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> #OK to post homework
  #Shreya Ghosh, 10-25-2021, Assignment 14
>
> read "/Users/shreyaghosh/Documents/M14.txt"
> Help14( )
                                     RevOp(n,k), RevOpTr(n,k)                (1)
> #4.
  RevOpTr(327, 3)
                                     [327, 495, 495]                    (2)
> RevOpTr(4567, 4)
                                     [4567, 3087, 8352, 6174, 6174]    (3)
> RevOpTr(444, 3)
                                     [444, 0, 0]                       (4)
> RevOpTr(5874, 4)
                                     [5874, 4176, 6174, 6174]          (5)
> #They are all fixed points. For T3 they are 495 and 0. For T4 they are 6174 and 0

```

```

> #5.
  rand( )
                                     395718860534                        (6)
> rand( )
                                     193139816415                        (7)
> rand( )
                                     22424170465                       (8)
> rand( )
                                     800187484459                       (9)
> rand( )
                                     427552056869                       (10)
> CollatzConj := proc(n) local l :
  if (type(n, even)) then
    l :=  $\frac{n}{2}$ 
  else
    l :=  $\frac{3 \cdot n + 1}{2}$ 
  fi:
end:
> CollatzConj(5)
                                     8                               (11)
> CollatzConjTr := proc(n) local L, n1 :

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```

L := [ ]:
n1 := n:
while not member(n1, L) do
L := [op(L), n1]:
n1 := CollatzConj(n1):
od:
L:
end:

```

> CollatzConjTr(395718860534)

[395718860534, 197859430267, 296789145401, 445183718102, 222591859051, 333887788577, (12)

500831682866, 250415841433, 375623762150, 187811881075, 281717821613,
422576732420, 211288366210, 105644183105, 158466274658, 79233137329,
118849705994, 59424852997, 89137279496, 44568639748, 22284319874, 11142159937,
16713239906, 8356619953, 12534929930, 6267464965, 9401197448, 4700598724,
2350299362, 1175149681, 1762724522, 881362261, 1322043392, 661021696, 330510848,
165255424, 82627712, 41313856, 20656928, 10328464, 5164232, 2582116, 1291058,
645529, 968294, 484147, 726221, 1089332, 544666, 272333, 408500, 204250, 102125,
153188, 76594, 38297, 57446, 28723, 43085, 64628, 32314, 16157, 24236, 12118, 6059,
9089, 13634, 6817, 10226, 5113, 7670, 3835, 5753, 8630, 4315, 6473, 9710, 4855, 7283,
10925, 16388, 8194, 4097, 6146, 3073, 4610, 2305, 3458, 1729, 2594, 1297, 1946, 973, 1460,
730, 365, 548, 274, 137, 206, 103, 155, 233, 350, 175, 263, 395, 593, 890, 445, 668, 334, 167,
251, 377, 566, 283, 425, 638, 319, 479, 719, 1079, 1619, 2429, 3644, 1822, 911, 1367, 2051,
3077, 4616, 2308, 1154, 577, 866, 433, 650, 325, 488, 244, 122, 61, 92, 46, 23, 35, 53, 80, 40,
20, 10, 5, 8, 4, 2, 1]

> CollatzConjTr(193139816415)

[193139816415, 289709724623, 434564586935, 651846880403, 977770320605, (13)

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12402867658, 6201433829, 9302150744, 4651075372, 2325537686, 1162768843,
1744153265, 2616229898, 1308114949, 1962172424, 981086212, 490543106, 245271553,
367907330, 183953665, 275930498, 137965249, 206947874, 103473937, 155210906,
