Dynamical Modeling HW 14 - Okay to Post

1) $f(x,y) = x + y^4 - \frac{1}{16}$

$g(x,y) = x^2 + y - \frac{1}{9}$

$x = \cancel{x} + y^4 - \frac{1}{16}$

$\sqrt[4]{y^4} = \sqrt[4]{\frac{1}{16}} \Rightarrow y = \pm\frac{1}{2}$

$\cancel{y} = x^2 + \cancel{y} - \frac{1}{9}$

$\sqrt{x^2} = \sqrt{\frac{1}{9}} \Rightarrow x = \pm\frac{1}{3}$

F.P.: $(\frac{1}{2}, \frac{1}{3}), (\frac{-1}{2}, \frac{1}{3}), (\frac{1}{2}, \frac{-1}{3}) (\frac{-1}{2}, \frac{-1}{3})$

Jacobian: $\begin{bmatrix} f_x(x_0,y_0) & f_y(x_0,y_0) \\ g_x(x_0,y_0) & g_y(x_0,y_0) \end{bmatrix}$

$f_x = 1$ $\quad g_x = 2x$ $\quad$ Jacobian: $\begin{bmatrix} 1 & 4y^3 \\ 2x & 1 \end{bmatrix}$

$f_y = 4y^3$ $\quad g_y = 1$

Point : $(\frac{1}{2}, \frac{1}{3})$

$\begin{bmatrix} 1 & 4(\frac{1}{3})^3 \\ 2(\frac{1}{2}) & 1 \end{bmatrix} = \begin{bmatrix} 1 & \frac{4}{27} \\ 1 & 1 \end{bmatrix} \Rightarrow \det\begin{bmatrix} 1-\lambda & 4/27 \\ 1 & 1-\lambda \end{bmatrix} = (1-\lambda)^2 - \frac{4}{27} = 0$

$\sqrt{(1-\lambda)^2} = \sqrt{\frac{4}{27}}$

$1-\lambda = \pm\frac{2}{3\sqrt{3}}$

$\lambda = 1 \pm \frac{2}{3\sqrt{3}}$

$\boxed{\text{*one of the eigenvalues is } > 1 \; (\lambda = 1 + \frac{2}{3\sqrt{3}}) \text{ so the point } (1/2, 1/3) \text{ is unstable}}$

Point: $(-1/2, 1/3)$

$$\begin{bmatrix} 1 & 4(1/3)^3 \\ 2(-1/2) & 1 \end{bmatrix} = \begin{bmatrix} 1 & 4/27 \\ -1 & 1 \end{bmatrix} \Rightarrow \det \begin{bmatrix} 1-\lambda & 4/27 \\ -1 & 1-\lambda \end{bmatrix} = (1-\lambda)^2 + \frac{4}{27} = 0$$

$$\sqrt{(1-\lambda)^2} = \sqrt{-\frac{4}{27}}$$

$$\lambda = \sqrt{\left(1^2 + \left(\frac{2}{3\sqrt{3}}\right)^2\right)} = 1.07 > 1 \qquad 1-\lambda = \pm\frac{2}{3\sqrt{3}}i$$

$\boxed{\text{The eigenvalues are } >1, \text{ so } (-1/2, 1/3) \text{ is unstable}}$ $\lambda = 1 \pm \frac{2}{3\sqrt{3}}i$

Point: $(1/2, -1/3)$

$$\begin{bmatrix} 1 & 4(-1/3)^3 \\ 2(1/2) & 1 \end{bmatrix} = \begin{bmatrix} 1 & -4/27 \\ 1 & 1 \end{bmatrix} \Rightarrow \det \begin{bmatrix} 1-\lambda & -4/27 \\ 1 & 1-\lambda \end{bmatrix} = (1-\lambda)^2 + 4/27 = 0$$

$$\sqrt{(1-\lambda)^2} = \sqrt{4/27}$$

$$\lambda = \sqrt{(1)^2 + \left(\frac{2}{3\sqrt{3}}\right)^2} = 1.07 > 1 \qquad 1-\lambda = \pm\frac{2}{3\sqrt{3}}i$$

$\boxed{\text{The eigenvalues are } >1, \text{ so } (1/2, -1/3) \text{ is unstable}}$ $\lambda = 1 \pm \frac{2}{3\sqrt{3}}i$

Point: $(-1/2, -1/3)$

$$\begin{bmatrix} 1 & 4(-1/3)^3 \\ 2(-1/2) & 1 \end{bmatrix} = \begin{bmatrix} 1 & -4/27 \\ -1 & 1 \end{bmatrix} \Rightarrow \det \begin{bmatrix} 1-\lambda & -4/27 \\ -1 & 1-\lambda \end{bmatrix} = (1-\lambda)^2 - 4/27 = 0$$

