

### Dynamical Modeling HW 14 - Okay to Post

$$1) \begin{aligned} f(x,y) &= x + y^4 - \frac{1}{16} \\ g(x,y) &= x^2 + y - \frac{1}{9} \\ x &= x + y^4 - \frac{1}{16} \\ 4\sqrt{y^4} = 4\sqrt{\frac{1}{16}} &\Rightarrow y = \pm \frac{1}{2} \\ y &= x^2 + y - \frac{1}{9} \\ \sqrt{x^2} = \sqrt{\frac{1}{9}} &\Rightarrow x = \pm \frac{1}{3} \end{aligned} \quad \left. \begin{array}{l} \text{F.P.: } (\frac{1}{2}, \frac{1}{3}), (-\frac{1}{2}, \frac{1}{3}), (\frac{1}{2}, -\frac{1}{3}), (-\frac{1}{2}, -\frac{1}{3}) \\ \text{Jacobian: } \begin{bmatrix} f_x(x_0, y_0) & f_y(x_0, y_0) \\ g_x(x_0, y_0) & g_y(x_0, y_0) \end{bmatrix} \\ f_x = 1 \quad g_x = 2x \quad \text{Jacobian: } \begin{bmatrix} 1 & 4y^3 \\ 2x & 1 \end{bmatrix} \\ f_y = 4y^3 \quad g_y = 1 \end{array} \right.$$

Point:  $(\frac{1}{2}, \frac{1}{3})$

$$\begin{bmatrix} 1 & 4(\frac{1}{3})^3 \\ 2(\frac{1}{2}) & 1 \end{bmatrix} = \begin{bmatrix} 1 & \frac{4}{27} \\ 1 & 1 \end{bmatrix} \Rightarrow \det \begin{bmatrix} 1-\lambda & \frac{4}{27} \\ 1 & 1-\lambda \end{bmatrix} = (1-\lambda)^2 - \frac{4}{27} = 0$$

$$1-\lambda = \pm \frac{2}{3\sqrt{3}}$$

$$\lambda = 1 \pm \frac{2}{3\sqrt{3}}$$

\*one of the eigenvalues is  $> 1$  ( $\lambda = 1 + \frac{2}{3\sqrt{3}}$ )  
so the point  $(\frac{1}{2}, \frac{1}{3})$  is unstable

Point:  $(-\frac{1}{2}, \frac{1}{3})$

$$\begin{bmatrix} 1 & 4\left(\frac{1}{3}\right)^3 \\ 2(-\frac{1}{2}) & 1 \end{bmatrix} = \begin{bmatrix} 1 & 4/27 \\ -1 & 1 \end{bmatrix} \Rightarrow \det \begin{bmatrix} 1-2 & 4/27 \\ -1 & 1-2 \end{bmatrix} = (1-2)^2 + \frac{4}{27} = 0$$

$$\lambda = \sqrt{(1-2)^2 + \left(\frac{4}{27}\right)^2} = \sqrt{1-4 + \frac{16}{729}} = \sqrt{\frac{-4}{27}} = \pm \frac{2}{3\sqrt{3}} i$$

The eigenvalues are  $> 1$ , so  $(-\frac{1}{2}, \frac{1}{3})$  is unstable

$$\lambda = 1 \pm \frac{2}{3\sqrt{3}} i$$

Point:  $(\frac{1}{2}, -\frac{1}{3})$

$$\begin{bmatrix} 1 & 4\left(-\frac{1}{3}\right)^3 \\ 2(\frac{1}{2}) & 1 \end{bmatrix} = \begin{bmatrix} 1 & -4/27 \\ 1 & 1 \end{bmatrix} \Rightarrow \det \begin{bmatrix} 1-2 & -4/27 \\ 1 & 1-2 \end{bmatrix} = (1-2)^2 - 4/27 = 0$$

$$\lambda = \sqrt{(1-2)^2 + \left(\frac{4}{27}\right)^2} = \sqrt{1-4 + \frac{16}{729}} = \sqrt{\frac{-4}{27}} = \pm \frac{2}{3\sqrt{3}} i$$

