> **read** `/Users/julianherman/Documents/Rutgers/Fall 2021/Dynamical Models In Biology/HW/M15.txt`

> *Help15*( )

$$HW3(u,v,w),\ HW2(u,v)\ ,\ Dis1(F,y,y0,h,A),\ ToSys(k,z,f,INI) \tag{1}$$

> *#OK to post*
> *#Julian Herman, October 25 th, 2021, Assignment 15*

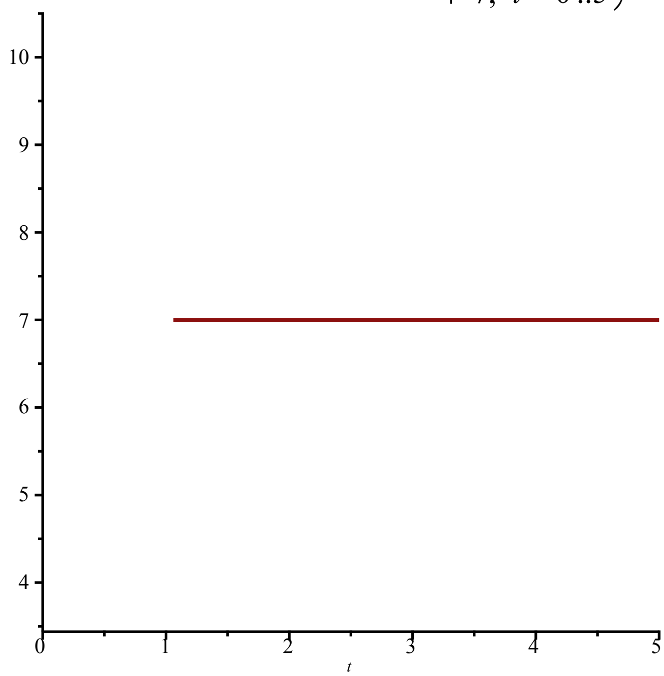
> *#2)*

> *#i)*

> $dsolve(\{diff(x(t),t) = (4 - x(t))\cdot(2 - x(t))\cdot(7 - x(t)), x(0) = 3\}, x(t))$

$$x(t) = \mathrm{e}^{RootOf\left(2\_Z + 30\,t - 4\ln(2) - 3\,\mathrm{I}\,\pi + \ln\left(\frac{(\mathrm{e}^{\_Z} + 5)^3}{(3 + \mathrm{e}^{\_Z})^5}\right)\right)} + 7 \tag{2}$$

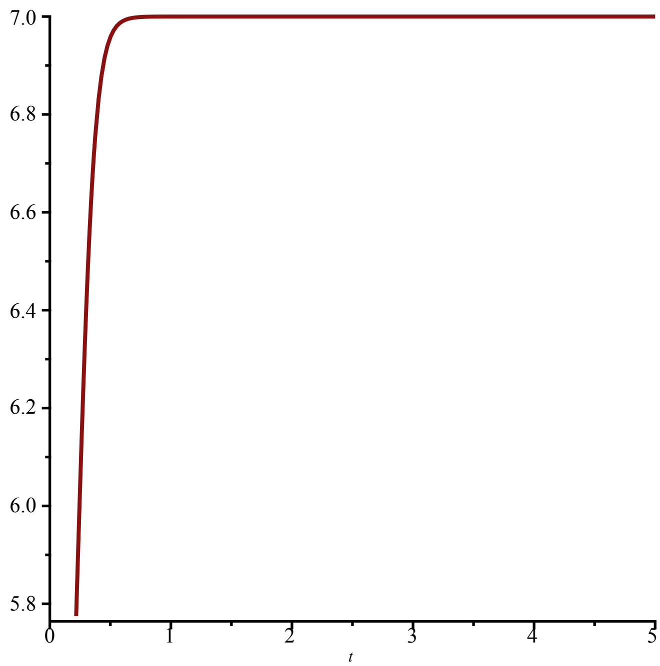
> $plot\left(\mathrm{e}^{RootOf\left(2\_Z + 30\,t - 4\ln(2) - 3\,\mathrm{I}\,\pi + \ln\left(\frac{(\mathrm{e}^{\_Z} + 5)^3}{(3 + \mathrm{e}^{\_Z})^5}\right)\right)} + 7,\ t = 0..5\right)$



> $dsolve\left(\left\{diff(x(t),t) = (4 - x(t))\cdot(2 - x(t))\cdot(7 - x(t)), x(0) = \frac{9}{2}\right\}, x(t)\right)$