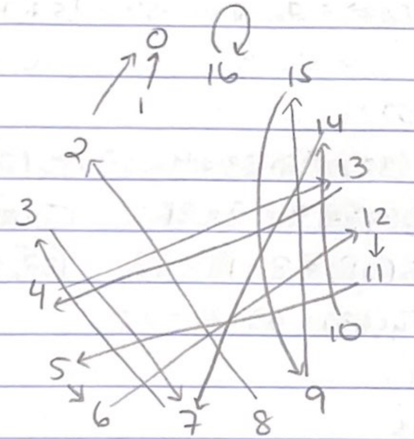
$$\sqrt{(1-\lambda)^2} = \sqrt{4/27}$$

$$1-\lambda = \pm\frac{2}{3\sqrt{3}}$$

$\boxed{\begin{array}{l} * \text{ one of the eigenvalues is } > 1 \\ \left(1+\frac{2}{3\sqrt{3}}\right), \text{ so the point } (-1/2, -1/3) \text{ is unstable} \end{array}}$ $\lambda = 1 \pm \frac{2}{3\sqrt{3}}$

2) $x \to x^3 \bmod 17$

x	$x^3 \bmod 17$
1	0
2	0
3	7
4	13
5	6
6	12
7	3
8	2
9	15
10	14
11	5
12	11
13	4
14	7
15	9
16	16



[1, 1], [2, 8, 2], [3, 7, 3], [4, 13, 4], [5, 6, 12, 11, 5], [6, 12, 11, 5, 6], [7, 3, 7], [8, 2, 0], [9, 15, 9], [10, 14, 7, 3, 7], [11, 5, 6, 12, 11], [12, 11, 5, 6, 12], [13, 4, 13], [14, 7, 3, 7], [15, 9, 15], [16, 16]

3) (i) ① 27

$T_1(27) = 72 - 27 = 45$

$T_2(45) = 54 - 45 = 9$

$T_3(09) = 90 - 9 = 81$

$T_4(81) = 81 - 18 = 63$

$T_5(63) = 63 - 36 = 27$ ✓

[27, 45, 09, 81, 63, 27]

② 42

$T_1(42) = 42 - 24 = 18$

$T_2(18) = 81 - 18 = 63$

$T_3(63) = 63 - 36 = 27$

$T_4(27) = 72 - 27 = 45$

$T_5(45) = 54 - 45 = 9$

$T_6(09) = 90 - 9 = 81$

$T_7(81) = 81 - 18 = 63$ ✓

[42, 18, 63, 27, 45, 9, 81, 63]

③ 57

$T_1(57) = 75 - 57 = 18$

$T_2(18) = 81 - 18 = 63$

$T_3(63) = 63 - 36 = 27$

$T_4(27) = 72 - 27 = 45$

$T_5(45) = 54 - 45 = 9$

$T_6(09) = 90 - 9 = 81$

$T_7(81) = 81 - 18 = 63$ ✓

[57, 18, 63, 27, 45, 9, 81, 63]

④ 32

$T_1(32) = 32 - 23 = 9$

$T_2(9) = 90 - 9 = 81$

$T_3(81) = 81 - 18 = 63$

$T_4(63) = 63 - 36 = 27$

$T_5(27) = 72 - 27 = 45$

$T_6(45) = 54 - 45 = 9$ ✓

[32, 9, 81, 63, 27, 45, 9]

⑤ 53

$T_1(53) = 53 - 35 = 18$

$T_2(18) = 81 - 18 = 63$

$T_3(63) = 63 - 36 = 27$

$T_4(27) = 72 - 27 = 45$

$T_5(45) = 54 - 45 = 9$

$T_6(9) = 90 - 9 = 81$

$T_7(81) = 81 - 18 = 63$ ✓

[53, 18, 63, 27, 45, 9, 81, 63]

⑥ 72

$T_1(72) = 72-27 = 45$ $T_5(63) = 63-36 = 27$

$T_2(45) = 54-45 = 9$ $T_6(27) = 72-27 = 45$ ✓

$T_3(9) = 90-9 = 81$ [72, 45, 9, 81, 63, 27, 45]

$T_4(81) = 81-18 = 63$

⑦ 68

$T_1(68) = 86-68 = 18$ $T_5(45) = 54-45 = 9$

$T_2(18) = 81-18 = 63$ $T_6(9) = 90-9 = 81$

$T_3(63) = 63-36 = 27$ $T_7(81) = 81-18 = 63$ ✓

$T_4(27) = 72-27 = 45$ [68, 18, 63, 27, 45, 9, 81, 63]

⑧ 47

$T_1(47) = 74-47 = 27$ $T_5(81) = 81-18 = 63$

$T_2(27) = 72-27 = 45$ $T_6(63) = 63-36 = 27$ ✓

$T_3(45) = 54-45 = 9$ [47, 27, 45, 9, 81, 63, 27]