The eigenvalues are  $> 1$ , so  $(\frac{1}{2}, -\frac{1}{3})$  is unstable

$$\lambda = 1 \pm \frac{2}{3\sqrt{3}} i$$

Point:  $(-\frac{1}{2}, -\frac{1}{3})$

$$\begin{bmatrix} 1 & 4\left(-\frac{1}{3}\right)^3 \\ 2(-\frac{1}{2}) & 1 \end{bmatrix} = \begin{bmatrix} 1 & -4/27 \\ -1 & 1 \end{bmatrix} \Rightarrow \det \begin{bmatrix} 1-2 & -4/27 \\ -1 & 1-2 \end{bmatrix} = (1-2)^2 - 4/27 = 0$$

$$\sqrt{(1-2)^2} = \sqrt{4/27}$$

\* one of the eigenvalues is  $> 1$   
 $(1 + \frac{2}{3\sqrt{3}})$ , so the point  $(\frac{1}{2}, -\frac{1}{3})$  is unstable

$$1-2 = \pm \frac{2}{3\sqrt{3}}$$

$$\lambda = 1 \pm \frac{2}{3\sqrt{3}} i$$

2)  $x \rightarrow x^3 \bmod 17$

$x$	$x^3 \bmod 17$
1	0
2	0
3	7
4	13
5	6
6	12
7	3
8	2
9	15
10	14
11	5
12	11
13	4
14	7
15	9
16	16