77605453, 116408180, 58204090, 29102045, 43653068, 21826534, 10913267, 16369901,
24554852, 12277426, 6138713, 9208070, 4604035, 6906053, 10359080, 5179540, 2589770,
1294885, 1942328, 971164, 485582, 242791, 364187, 546281, 819422, 409711, 614567,
921851, 1382777, 2074166, 1037083, 1555625, 2333438, 1166719, 1750079, 2625119,

3937679, 5906519, 8859779, 13289669, 19934504, 9967252, 4983626, 2491813, 3737720, 1868860, 934430, 467215, 700823, 1051235, 1576853, 2365280, 1182640, 591320, 295660, 147830, 73915, 110873, 166310, 83155, 124733, 187100, 93550, 46775, 70163, 105245, 157868, 78934, 39467, 59201, 88802, 44401, 66602, 33301, 49952, 24976, 12488, 6244, 3122, 1561, 2342, 1171, 1757, 2636, 1318, 659, 989, 1484, 742, 371, 557, 836, 418, 209, 314, 157, 236, 118, 59, 89, 134, 67, 101, 152, 76, 38, 19, 29, 44, 22, 11, 17, 26, 13, 20, 10, 5, 8, 4, 2, 1]

> *CollatzConjTr*(22424170465)

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(14)

4123172, 2061586, 1030793, 1546190, 773095, 1159643, 1739465, 2609198, 1304599,
1956899, 2935349, 4403024, 2201512, 1100756, 550378, 275189, 412784, 206392, 103196,
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154, 77, 116, 58, 29, 44, 22, 11, 17, 26, 13, 20, 10, 5, 8, 4, 2, 1]

> *CollatzConjTr*(800187484459)

[800187484459, 1200281226689, 1800421840034, 900210920017, 1350316380026, (15)
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41914, 20957, 31436, 15718, 7859, 11789, 17684, 8842, 4421, 6632, 3316, 1658, 829, 1244,
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425, 638, 319, 479, 719, 1079, 1619, 2429, 3644, 1822, 911, 1367, 2051, 3077, 4616, 2308,
1154, 577, 866, 433, 650, 325, 488, 244, 122, 61, 92, 46, 23, 35, 53, 80, 40, 20, 10, 5, 8, 4, 2,
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> *CollatzConjTr*(427552056869)

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43387461998, 21693730999, 32540596499, 48810894749, 73216342124, 36608171062,
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87958817270, 43979408635, 65969112953, 98953669430, 49476834715, 74215252073,
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140893017610, 70446508805, 105669763208, 52834881604, 26417440802, 13208720401,
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1914992, 957496, 478748, 239374, 119687, 179531, 269297, 403946, 201973, 302960,
151480, 75740, 37870, 18935, 28403, 42605, 63908, 31954, 15977, 23966, 11983, 17975,
26963, 40445, 60668, 30334, 15167, 22751, 34127, 51191, 76787, 115181, 172772, 86386,
43193, 64790, 32395, 48593, 72890, 36445, 54668, 27334, 13667, 20501, 30752, 15376,
7688, 3844, 1922, 961, 1442, 721, 1082, 541, 812, 406, 203, 305, 458, 229, 344, 172, 86, 43,
65, 98, 49, 74, 37, 56, 28, 14, 7, 11, 17, 26, 13, 20, 10, 5, 8, 4, 2, 1]

> *CollatzConjTr*(76)

[76, 38, 19, 29, 44, 22, 11, 17, 26, 13, 20, 10, 5, 8, 4, 2, 1]

(17)

> #*The trajectory always ends at 1*

HW 14

$$1. x(n) = x(n-1) + y(n-1)^4 - \frac{1}{16}$$

$$y(n) = x(n-1)^2 + y(n-1) - \frac{1}{9}$$

$$x = x + y^4 - \frac{1}{16}$$

$$y = x^2 + y - \frac{1}{9}$$

$$0 = y^4 - \frac{1}{16}$$

$$0 = x^2 - \frac{1}{9}$$

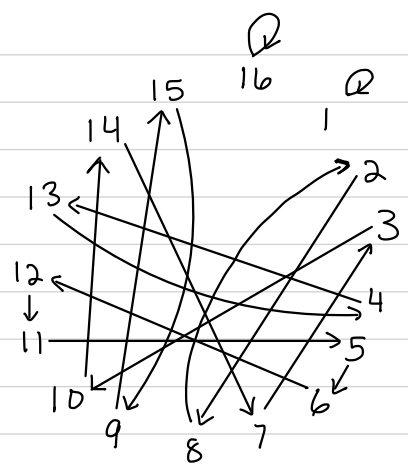
$$0 = (y^2 + \frac{1}{4})(y + \frac{1}{2})(y - \frac{1}{2}) \quad 0 = (x + \frac{1}{3})(x - \frac{1}{3})$$

