OK to post

Anne Somalwar, hw14, 10/21/2021

1.

$$f(x) = x + y^4 - 1/16$$

$$g(x) = x^2 + y - 1/9$$

$$x = x + y^4 - 1/16$$

$$y = x^2 + y - 1/9$$

$$y^4 = 1/16$$

$$x^2 = 1/9$$

$$y = \pm 1/2$$

$$x = \pm 1/3$$

$$\boxed{\text{FIXED PTS: } (1/3, 1/2),\ (1/3, -1/2),\ (-1/3, 1/2),\ (-1/3, -1/2)}$$

$$J = \begin{pmatrix} f_x & f_y \\ g_x & g_y \end{pmatrix} = \begin{pmatrix} 1 & 4y^3 \\ 2x & 1 \end{pmatrix}$$

@ $(1/3, 1/2)$

$$J = \begin{pmatrix} 1 & 1/2 \\ 2/3 & 1 \end{pmatrix}$$

$$\Rightarrow (1-\lambda)(1-\lambda) - 1/3 = 0$$

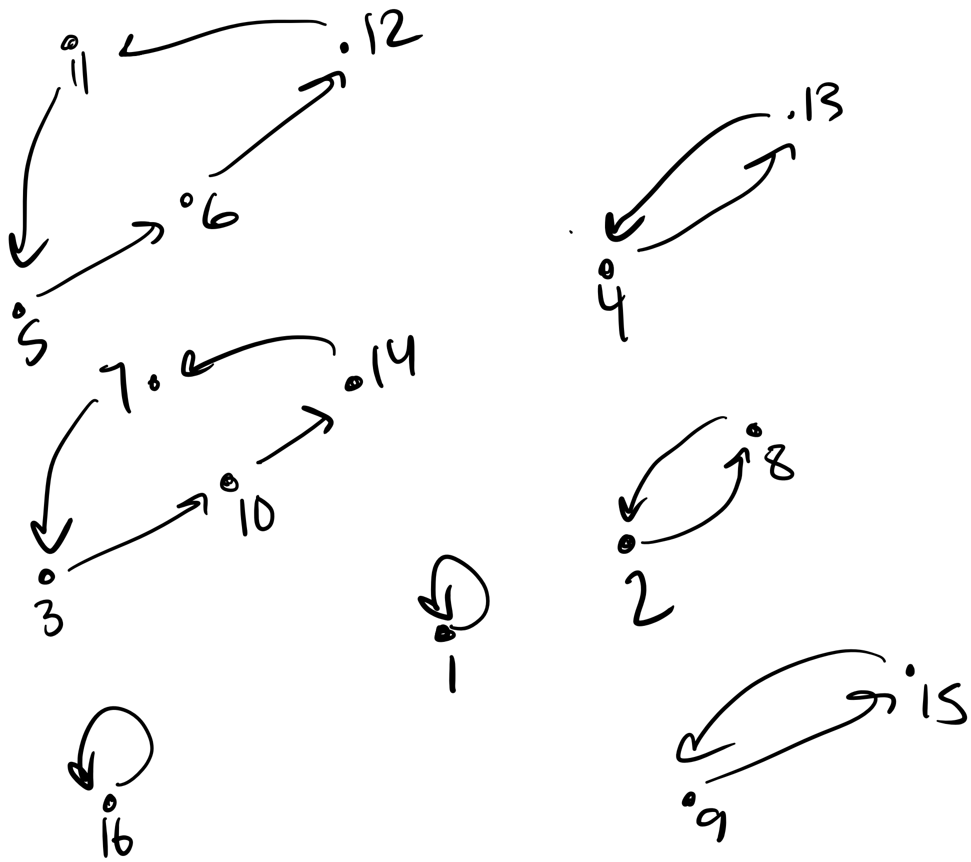
$$\lambda^2 - 2\lambda + 2/3 = 0$$

$$\frac{2 \pm \sqrt{4 - 4(\frac{2}{3})}}{2} = 1 \pm \sqrt{1/3}$$

$1+\sqrt{1/3} > 1$, so $(1/3, 1/2)$ is not stable.

I'd do it similarly for the other fixed pts.

3.



Period 1 : [1] , [16]

Period 2 : [9, 15] , [2, 8] , [4, 13] ,

Period 4 : [3, 10, 14, 7] , [5, 6, 12, 11]

Trajectories

[1, 1]

[2, 8, 2]

[3, 10, 14, 7, 3]

[4, 13, 4]

[5, 6, 12, 11, 5]

[6, 12, 11, 5, 6]

[7, 3, 10, 14, 7]

[8, 2, 8]

[9, 15, 9]

[10, 14, 7, 3, 10]

[11, 5, 6, 12, 11]

[12, 11, 5, 6, 12]

[13, 4, 13]

[14, 7, 3, 10, 14]

[15, 9, 15]

[16, 16]