3) (i) ① 27

$$T_1(27) = 72 - 27 = 45$$

$$T_2(45) = 54 - 45 = 9$$

$$T_3(9) = 90 - 9 = 81$$

$$T_4(81) = 81 - 18 = 63$$

$$T_5(63) = 63 - 36 = 27 \checkmark$$

$$[27, 45, 9, 81, 63, 27]$$

$$T_6(27) = 27 - 9 = 18$$

$$T_7(18) = 81 - 18 = 63$$

$$T_8(63) = 63 - 36 = 27$$

$$T_9(27) = 72 - 27 = 45$$

$$T_{10}(45) = 54 - 45 = 9$$

$$T_{11}(9) = 90 - 9 = 81$$

$$T_{12}(81) = 81 - 18 = 63 \checkmark$$

$$T_{13}(63) = 63 - 36 = 27$$

$$T_{14}(27) = 72 - 27 = 45$$

$$T_{15}(45) = 54 - 45 = 9$$

$$T_{16}(9) = 90 - 9 = 81$$

$$T_{17}(81) = 81 - 18 = 63 \checkmark$$

$$T_{18}(63) = 63 - 36 = 27$$

$$T_{19}(27) = 72 - 27 = 45$$

$$T_{20}(45) = 54 - 45 = 9$$

$$T_{21}(9) = 90 - 9 = 81$$

$$T_{22}(81) = 81 - 18 = 63 \checkmark$$

$$T_{23}(63) = 63 - 36 = 27$$

$$T_{24}(27) = 72 - 27 = 45$$

$$T_{25}(45) = 54 - 45 = 9$$

$$T_{26}(9) = 90 - 9 = 81$$

$$T_{27}(81) = 81 - 18 = 63 \checkmark$$

$$T_{28}(63) = 63 - 36 = 27$$

$$T_{29}(27) = 72 - 27 = 45$$

$$T_{30}(45) = 54 - 45 = 9$$

$$T_{31}(9) = 90 - 9 = 81$$

$$T_{32}(81) = 81 - 18 = 63 \checkmark$$

$$T_{33}(63) = 63 - 36 = 27$$

$$T_{34}(27) = 72 - 27 = 45$$

$$T_{35}(45) = 54 - 45 = 9$$

$$T_{36}(9) = 90 - 9 = 81$$

$$T_{37}(81) = 81 - 18 = 63 \checkmark$$

$$T_{38}(63) = 63 - 36 = 27$$

$$T_{39}(27) = 72 - 27 = 45$$

$$T_{40}(45) = 54 - 45 = 9$$

$$T_{41}(9) = 90 - 9 = 81$$

$$T_{42}(81) = 81 - 18 = 63 \checkmark$$

$$T_{43}(63) = 63 - 36 = 27$$

$$T_{44}(27) = 72 - 27 = 45$$

$$T_{45}(45) = 54 - 45 = 9$$

$$T_{46}(9) = 90 - 9 = 81$$

$$T_{47}(81) = 81 - 18 = 63 \checkmark$$

$$T_{48}(63) = 63 - 36 = 27$$

$$T_{49}(27) = 72 - 27 = 45$$

$$T_{50}(45) = 54 - 45 = 9$$

$$T_{51}(9) = 90 - 9 = 81$$

$$T_{52}(81) = 81 - 18 = 63 \checkmark$$

$$T_{53}(63) = 63 - 36 = 27$$

$$T_{54}(27) = 72 - 27 = 45$$

$$T_{55}(45) = 54 - 45 = 9$$

$$T_{56}(9) = 90 - 9 = 81$$

$$T_{57}(81) = 81 - 18 = 63 \checkmark$$

$$T_{58}(63) = 63 - 36 = 27$$

$$T_{59}(27) = 72 - 27 = 45$$

$$T_{60}(45) = 54 - 45 = 9$$

$$T_{61}(9) = 90 - 9 = 81$$

$$T_{62}(81) = 81 - 18 = 63 \checkmark$$

$$T_{63}(63) = 63 - 36 = 27$$

$$T_{64}(27) = 72 - 27 = 45$$

$$T_{65}(45) = 54 - 45 = 9$$

$$T_{66}(9) = 90 - 9 = 81$$

$$T_{67}(81) = 81 - 18 = 63 \checkmark$$

$$T_{68}(63) = 63 - 36 = 27$$

$$T_{69}(27) = 72 - 27 = 45$$

$$T_{70}(45) = 54 - 45 = 9$$

$$T_{71}(9) = 90 - 9 = 81$$

$$T_{72}(81) = 81 - 18 = 63 \checkmark$$

$$T_{73}(63) = 63 - 36 = 27$$

$$T_{74}(27) = 72 - 27 = 45$$

$$T_{75}(45) = 54 - 45 = 9$$

