HW 14

#1

$x(n) = x(n-1) + y(n-1)^4 - 1/16$

$y(n) = x(n-1)^2 + y(n-1) - 1/9$

$f(x,y) = x + y^4 - 1/16$

$g(x,y) = x^2 + y - 1/9$

$J = \begin{bmatrix} 1 & 4y^3 \\ 2x & 1 \end{bmatrix}$

$x = x + y^4 - 1/16$ $\quad y^4 = 1/16$ $\quad y = \pm 1/2$

$y = x^2 + y - 1/9$ $\quad x^2 = 1/9$ $\quad x = +/- 1/3$

$(-1/3, -1/2) = p1$

$(-1/3, 1/2) = p2$

$(1/3, 1/2) = p3$

$(1/3, -1/2) = p4$

$J_{P1} = \begin{bmatrix} 1 & -1/2 \\ -2/3 & 1 \end{bmatrix}$ $\quad (1-\lambda)^2 + \frac{1}{3} = 0$ $\quad \lambda = \frac{3 \pm \sqrt{3}}{3}$

$(1-\lambda) = \sqrt{1/3}\, i$

$\lambda = \frac{3+\sqrt{3}}{3} > 1$, not stable!

$J_{P2} = \begin{bmatrix} 1 & 1/2 \\ -2/3 & 1 \end{bmatrix}$ $\quad (1-\lambda^2) + 1/3 = 0$ $\quad \lambda = 1 \pm i\sqrt{3}/3$ Not stable

$J_{P3} = \begin{bmatrix} 1 & 1/2 \\ 2/3 & 1 \end{bmatrix}$ same eigenvalues as J_{P1}, not stable

$J_{P4} = \begin{bmatrix} 1 & -1/2 \\ 2/3 & 1 \end{bmatrix}$ same eigenvalues as J_{P2}, not stable

2

$x \to x^3 \mod 17$

[1,1] [5,6,12,11,5] [8,2,8]

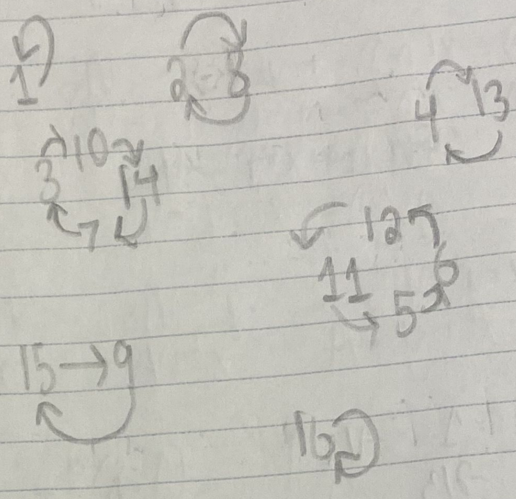
[2,8,2]

[3,9,15,9] [6,12,11,5,6] [9,15,9]

[4,13,4] [7,3,9,15,9] [10,14,7,3,9,15,9]

2. ~~mo~~ $x \to x^3 \bmod 17$

[1,1]
[2,8,2]
[3,10,14,7,3]
[4,13,4]
[5,6,12,11,5]
[6,12,11,5,6]
[7,3,10,14,7]
[8,2,8]
[9,15,9]
[10,14,7,3,10]
[11,5,6,12,11]
[12,11,5,6,12]
[13,4,13]
[14,7,3,10,14]
[15,9,15]
[16,16]



3. Random 2 Digit #'s [15,31, 26, 45, 93, 72, 41, 69, 25, 12]

i #1 51-15 83-36 72=27 54 45 90-09 81-18

[15, 36, 27, 45, 09, 81, 63, 27]

#2

[31, 18, 63, 27, 45, 09, 81, 63]

#3 [26, 36, 27, 45, 9, 81, 63, 27]
#4 [45, 09, 81, 63, 27, 45]
#5 [93, 54, 09, 81, 63, 27, 45, 09]
#6 [72, 45, 09, 81, 63, 27, 45]
#7 [41, 27, 45, 9, 81, 63, 27]
#8 [69, 27, 45, 9, 81, 63, 27]
#9 [25, 27, 45, 9, 81, 63, 27]
#10 [12, 9, 81, 63, 27, 45, 9]

ii Random 3 Digits [412, 362, 420, 812, 069]

421-124 972-279 963-369 954-459 954-459

#1 [412, 297, 693, 594, 495, 495]

632-236 963-369

#2 [362, 396, 594, 495, 495]

420-24

#3 [420, 396, 594, 495, 495]

821-128

#4 [812, 693 594, 495, 495]

960-069 981-189

#5 [069, 891, 792, 693, 594, 495 495]

iii 3 random 4 Digits [1234, 8210, 2412]

4321-1234 8730-0378, 8532-2358, 7641-1467

#1 [1234, 3087, 8352, 6174, 6174]

8210-0128 8820-0288

#2 [8210, 8082, 8532, 6174, 6174]

4221-1224 9972-2799 7731-1377 6543-3456

#3 [2412, 2997, 7173, 6354, 3087, 8352, 6174, 6174]

#4

The only fixed point for $T_3(n) = 495$

a " for $T_4(n) = 6174$

$n \to \frac{n}{2}$ if n is even $\qquad n \to \frac{3n+1}{2}$ if n is odd

5 random #'s $\qquad$ 2, 42, 9, 21, 469

n=2 [2, 1, 2]

n=42 [42, 21, 64, 32, 16, 8, 4, 2, 1]

n=9 [9, 14, 7, 11, 17, 26, 13, 20, 10, 5, 8, 4, 2, 1, 2]

n=21 [21, 32, 16, 8, 4, 2, 1, 2]

n=464 [464, 1408, 704, 352, 176, 88, 44, 22, 11, 17, 26, 13, 20, 10, 5, 8, 4, 2, 1, 2]

6. The only periodic orbit is [2, 1, 2]

$$f(n) = \begin{cases} \frac{n}{2} & \text{if } n \equiv 0 \bmod 2 \\ \frac{3n+1}{2} & \text{if } n \equiv 1 \bmod 2 \end{cases}$$

if n is even

$\frac{n}{2}$ will result in an odd number when the digit become 2, 6, 0

if n is odd $\frac{3n+1}{2}$ results in an even number over time

Since if both processes divide by 2, over time every number will approach $n = 2^k$, $k \leq 3$ which simplifies to 2 by $\frac{n}{2}$, and then 1, and oscillates as that cycle.