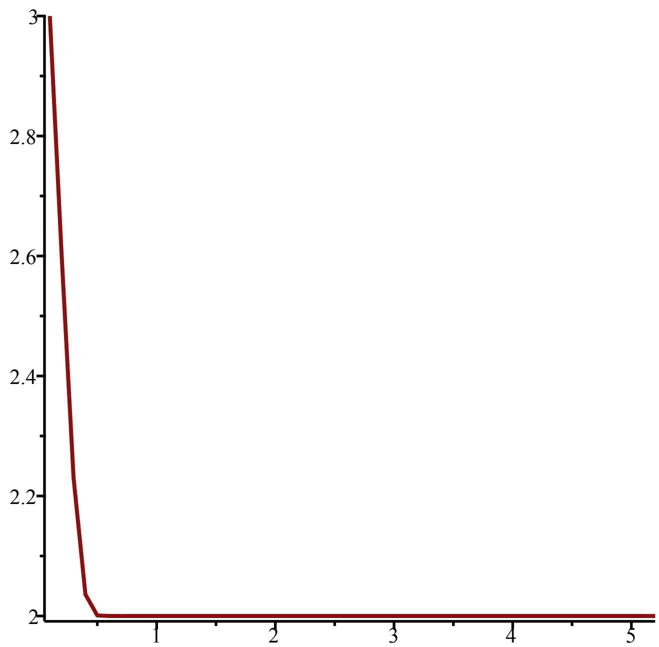
$$x(t) = \mathrm{e}^{RootOf\left(2\_Z + 30\,t - 2\ln\left(\frac{5}{2}\right) - 2\,\mathrm{I}\,\pi - \ln(500) + \ln\left(\frac{(\mathrm{e}^{\_Z} + 5)^3}{(3 + \mathrm{e}^{\_Z})^5}\right)\right)} + 7 \tag{3}$$

> $plot\left(\mathrm{e}^{RootOf\left(2\_Z + 30\,t - 2\ln\left(\frac{5}{2}\right) - 2\,\mathrm{I}\,\pi - \ln(500) + \ln\left(\frac{(\mathrm{e}^{\_Z} + 5)^3}{(3 + \mathrm{e}^{\_Z})^5}\right)\right)} + 7, t = 0..5\right)$

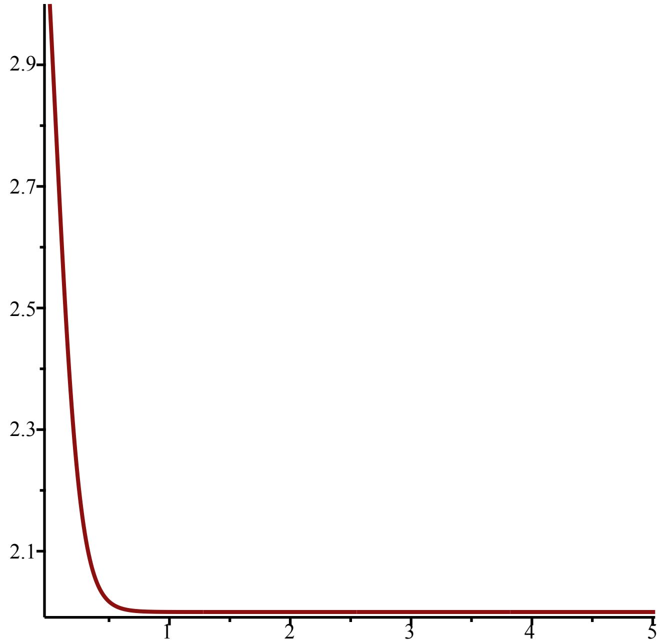


> *#ii)*

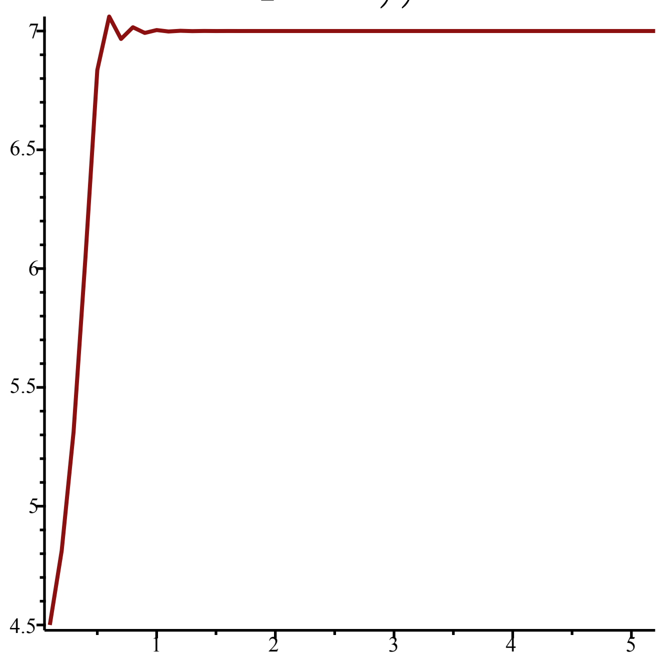
> $plot(Dis1((4-y)\cdot(2-y)\cdot(7-y), y, 3, 0.1, 5))$