$$T_{76}(9) = 90 - 9 = 81$$

$$T_{77}(81) = 81 - 18 = 63 \checkmark$$

$$T_{78}(63) = 63 - 36 = 27$$

$$T_{79}(27) = 72 - 27 = 45$$

$$T_{80}(45) = 54 - 45 = 9$$

$$T_{81}(9) = 90 - 9 = 81$$

$$T_{82}(81) = 81 - 18 = 63 \checkmark$$

$$T_{83}(63) = 63 - 36 = 27$$

$$T_{84}(27) = 72 - 27 = 45$$

$$T_{85}(45) = 54 - 45 = 9$$

$$T_{86}(9) = 90 - 9 = 81$$

$$T_{87}(81) = 81 - 18 = 63 \checkmark$$

$$T_{88}(63) = 63 - 36 = 27$$

$$T_{89}(27) = 72 - 27 = 45$$

$$T_{90}(45) = 54 - 45 = 9$$

$$T_{91}(9) = 90 - 9 = 81$$

$$T_{92}(81) = 81 - 18 = 63 \checkmark$$

$$T_{93}(63) = 63 - 36 = 27$$

$$T_{94}(27) = 72 - 27 = 45$$

$$T_{95}(45) = 54 - 45 = 9$$

$$T_{96}(9) = 90 - 9 = 81$$

$$T_{97}(81) = 81 - 18 = 63 \checkmark$$

$$T_{98}(63) = 63 - 36 = 27$$

$$T_{99}(27) = 72 - 27 = 45$$

$$T_{100}(45) = 54 - 45 = 9$$

$$T_{101}(9) = 90 - 9 = 81$$

$$T_{102}(81) = 81 - 18 = 63 \checkmark$$

$$T_{103}(63) = 63 - 36 = 27$$

$$T_{104}(27) = 72 - 27 = 45$$

$$T_{105}(45) = 54 - 45 = 9$$

$$T_{106}(9) = 90 - 9 = 81$$

$$T_{107}(81) = 81 - 18 = 63 \checkmark$$

$$T_{108}(63) = 63 - 36 = 27$$

$$T_{109}(27) = 72 - 27 = 45$$

$$T_{110}(45) = 54 - 45 = 9$$

$$T_{111}(9) = 90 - 9 = 81$$

$$T_{112}(81) = 81 - 18 = 63 \checkmark$$

$$T_{113}(63) = 63 - 36 = 27$$

$$T_{114}(27) = 72 - 27 = 45$$

$$T_{115}(45) = 54 - 45 = 9$$

$$T_{116}(9) = 90 - 9 = 81$$

$$T_{117}(81) = 81 - 18 = 63 \checkmark$$

$$T_{118}(63) = 63 - 36 = 27$$

$$T_{119}(27) = 72 - 27 = 45$$

$$T_{120}(45) = 54 - 45 = 9$$

$$T_{121}(9) = 90 - 9 = 81$$

$$T_{122}(81) = 81 - 18 = 63 \checkmark$$

$$T_{123}(63) = 63 - 36 = 27$$

$$T_{124}(27) = 72 - 27 = 45$$

$$T_{125}(45) = 54 - 45 = 9$$

$$T_{126}(9) = 90 - 9 = 81$$

$$T_{127}(81) = 81 - 18 = 63 \checkmark$$

$$T_{128}(63) = 63 - 36 = 27$$

$$T_{129}(27) = 72 - 27 = 45$$

$$T_{130}(45) = 54 - 45 = 9$$

$$T_{131}(9) = 90 - 9 = 81$$

$$T_{132}(81) = 81 - 18 = 63 \checkmark$$

$$T_{133}(63) = 63 - 36 = 27$$

$$T_{134}(27) = 72 - 27 = 45$$

$$T_{135}(45) = 54 - 45 = 9$$

$$T_{136}(9) = 90 - 9 = 81$$

$$T_{137}(81) = 81 - 18 = 63 \checkmark$$

$$T_{138}(63) = 63 - 36 = 27$$

$$T_{139}(27) = 72 - 27 = 45$$

$$T_{140}(45) = 54 - 45 = 9$$

$$T_{141}(9) = 90 - 9 = 81$$

$$T_{142}(81) = 81 - 18 = 63 \checkmark$$

$$T_{143}(63) = 63 - 36 = 27$$

$$T_{144}(27) = 72 - 27 = 45$$

$$T_{145}(45) = 54 - 45 = 9$$

$$T_{146}(9) = 90 - 9 = 81$$

$$T_{147}(81) = 81 - 18 = 63 \checkmark$$

$$T_{148}(63) = 63 - 36 = 27$$

$$T_{149}(27) = 72 - 27 = 45$$