$T_4(9) = 90-9 = 81$

⑨ 49

$T_1(49) = 94-49 = 45$ $T_5(63) = 63-36 = 27$

$T_2(45) = 54-45 = 9$ $T_6(27) = 72-27 = 45$ ✓

$T_3(9) = 90-9 = 81$ [49, 45, 9, 81, 63, 27, 45]

$T_4(81) = 81-18 = 63$

⑩ 17

$T_1(17) = 71-17 = 54$ $T_5(63) = 63-36 = 27$

$T_2(54) = 54-45 = 9$ $T_6(27) = 72-27 = 45$

$T_3(9) = 90-9 = 81$ $T_7(45) = 54-45 = 9$ ✓

$T_4(81) = 81-18 = 63$ [17, 54, 9, 81, 63, 27, 45, 9]

(ii) ① 412

$T_1(412) = 421 - 124 = 297$ [412, 297, 693, 594, 495, 495]

$T_2(297) = 972 - 279 = 693$

$T_3(693) = 963 - 369 = 594$

$T_4(594) = 954 - 459 = 495$

$T_5(495) = 954 - 459 = 495$ ✓

② 273

$T_1(273) = 732 - 237 = 495$ [273, 495, 495]

$T_2(495) = 954 - 459 = 495$ ✓

③ 538

$T_1(538) = 853 - 358 = 495$ [538, 495, 495]

$T_2(495) = 954 - 459 = 495$ ✓

④ 796

$T_1(796) = 976 - 679 = 297$ [796, 297, 693, 594, 495, 495]

$T_2(297) = 972 - 279 = 693$

$T_3(693) = 963 - 369 = 594$

$T_4(594) = 954 - 459 = 495$

$T_5(495) = 954 - 459 = 495$ ✓

⑤ 321

$T_1(321) = 321 - 123 = 198$ [321, 198, 792, 693, 594, 495, 495]

$T_2(198) = 981 - 189 = 792$

$T_3(792) = 792 - 297 = 693$

$T_4(693) = 963 - 369 = 594$

$T_5(594) = 954 - 459 = 495$

$T_6(495) = 954 - 459 = 495$

(iii) ① 4127

$T_1(4127) = 7421 - 1247 = 6174$ [4127, 6174, 6174]

$T_2(6174) = 7641 - 1467 = 6174$ ✓

② 8811

$T_1(8811) = 8811 - 1188 = 7623$ [8811, 7623, 5265, 3996, 6264, 4176, 6174, 6174]

$T_2(7623) = 7632 - 2367 = 5265$

$T_3(5265) = 6552 - 2556 = 3996$

$T_4(3996) = 9963 - 3699 = 6264$

$T_5(6264) = 6642 - 2466 = 4176$

$T_6(4176) = 7641 - 1467 = 6174$

$T_7(6174) = 7641 - 1467 = 6174$ ✓

③ 2442

$T_1(2442) = 4422 - 2244 = 2178$ [2442, 2178, 7443, 3996, 6264, 4176, 6174]

$T_2(2178) = 8721 - 1278 = 7443$

$T_3(7443) = 7443 - 3447 = 3996$

$T_4(3996) = 9963 - 3699 = 6264$

$T_5(6264) = 6642 - 2466 = 4176$

$T_6(4176) = 7641 - 1467 = 6174$

$T_7(6174) = 7641 - 1467 = 6174$ ✓

5) ① $n = 10 \Rightarrow \frac{10}{2} = 5$ [10, 5, 8, 4, 2, 1, 2]

$n = 5 \Rightarrow \frac{3(5)+1}{2} = 8$

$n = 8 \Rightarrow \frac{8}{2} = 4$

$n = 4 \Rightarrow \frac{4}{2} = 2$

$n = 2 \Rightarrow \frac{2}{2} = 1$

$n = 1 \Rightarrow \frac{3(1)+1}{2} = 2$

② $n = 12 \Rightarrow \frac{12}{2} = 6$ [12, 6, 3, 5, 4, 2, 1, 2]

