{24}\left(\sqrt{-\frac{1}{-\frac{\left(e^{t}\right)^{24}}{27}-1}}+1\right)\left(-\frac{\left(e^{t}\right)^{24}}{27}-1\right)^{2}}{27}\right)^{1/3}} \tag{2}$$

$$+ \mathrm{I}\sqrt{3}\left(\frac{\left(-\frac{\left(\mathrm{e}^{t}\right)^{24}\left(\sqrt{-\frac{1}{-\frac{\left(\mathrm{e}^{t}\right)^{24}}{27}-1}}+1\right)\left(-\frac{\left(\mathrm{e}^{t}\right)^{24}}{27}-1\right)^{2}}{27}\right)^{1/3}}{-\frac{\left(\mathrm{e}^{t}\right)^{24}}{27}-1}\right.$$

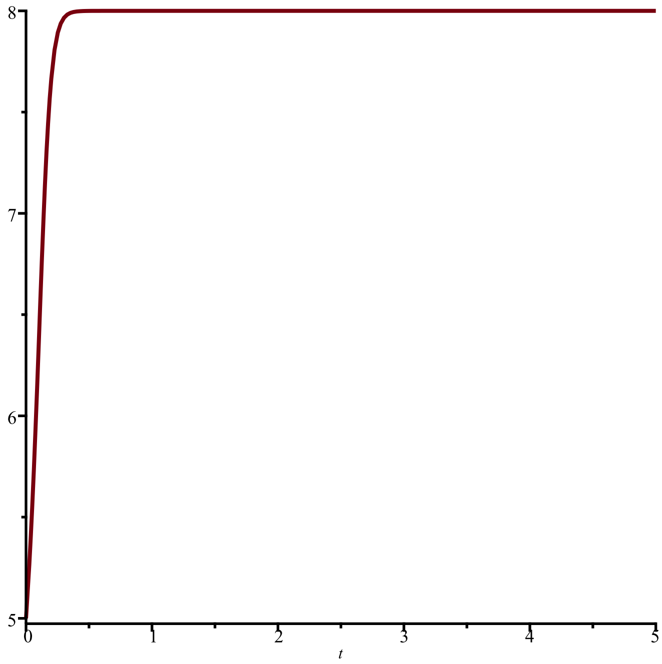
$$\left.+\frac{\left(\mathrm{e}^{t}\right)^{24}}{27\left(-\frac{\left(\mathrm{e}^{t}\right)^{24}\left(\sqrt{-\frac{1}{-\frac{\left(\mathrm{e}^{t}\right)^{24}}{27}-1}}+1\right)\left(-\frac{\left(\mathrm{e}^{t}\right)^{24}}{27}-1\right)^{2}}{27}\right)^{1/3}}\right)+4$$

$$> plot\left(-\frac{\left(-\frac{\left(\mathrm{e}^{t}\right)^{24}\left(\sqrt{-\frac{1}{-\frac{\left(\mathrm{e}^{t}\right)^{24}}{27}-1}}+1\right)\left(-\frac{\left(\mathrm{e}^{t}\right)^{24}}{27}-1\right)^{2}}{27}\right)^{1/3}}{-\frac{\left(\mathrm{e}^{t}\right)^{24}}{27}-1}\right.$$

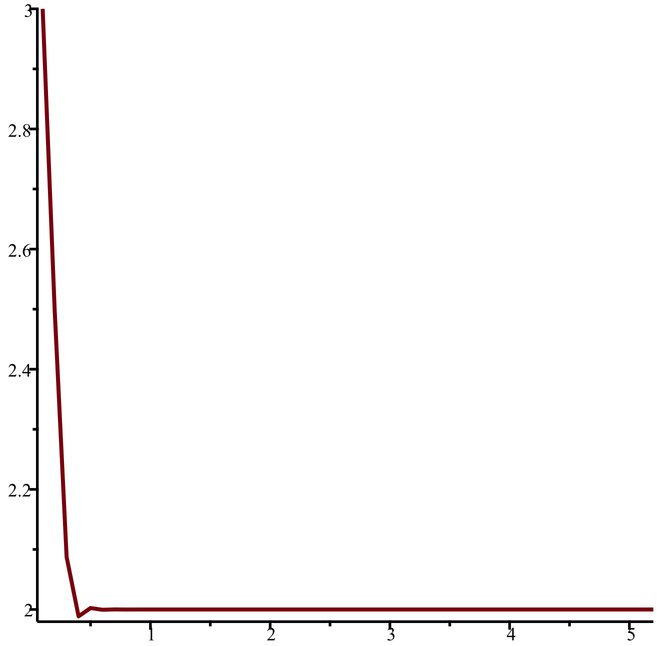
$$+\frac{\left(\mathrm{e}^{t}\right)^{24}}{27\left(-\frac{\left(\mathrm{e}^{t}\right)^{24}\left(\sqrt{-\frac{1}{-\frac{\left(\mathrm{e}^{t}\right)^{24}}{27}-1}}+1\right)\left(-\frac{\left(\mathrm{e}^{t}\right)^{24}}{27}-1\right)^{2}}{27}\right)^{1/3}}$$