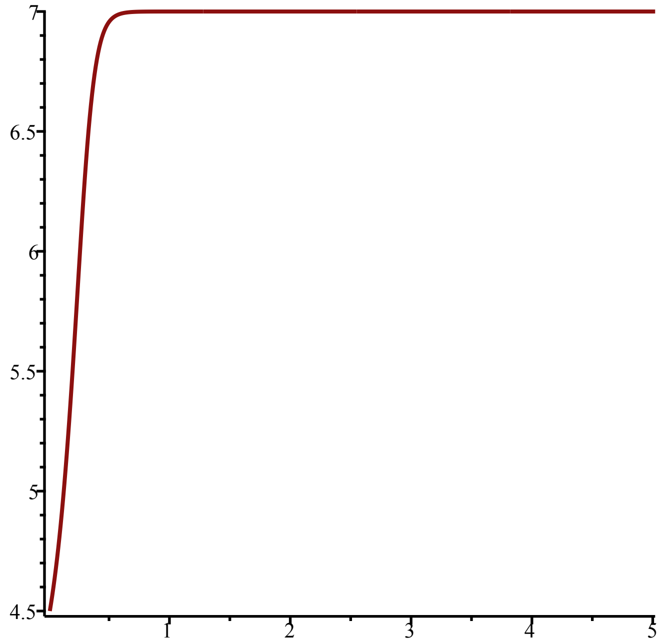
> $plot(Dis1((4-y)\cdot(2-y)\cdot(7-y), y, 3, 0.01, 5))$



> $plot\left(Dis1\left((4-y)\cdot(2-y)\cdot(7-y), y, \frac{9}{2}, 0.1, 5\right)\right)$



> $plot\left(Dis1\left((4-y)\cdot(2-y)\cdot(7-y), y, \frac{9}{2}, 0.01, 5\right)\right)$



> #the smaller the step sizes (h), the more accurate the discretization is to the continuous solution

> #2)

> #ii)

> $ToSys\left(4, z, \frac{z[1] + 2\cdot z[2] + 3\cdot z[3] + 11\cdot z[4]}{z[1] + z[3]}, [1, 5, 5, 2]\right)$

$$\left[\frac{z_1 + 2\,z_2 + 3\,z_3 + 11\,z_4}{z_1 + z_3}, z_1, z_2, z_3\right] \tag{4}$$

> #3)

> $Help11(\ )$

SFPe(f,x), Orbk(k,z,f,INI,K1,K2) **(5)**

> $convert(Orbk(2, z, (1 - z[1])\cdot(1 - z[2]), [2.5, 2.7], 1000, 1010), set)$

$$\{0.3819660112, 0.3819660113\} \tag{6}$$

> #Fixed point at 0.3819660112. There are two values in the set above due to rounding.

> $convert(Orbk(2, z, (1 - z[1])\cdot(1 - z[2]), [2.4, 2.6], 1000, 1010), set)$

$$\{0.3819660112, 0.3819660113\} \tag{7}$$

> #Stable fixed point at 0.3819660112. Small changes in initial conditions still result in the same fixed point, therefore, it is stable.

> $f := ToSys(2, z, (1 - z[1])\cdot(1 - z[2]), [2.5, 2.7])$

$$f := \left[\left(1 - z_1\right)\left(1 - z_2\right), z_1\right] \tag{8}$$

> $SFP2(f, z[1], z[2])$

$$[[0.3819660113, 0.3819660113]] \tag{9}$$

> #This shows the stable fixed point as being of R^2, when it is really of R^1. This is because we **use** *ToSys*( ) **to** *describe the second − order recurrence* **in** *terms of a system* (*two different sequences of first − order*) . *This result is showing the solutions* **to** *the system of the two different sequences*, *when* **in** *fact the sequences are derived* **from** *the same sequence*, *just offset* **in** *values of n*. *Therefore*,

this needs **to** *be interpreted as the stable fixed point* = 0.3819660113.