$$T_{150}(45) = 54 - 45 = 9$$

$$T_{151}(9) = 90 - 9 = 81$$

$$T_{152}(81) = 81 - 18 = 63 \checkmark$$

$$T_{153}(63) = 63 - 36 = 27$$

$$T_{154}(27) = 72 - 27 = 45$$

$$T_{155}(45) = 54 - 45 = 9$$

$$T_{156}(9) = 90 - 9 = 81$$

$$T_{157}(81) = 81 - 18 = 63 \checkmark$$

$$T_{158}(63) = 63 - 36 = 27$$

$$T_{159}(27) = 72 - 27 = 45$$

$$T_{160}(45) = 54 - 45 = 9$$

$$T_{161}(9) = 90 - 9 = 81$$

$$T_{162}(81) = 81 - 18 = 63 \checkmark$$

$$T_{163}(63) = 63 - 36 = 27$$

$$T_{164}(27) = 72 - 27 = 45$$

$$T_{165}(45) = 54 - 45 = 9$$

$$T_{166}(9) = 90 - 9 = 81$$

$$T_{167}(81) = 81 - 18 = 63 \checkmark$$

$$T_{168}(63) = 63 - 36 = 27$$

$$T_{169}(27) = 72 - 27 = 45$$

$$T_{170}(45) = 54 - 45 = 9$$

$$T_{171}(9) = 90 - 9 = 81$$

$$T_{172}(81) = 81 - 18 = 63 \checkmark$$

$$T_{173}(63) = 63 - 36 = 27$$

$$T_{174}(27) = 72 - 27 = 45$$

$$T_{175}(45) = 54 - 45 = 9$$

$$T_{176}(9) = 90 - 9 = 81$$

$$T_{177}(81) = 81 - 18 = 63 \checkmark$$

$$T_{178}(63) = 63 - 36 = 27$$

$$T_{179}(27) = 72 - 27 = 45$$

$$T_{180}(45) = 54 - 45 = 9$$

$$T_{181}(9) = 90 - 9 = 81$$

$$T_{182}(81) = 81 - 18 = 63 \checkmark$$

$$T_{183}(63) = 63 - 36 = 27$$

$$T_{184}(27) = 72 - 27 = 45$$

$$T_{185}(45) = 54 - 45 = 9$$

⑥ 72

$$\begin{aligned} T_1(72) &= 72 - 27 = 45 & T_5(63) &= 63 - 36 = 27 \\ T_2(45) &= 54 - 45 = 9 & T_6(27) &= 72 - 27 = 45 \checkmark \\ T_3(9) &= 90 - 9 = 81 & [72, 45, 9, 81, 63, 27, 45] \\ T_4(81) &= 81 - 18 = 63 \end{aligned}$$

⑦ 68

$$\begin{aligned} T_1(68) &= 86 - 68 = 18 & T_5(45) &= 54 - 45 = 9 \\ T_2(18) &= 81 - 18 = 63 & T_6(9) &= 90 - 9 = 81 \\ T_3(63) &= 63 - 36 = 27 & T_7(81) &= 81 - 18 = 63 \checkmark \\ T_4(27) &= 72 - 27 = 45 & [68, 18, 63, 27, 45, 9, 81, 63] \end{aligned}$$

⑧ 47

$$\begin{aligned} T_1(47) &= 74 - 47 = 27 & T_5(81) &= 81 - 18 = 63 \\ T_2(27) &= 72 - 27 = 45 & T_6(63) &= 63 - 36 = 27 \checkmark \\ T_3(45) &= 54 - 45 = 9 & [47, 27, 45, 9, 81, 63, 27] \\ T_4(9) &= 90 - 9 = 81 \end{aligned}$$

⑨ 49

$$\begin{aligned} T_1(49) &= 94 - 49 = 45 & T_5(63) &= 63 - 36 = 27 \\ T_2(45) &= 54 - 45 = 9 & T_6(27) &= 72 - 27 = 45 \checkmark \\ T_3(9) &= 90 - 9 = 81 & [49, 45, 9, 81, 63, 27, 45] \\ T_4(81) &= 81 - 18 = 63 \end{aligned}$$

⑩ 17

$$\begin{aligned} T_1(17) &= 71 - 17 = 54 & T_5(63) &= 63 - 36 = 27 \\ T_2(54) &= 54 - 45 = 9 & T_6(27) &= 72 - 27 = 45 \\ T_3(9) &= 90 - 9 = 81 & T_7(45) &= 54 - 45 = 9 \checkmark \\ T_4(81) &= 81 - 18 = 63 & [17, 54, 9, 81, 63, 27, 45, 9] \end{aligned}$$