$$(\frac{1}{3}, \frac{1}{2}) \quad (\frac{1}{3}, -\frac{1}{2}) \quad (-\frac{1}{3}, \frac{1}{2}) \quad (-\frac{1}{3}, -\frac{1}{2})$$

$$\text{Jac} = \begin{pmatrix} 1 & 4y^3 \\ 2x & 1 \end{pmatrix}$$

$$\text{Jac}(\frac{1}{3}, \frac{1}{2}) = \begin{pmatrix} 1 & \frac{1}{2} \\ \frac{2}{3} & 1 \end{pmatrix} \Rightarrow 0 = (1-\lambda)^2 - \frac{1}{3} = 1 - 2\lambda + \lambda^2 - \frac{1}{3} = \lambda^2 - 2\lambda + \frac{2}{3}$$
$$\frac{2 \pm \sqrt{4 - 4(\frac{2}{3})}}{2} = \frac{2 \pm \sqrt{4 - \frac{8}{3}}}{2} = \frac{2 \pm \sqrt{\frac{4}{3}}}{2} \Rightarrow \frac{2 + \sqrt{\frac{4}{3}}}{2} > 1 \text{ so not stable}$$

x	$x^2 \pmod{17}$
1	1
2	8
3	10
4	13
5	6
6	12
7	3
8	2
9	15
10	14
11	5
12	11
13	4
14	7
15	9
16	16



Trajectories:

[1, 1] [2, 8, 2] [3, 10, 14, 7, 3] [4, 13, 4]

[5, 6, 12, 11, 5] [6, 12, 11, 5, 6] [7, 3, 10, 14, 7]

[8, 2, 8] [9, 15, 9] [10, 14, 7, 3, 10] [11, 5, 6, 12, 11]

[12, 11, 5, 6, 12] [13, 4, 13] [14, 7, 3, 10, 14] [15, 9, 15]

[16, 16]

$$3. T_k(n) = L(n) - S(n)$$

$$i) 75, 98, 62, 47, 37, 55, 46, 67, 41, 40$$

$$T_2(75) = 75 - 57 = 18 \rightarrow 81 - 18 = 63 \rightarrow 63 - 36 = 27 \rightarrow 72 - 27 = 45 \rightarrow$$

$$54 - 45 = 9 \rightarrow 90 - 9 = 81 \rightarrow 81 - 18 = 63 \dots$$

$$[75, 18, 63, 27, 45, 9, 81, 63]$$

$$T_2(98) = 98 - 89 = 9 \rightarrow 90 - 9 = 81 \rightarrow 81 - 18 = 63 \dots$$

$$[98, 9, 81, 63, 27, 45, 9, 81]$$

$$T_2(62) = 62 - 26 = 36 \rightarrow 63 - 36 = 27 \dots$$

$$[62, 36, 27, 45, 9, 81, 63, 27]$$

$$T_2(47) = 74 - 47 = 27 \rightarrow 72 - 27 = 45 \dots$$

$$[47, 27, 45, 9, 81, 63, 27]$$

$$T_2(37) = 73 - 37 = 36 \rightarrow 63 - 36 = 27 \dots$$

$$[37, 36, 27, 45, 9, 81, 63, 27]$$

$$T_2(55) = 55 - 55 = 0 \rightarrow 0 - 0 = 0$$

$$[55, 0, 0 \dots]$$

$$T_2(46) = 64 - 46 = 18 \rightarrow 81 - 18 = 63 \dots$$

$$[46, 18, 63, 27, 45, 9, 81, 63]$$

$$T_2(67) = 76 - 67 = 9 \rightarrow 90 - 9 = 81 \dots$$

$$[67, 9, 81, 63, 27, 45, 9]$$

$$T_2(41) = 41 - 14 = 27 \dots$$

$$[41, 27, 45, 9, 81, 63, 27]$$

$$T_2(40) = 40 - 4 = 36 \rightarrow 63 - 36 = 27 \dots$$

$$[40, 36, 27, 45, 9, 81, 63, 27]$$

3. ii) 273, 958, 705, 211, 962

$$T_3(273) = 732 - 237 = 495 \rightarrow 954 - 459 = 495 \rightarrow 954 - 459 = 495$$

[273, 495]

$$T_3(958) = 985 - 589 = 396 \rightarrow 963 - 369 = 594 \rightarrow 954 - 459 = 495$$

[958, 396, 594, 495]

$$T_3(705) = 750 - 057 = 693 \rightarrow 963 - 369 = 594 \rightarrow 954 - 459 = 495$$

[705, 693, 594, 495]

$$T_3(211) = 211 - 112 = 99 \rightarrow 990 - 099 = 891 \rightarrow 981 - 189 = 792 \rightarrow 972 - 279 = 693$$

[211, 99, 891, 792, 693, 594, 495]

$$T_3(962) = 962 - 269 = 693$$

[962, 693, 594, 495]

iii) 9640, 3932, 3257

$$T_4(9640) = 9640 - 0469 = 9171 \rightarrow 9711 - 1179 = 8532 \rightarrow 8532 - 2358 = 6174 \rightarrow 7641 - 1467 = 6174$$

[9640, 9171, 8532, 6174]

$$T_4(3932) = 9332 - 2339 = 6993 \rightarrow 9963 - 3699 = 6264 \rightarrow 6642 - 2466 = 4176 \rightarrow 7641 - 1467 = 6174 \rightarrow 7641 - 1467 = 6174$$

[3932, 6993, 6264, 4176, 6174]

$$T_4(3257) = 7532 - 2357 = 5175 \rightarrow 7551 - 1557 = 5994 \rightarrow 9954 - 4599 = 5355 \rightarrow 5553 - 3555 = 1998 \rightarrow 9981 - 1899 = 8082 \rightarrow 8820 - 0288 = 8532 \rightarrow$$

$$8532 - 2358 = 6174$$

[3257, 5175, 5994, 5355, 1998, 8082, 8532, 6174]