$$+ \mathrm{I}\sqrt{3}\left(\frac{\left(-\frac{\left(\mathrm{e}^{t}\right)^{24}\left(\sqrt{-\frac{1}{-\frac{\left(\mathrm{e}^{t}\right)^{24}}{27}-1}}+1\right)\left(-\frac{\left(\mathrm{e}^{t}\right)^{24}}{27}-1\right)^{2}}{27}\right)^{1/3}}{-\frac{\left(\mathrm{e}^{t}\right)^{24}}{27}-1}\right.$$

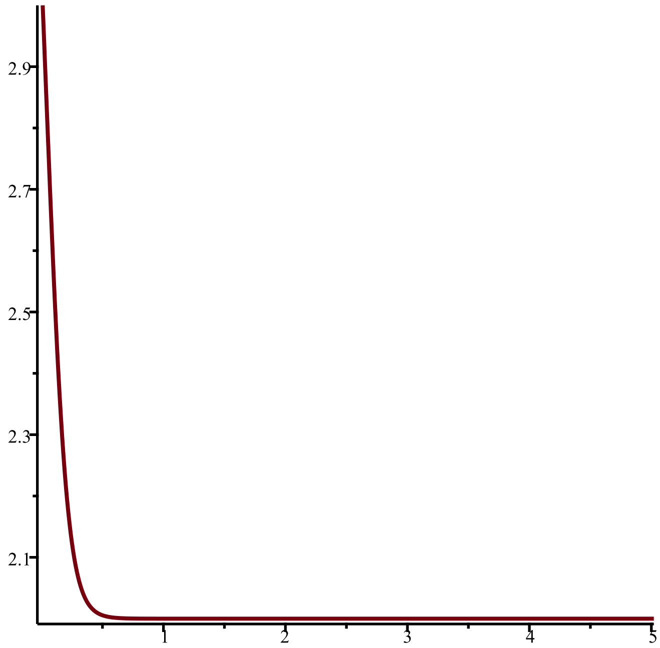
$$\left.+\frac{\left(\mathrm{e}^{t}\right)^{24}}{27\left(-\frac{\left(\mathrm{e}^{t}\right)^{24}\left(\sqrt{-\frac{1}{-\frac{\left(\mathrm{e}^{t}\right)^{24}}{27}-1}}+1\right)\left(-\frac{\left(\mathrm{e}^{t}\right)^{24}}{27}-1\right)^{2}}{27}\right)^{1/3}}\right)+4, t=0..5\Bigg);$$