3) (i) $\overset{29}{\overline{T_2}}(29) = L(29) - S(29) = 92 - 29 = 63$

$T_2(63) = 63 - 36 = 27$

$T_2(27) = 72 - 27 = 45$

$T_2(45) = 54 - 45 = 9$

$T_2(09) = 90 - 9 = 81$

$T_2(81) = 81 - 18 = 63 \rightarrow$ repeat

$\boxed{\text{cycle} \Rightarrow [63, 27, 45, 9, 81, 63]}$

$\overset{12}{\overline{T_2}}(12) = 21 - 12 = 9 \rightarrow 81 \rightarrow 63$

$\Rightarrow$ same as last cycle

$\Rightarrow [63, 27, 45, 9, 81, 63]$

$\underline{14}$

$T_2(14) = 41 - 14 = 27 \to 45 \to 9 \to 81 \to 63$

$\Rightarrow$ same as last cycle

$\Rightarrow [63, 27, 45, 9, 81, 63]$

$\underline{31}$

$T_2(31) = 31 - 13 = 18$

$T_2(18) = 81 - 18 = 63$

$\Rightarrow$ same as last cycle

$\Rightarrow [63, 27, 45, 9, 81, 63]$

51

$T_2(51) = 51 - 15 = 36$

$T_2(36) = 63 - 36 = 27 \to 45 \to 9 \to 81 \to 63$

$\Rightarrow$ same as last cycle

$\Rightarrow [63, 27, 45, 9, 81, 63]$

$\underline{19}$

$T_2(19) = 91 - 19 = 72$

$T_2(72) = 72 - 27 = 45 \to 9 \to 81 \to 63$

$\Rightarrow$ same as last cycle

$\Rightarrow [63, 27, 45, 9, 81, 63]$

$\underline{23}$

$T_2(23) = 32 - 23 = 9 \to 81 \to 63$

$\Rightarrow$ same as last cycle

$\Rightarrow [63, 27, 45, 9, 81, 63]$

98

$T_2(97) = 97 - 79 = 18$

$T_2(18) = 81 - 18 = 63$

$\Rightarrow$ same as last cycle

$\Rightarrow [63, 27, 45, 9, 81, 63]$

96

$T_2(96) = 96 - 69 = 27 \to 45 \to 9 \to 81 \to 63$

$\Rightarrow$ same as last cycle

$\Rightarrow [63, 27, 45, 9, 81, 63]$

61

$T_2(61) = 61 - 16 = 45 \to 9 \to 81 \to 63$

$\Rightarrow$ same as last cycle

$\Rightarrow [63, 27, 45, 9, 81, 63]$

(ii) 364

$T_3(364) = 643 - 346 = 297$

$T_3(297) = 972 - 279 = 639$

$T_3(639) = 963 - 369 = 594$

$T_3(594) = 954 - 459 = 495$

$T_3(495) = 954 - 459 = 495$

$\Rightarrow$ cycle: [495, 495]

421

$T_3(421) = 421 - 124 = 297$

$T_4(297) = 972 - 279 = 693$

$T_4(693) = T_4(639) \rightarrow 594 \rightarrow 495$

$\Rightarrow$ same as last cycle

[495, 495]

$\underline{672}$

$T_3(672) = 762 - 267 = 495$

$\Rightarrow$ same as last cycle

$[495, 495]$

$\underline{983}$

$T_3(983) = 983 - 389 = 594$

$T_3(594) = T_3(495)$

$\Rightarrow$ same as last cycle

$[495, 495]$

$\underline{137}$

$T_3(137) = 731 - 137 = 594 \rightarrow 495$

$\Rightarrow$ same as last cycle

$[495, 495]$

(iii)

$\underline{1246}$

$T_4(1246) = 6421 - 1246 = 5175$

$T_4(5175) = 7551 - 1557 = 5994$

$T_4(5994) = 9954 - 4599 = 5355$

$T_4(5355) = 5553 - 3555 = 1998$

$T_4(1998) = 9981 - 1899 = 8082$

$T_4(8082) = 8820 - 288 = 8532$

$T_4(8532) = 8532 - 2358 = 6174$

$T_4(6174) = 7641 - 1467 = 6174$

$\Rightarrow$ cycle: [6174, 6174]

<u>3564</u>

$T_4(3564) = 6543 - 3456 = 3087$

$T_4(3087) = 8730 - 378 = 8352$

$T_4(8352) = 8532 - 2358 = 6174$

$\Rightarrow$ same as last cycle

[6174, 6174]

9884

$T_4(9884) = 9884 - 4889 = 4995$

$T_4(4995) = 9954 - 4599 = 5355$

$T_4(5355) = 5553 - 3555 = 1998$

$\to 8082 \to 8532 \to 6174$

$\Rightarrow$ same as last cycle

$[6174, 6174]$

4) See Maple Code

5)

$\underline{32}$

$32 \to 16 \to 8 \to 4 \to 2 \to 1 \to 2$

$\boxed{\Rightarrow \text{cycle } [1,2]}$

$\underline{17}$

$17 \to 26 \to 13 \to 20$

$\to 10 \to 5 \to 8 \to 4 \to 2$

$\boxed{\Rightarrow \text{cycle } [1,2]}$

$\underline{23}$

$23 \to 35 \to 53 \to 80 \to 40 \to 20$

$\to 10 \to 5 \to 8 \to 4 \to 2$

$\Rightarrow$ cycle [1, 2]

7

7 → 11 → 17 → 26 → 13 → 20
→ 10 → 5 → 8 → 4 → 2

$\Rightarrow$ cycle [1, 2]

18

18 → 9 → 14 → 7
→ 11 → 17 → 26 → 13 → 20
→ 10 → 5 → 8 → 4 → 2

$\Rightarrow$ cycle [1, 2]

6) I guess that [1, 2, 1] is the only cycle because, if you start at a point x, if x is odd, it maps approximately to $\frac{3}{2}x$. If x is even, it maps to $\frac{1}{2}x$. Thus, since there will be approximately $\frac{1}{2}n$ odds and $\frac{1}{2}n$ evens in n iterations, x will map to $\sim \left(\frac{3}{2}\right)^{\frac{1}{2}n}\left(\frac{1}{2}\right)^{\frac{1}{2}n} x = \left(\frac{3}{4}\right)^{\frac{1}{2}n} x$ which deteriorates as n gets large. Thus, everything will fall back down to [1, 2, 1].