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Julian Herman, 10/25/21, Assignment 15

2) i) $$x(n) = \frac{x(n-1) + 2x(n-2) + 3x(n-3) + 11x(n-4)}{x(n-1) + x(n-3)}$$

Let: $x_1(n) = x(n)$

$x_2(n) = x_1(n-1)$

$x_3(n) = x_2(n-1) = x_1(n-2)$

$x_4(n) = x_3(n-1) = x_2(n-2) = x_1(n-3)$

$$x_1(n) = \frac{x_1(n-1) + 2 \cdot x_2(n-1) + 3 \cdot x_3(n-1) + 11 \cdot x_4(n-1)}{x_1(n-1) + x_3(n-1)}$$

where:

$x_1(0) = 1$

$x_1(1) = 5 \Rightarrow x_2(1-1=0) = 5$

$x_1(2) = 5 \Rightarrow x_3(2-2=0) = 5$

$x_1(3) = 2 \Rightarrow x_4(3-3=0) = 2$

ii) refer to PDF: $k=4$, $f = \dfrac{z[1] + 2z[2] + 3z[3] + 11z[4]}{z[1] + z[3]}$

$INI = [1, 5, 5, 2]$

The output: $\left[\dfrac{z_1 + 2z_2 + 3z_3 + 4z_4}{z_1 + z_3}, z_1, z_2, z_3\right]$

is the transformation in terms of $(n-1)$:

$$\left[z_1 = \dots,\ z_2 = \dots,\ z_3 = \dots,\ z_4 = \dots\right]$$

4) MATING-TABLE:

			Fathers		
Genotype			AA	Aa	aa
		Frequency %	u	v	w
Mothers	AA	u	u^2	uv	uw
	Aa	v	vu	v^2	vw
	aa	w	wu	wv	w^2

♀ Mother \ ♂ Father	AA	Aa	aa
AA	Frequency: u^2 AA: u^2 | Aa: 0 | aa: 0	Frequency: uv AA: $\frac{uv}{2}$ | Aa: $\frac{uv}{2}$ | aa: 0	Frequency: uw AA: 0 | Aa: uw | aa: 0
Aa	Frequency: vu AA: $\frac{vu}{2}$ | Aa: $\frac{vu}{2}$ | aa: 0	Frequency: v^2 AA: $\frac{v^2}{4}$ | Aa: $\frac{v^2}{2}$ | aa: $\frac{v^2}{4}$	Frequency: vw AA: 0 | Aa: $\frac{vw}{2}$ | aa: $\frac{vw}{2}$
aa	Frequency: wu AA: 0 | Aa: wu | aa: 0	Frequency: wv AA: 0 | Aa: $\frac{wv}{2}$ | aa: $\frac{wv}{2}$	Frequency: w^2 AA: 0 | Aa: 0 | aa: w^2

Frequency of the parents having the specified genotypes mating

Frequency of the offspring of those parents having the specified genotypes

$$\sum AA's = u^2 + \frac{uv}{2} + 0 + \frac{vu}{2} + \frac{v^2}{4} + 0 + 0 + 0 + 0$$

Frequency of AA's in next generation $= U_{n+1} = U_n^2 + U_n V_n + \frac{V_n^2}{4}$

$$\sum Aa's = 0 + \frac{uv}{2} + uw + \frac{vu}{2} + \frac{v^2}{2} + \frac{vw}{2} + wu + \frac{wv}{2} + 0$$

Frequency of Aa's in next generation $= V_{n+1} = U_n V_n + 2U_n W_n + \frac{V_n^2}{2} + V_n W_n$

$$\sum aa's = 0 + 0 + 0 + 0 + \frac{v^2}{4} + \frac{vw}{2} + 0 + \frac{wv}{2} + w^2$$

Frequency of aa's in next generation $= W_{n+1} = \frac{V_n^2}{4} + V_n W_n + W_n^2$

This yields the dynamical system representing the n^{th} generation allele frequencies:

$$\begin{cases} u_{n+1} = u_n^2 + u_n v_n + \frac{1}{4} v_n^2 \\ v_{n+1} = u_n v_n + 2 u_n w_n + \frac{1}{2} v_n^2 + v_n w_n \\ w_{n+1} = \frac{1}{4} v_n^2 + v_n w_n + w_n^2 \end{cases}$$