$n = 6 \Rightarrow \frac{6}{2} = 3$

$n = 3 \Rightarrow \frac{3(3)+1}{2} = 5$

$n = 5 \Rightarrow 8$

$n = 8 \Rightarrow 4$

$n = 4 \Rightarrow 2$

$n = 2 \Rightarrow 1$

$n = 1 \Rightarrow 2$

③ $n = 13 \Rightarrow \frac{3(13)+1}{2} = 20$ $\quad n = 8 \Rightarrow 4$

$n = 20 \Rightarrow \frac{20}{2} = 10$ $\quad n = 4 \Rightarrow 2$

$n = 10 \Rightarrow \frac{10}{2} = 5$ $\quad n = 2 \Rightarrow 1$

$n = 5 \Rightarrow \frac{3(5)+1}{2} = 8$ $\quad n = 1 \Rightarrow 2$

They all end in the periodic cycle [2, 1]

```
> #Nikita John, Assignment 14
  #October 25th, 2021
> #M14.txt: Maple code for Lecture 14 of Dynamical Modesl in Biology, Fall 2021 (taught by Dr.
    Z.)

  Help14 :=proc( ) : print( ` RevOp(n,k), RevOpTr(n,k) `) :end:


    #RevOp(n,k): The operation of taking a k-digit number, sorting its digits from large to small,
    and subtractiong it from the revers. For example
  #RevOp(39,2) should give 93-39=54
  RevOp :=proc(n, k) local L, L1, L2, i :
  if not (type(n, integer) and n ≥ 0 and n < 10^k) then
  print( `Bad input`) :
  RETURN(FAIL) :
  fi:
  L := convert(n, base, 10) :
  L1 := sort( [op(L), 0$(k-nops(L))]) :
  L2 := [seq(L1[k + 1-i], i=1 ..k)] :
  add(L1[i] * 10^(i-1), i=1 ..k)-add(L2[i] * 10^(i-1), i=1 ..k) :
  end:


    #RevOpTr(n,k): The trajectory of the dynamical system RevOp(n,k) until it hits the first
    repetition (and then it keeps cycling for ever)
  RevOpTr :=proc(n, k) local L, n1 :
  if not (type(n, integer) and n ≥ 0 and n < 10^k) then
  RETURN(FAIL) :
  fi:

  L := [ ]:
  n1 := n :

  while not member(n1, L) do
   L := [op(L), n1]:
   n1 := RevOp(n1, k) :
  od:

  [op(L), n1] :

  end:

> #T3(n)
   RevOpTr(556, 3);
                 [556, 99, 891, 792, 693, 594, 495, 495]                (1)
>  RevOpTr(618, 3);
                 [618, 693, 594, 495, 495]                              (2)
```

> $RevOpTr(978, 3)$;

$$[978, 198, 792, 693, 594, 495, 495] \quad (3)$$

> $RevOpTr(999, 3)$;

$$[999, 0, 0] \quad (4)$$

> $RevOpTr(415, 3)$;

$$[415, 396, 594, 495, 495] \quad (5)$$

> *#From this, it can be there are two fixed point at 495 and another at 0*

> *#T4(n)*
> $RevOpTr(4895, 4)$;

$$[4895, 5265, 3996, 6264, 4176, 6174, 6174] \quad (6)$$

> $RevOpTr(5562, 4)$;

$$[5562, 3996, 6264, 4176, 6174, 6174] \quad (7)$$

> $RevOpTr(2121, 4)$;

$$[2121, 1089, 9621, 8352, 6174, 6174] \quad (8)$$

> $RevOpTr(6789, 4)$;

$$[6789, 3087, 8352, 6174, 6174] \quad (9)$$

> $RevOpTr(5555, 4)$;

$$[5555, 0, 0] \quad (10)$$

> $RevOpTr(5551, 4)$;

$$[5551, 3996, 6264, 4176, 6174, 6174] \quad (11)$$

> $RevOpTr(6625, 4)$;
> *#From this, it can be seen that there are two fixed points, one at 0 and one at 6174*

$$[6625, 4086, 8172, 7443, 3996, 6264, 4176, 6174, 6174] \quad (12)$$

> *#6 Brownie Points*
> $RevOpTr(65235, 5)$;

$$[65235, 41976, 82962, 75933, 63954, 61974, 82962] \quad (13)$$

> $RevOpTr(59874, 5)$;

$$[59874, 52965, 70983, 94941, 84942, 73953, 63954, 61974, 82962, 75933, 63954] \quad (14)$$

> $RevOpTr(88888, 5)$;

$$[88888, 0, 0] \quad (15)$$

> $RevOpTr(65552, 5)$;

$$[65552, 39996, 62964, 71973, 83952, 74943, 62964] \quad (16)$$

> $RevOpTr(12345, 5)$;

$$[12345, 41976, 82962, 75933, 63954, 61974, 82962] \quad (17)$$

> $RevOpTr(20125, 5)$;

$$[20125, 50985, 92961, 86922, 75933, 63954, 61974, 82962, 75933] \quad (18)$$

> $RevOpTr(66678, 5)$;

$$[66678, 20988, 95931, 85932, 74943, 62964, 71973, 83952, 74943] \quad (19)$$

> $RevOpTr(77774, 5)$;

$$[77774, 29997, 71973, 83952, 74943, 62964, 71973] \quad (20)$$

> *#There seems to be one fixed point at 0, and two main cycles [63954,61974,82962,75933,63954]*

and its permutations (a.k.a starting and ending with different numbers, but its one of the 4 numbers in the list and the other three numbers occur once before the repeat), as well as [62964,71973,83952,74943,62964] and its permutations .