(ii) ① 412

$$T_1(412) = 421 - 124 = 297 \quad [412, 297, 693, 594, 495, 495]$$

$$T_2(297) = 972 - 279 = 693$$

$$T_3(693) = 963 - 369 = 594$$

$$T_4(594) = 954 - 459 = 495$$

$$T_5(495) = 954 - 459 = 495 \checkmark$$

② 273

$$T_1(273) = 732 - 237 = 495 \quad [273, 495, 495]$$

$$T_2(495) = 954 - 459 = 495 \checkmark$$

③ 538

$$T_1(538) = 853 - 358 = 495 \quad [538, 495, 495]$$

$$T_2(495) = 954 - 459 = 495 \checkmark$$

④ 796

$$T_1(796) = 976 - 679 = 297 \quad [796, 297, 693, 594, 495, 495]$$

$$T_2(297) = 972 - 279 = 693$$

$$T_3(693) = 963 - 369 = 594$$

$$T_4(594) = 954 - 459 = 495$$

$$T_5(495) = 954 - 459 = 495 \checkmark$$

⑤ 321

$$T_1(321) = 321 - 123 = 198 \quad [321, 198, 792, 693, 594, 495, 495]$$

$$T_2(198) = 981 - 189 = 792$$

$$T_3(792) = 792 - 297 = 693$$

$$T_4(693) = 963 - 369 = 594$$

$$T_5(594) = 954 - 459 = 495$$

$$T_6(495) = 954 - 459 = 495$$

(iii) ① 4127

$$T_1(4127) = 7421 - 1247 = 6174 \quad [4127, 6174, 6174]$$

$$T_2(6174) = 7641 - 1467 = 6174 \checkmark$$

② 8811

$$T_1(8811) = 8811 - 1188 = 7623 \quad [8811, 7623, 5265, 3996,$$

$$T_2(7623) = 7632 - 2367 = 5265 \quad 6264, 4176, 6174, 6174]$$

$$T_3(5265) = 6552 - 2556 = 3996$$

$$T_4(3996) = 9963 - 3699 = 6264$$

$$T_5(6264) = 6642 - 2466 = 4176$$

$$T_6(4176) = 7641 - 1467 = 6174$$

$$T_7(6174) = 7641 - 1467 = 6174 \checkmark$$

③ 2442

$$T_1(2442) = 4422 - 2244 = 2178 \quad [2442, 2178, 7443, 3996,$$

$$T_2(2178) = 8721 - 1278 = 7443 \quad 6264, 4176, 6174]$$

$$T_3(7443) = 7443 - 3447 = 3996$$

$$T_4(3996) = 9963 - 3699 = 6264$$

$$T_5(6264) = 6642 - 2466 = 4176$$

$$T_6(4176) = 7641 - 1467 = 6174$$

$$T_7(6174) = 7641 - 1467 = 6174 \checkmark$$

5) ①  $n = 10 \Rightarrow \frac{10}{2} = 5 \quad [10, 5, 8, 4, 3, 1, 2]$

$$n = 5 \Rightarrow \frac{2(5)+1}{2} = 8$$

$$n = 8 \Rightarrow \frac{8}{2} = 4$$

$$n = 4 \Rightarrow \frac{4}{2} = 2$$

$$n = 2 \Rightarrow \frac{2}{2} = 1$$

$$n = 1 \Rightarrow \frac{3(1)+1}{2} = 2$$

②  $n = 12 \Rightarrow \frac{12}{2} = 6 \rightarrow n = 8 \Rightarrow 4 \quad [12, 6, 3, 5, 4, 2, 1, 2]$