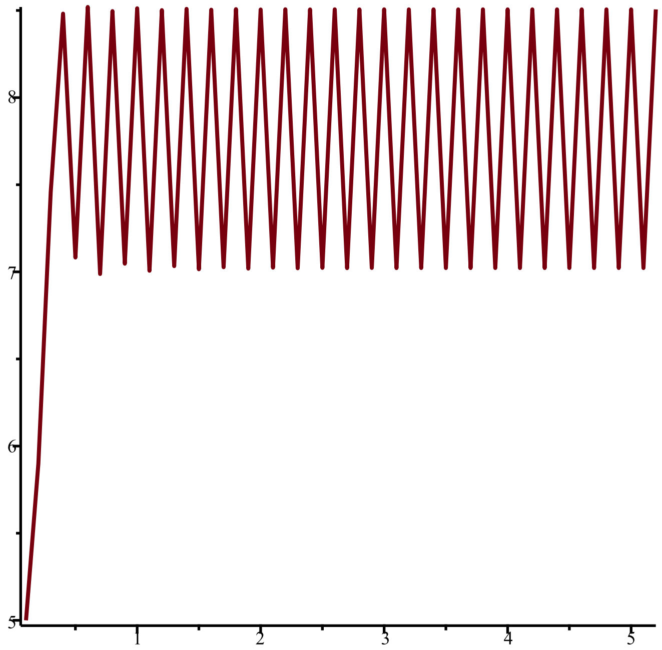
> *#1 (ii)*
#first eq
$Dis1\left((4-y)*(2-y)*(8-y), y, \frac{(4+2)}{2}, 0.1, 5\right):$
plot(%);



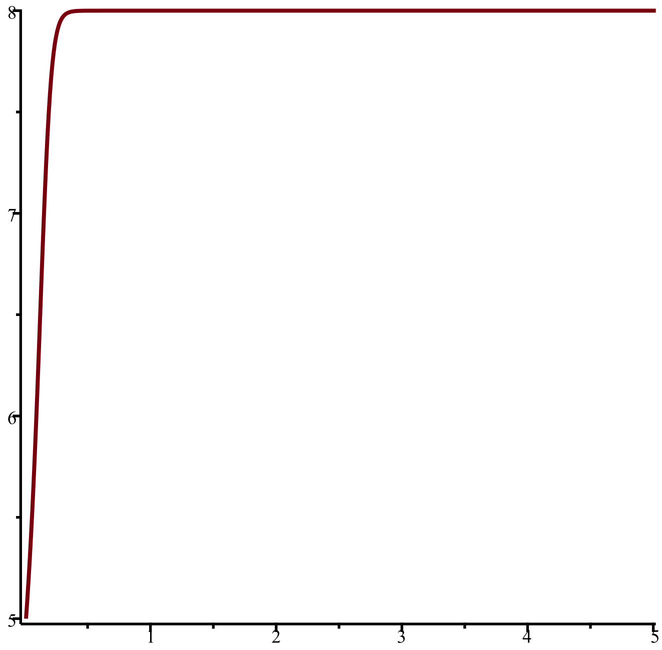
> $Dis1\left((4-y)*(2-y)*(8-y), y, \frac{(4+2)}{2}, 0.01, 5\right):$
$plot(\%);$



> *#second eq*
$Dis1\left((4-y)*(2-y)*(8-y), y, \frac{(8+2)}{2}, 0.1, 5\right):$
$plot(\%);$



> $Dis1\left((4 - y) * (2 - y) * (8 - y), y, \frac{(8+2)}{2}, 0.01, 5\right)$:
> $plot(\%)$;



> *#2 (ii) confirming converting fourth order to first order set using Maple*
> $ToSys\left(4, z, \frac{(z[1] + 2 \cdot z[2] + 3 \cdot z[3] + 11 \cdot z[4])}{z[1] + z[3]}, [1, 5, 5, 2]\right)$;

$$\left[\frac{z_1 + 2\, z_2 + 3\, z_3 + 11\, z_4}{z_1 + z_3}, z_1, z_2, z_3\right], [1, 5, 5, 2] \qquad \textbf{(3)}$$

> *#3*
> $Orbk(2, z, (1 - z[1]) \cdot (1 - z[2]), [2.5, 2.7], 1000, 1010)$;
> *#The fixed point given is (0.381966, 0.381966)*

$$[0.3819660113, 0.3819660113, 0.3819660112, 0.3819660113, 0.3819660113, 0.3819660112, 0.3819660113, 0.3819660113, 0.3819660112, 0.3819660113, 0.3819660113] \tag{4}$$

```
> ToSys(2, z, (1 - z[1])·(1 - z[2]), [2.5, 2.7]);
```

$$\left[\left(1-z_1\right)\left(1-z_2\right), z_1\right], [2.5, 2.7] \tag{5}$$

```
> SFP2([(1 - z[1])·(1 - z[2]), z[1]], z[1], z[2]);
```

$$[[0.3819660113, 0.3819660113]] \tag{6}$$

```
>
```