$$n = 6 \Rightarrow \frac{6}{2} = 3$$

$$n = 3 \Rightarrow \frac{3(3)+1}{2} = 5$$

$$n = 5 \Rightarrow 8$$

$$n = 4 \Rightarrow 2$$

$$n = 2 \Rightarrow 1$$

$$n = 1 \Rightarrow 2$$

$$\textcircled{3} \quad n = 13 \Rightarrow \frac{3(13)+1}{2} = 20 \quad \left. \begin{array}{l} n = 20 \Rightarrow \frac{20}{2} = 10 \\ n = 10 \Rightarrow \frac{10}{2} = 5 \\ n = 5 \Rightarrow \frac{3(5)+1}{2} = 8 \end{array} \right\} \quad \begin{array}{l} n = 8 \Rightarrow 4 \\ n = 4 \Rightarrow 2 \\ n = 2 \Rightarrow 1 \\ n = 1 \Rightarrow 2 \end{array}$$

They all end in the periodic cycle [2, 1]

> #Nikita John, Assignment 14  
#October 25th, 2021  
> #M14.txt: Maple code for Lecture 14 of Dynamical Models in Biology, Fall 2021 (taught by Dr. Z.)

*Help14 :=proc( ) :print(`RevOp(n,k), RevOpTr(n,k)` ) :end:*

#RevOp(n,k): The operation of taking a k-digit number, sorting its digits from large to small, and subtractiong it from the revers. For example  
#RevOp(39,2) should give 93-39=54  
*RevOp :=proc(n, k) local L, L1, L2, i :*  
**if not** (type(n, integer) **and** n ≥ 0 **and** n < 10^k) **then**  
*print(`Bad input`):*  
*RETURN(FAIL) :*  
**fi:**  
*L := convert(n, base, 10) :*  
*L1 := sort([op(L), 0\$(k-nops(L))]) :*  
*L2 := [seq(L1[k+1-i], i=1..k)] :*  
*add(L1[i]\*10^(i-1), i=1..k) - add(L2[i]\*10^(i-1), i=1..k) :*  
**end:**

#RevOpTr(n,k): The trajectory of the dynamical system RevOp(n,k) until it hits the first repetition (and then it keeps cycling for ever)  
*RevOpTr :=proc(n, k) local L, n1 :*  
**if not** (type(n, integer) **and** n ≥ 0 **and** n < 10^k) **then**  
*RETURN(FAIL) :*  
**fi:**  
*L := [ ]:*  
*n1 := n :*  
**while not** member(n1, L) **do**  
*L := [op(L), n1] :*  
*n1 := RevOp(n1, k) :*  
**od:**  
*[op(L), n1] :*  
**end:**

> #T3(n)  
*RevOpTr(556, 3);* [556, 99, 891, 792, 693, 594, 495, 495] (1)  
> *RevOpTr(618, 3);* [618, 693, 594, 495, 495] (2)

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> RevOpTr(978, 3);
[978, 198, 792, 693, 594, 495, 495] (3)

> RevOpTr(999, 3);
[999, 0, 0] (4)

> RevOpTr(415, 3);
[415, 396, 594, 495, 495] (5)

> #From this, it can be seen that there are two fixed points at 495 and another at 0
> #T4(n)
RevOpTr(4895, 4);
[4895, 5265, 3996, 6264, 4176, 6174, 6174] (6)

> RevOpTr(5562, 4);
[5562, 3996, 6264, 4176, 6174, 6174] (7)

> RevOpTr(2121, 4);
[2121, 1089, 9621, 8352, 6174, 6174] (8)

> RevOpTr(6789, 4);
[6789, 3087, 8352, 6174, 6174] (9)

> RevOpTr(5555, 4);
[5555, 0, 0] (10)

> RevOpTr(5551, 4);
[5551, 3996, 6264, 4176, 6174, 6174] (11)

> RevOpTr(6625, 4);
#From this, it can be seen that there are two fixed points, one at 0 and one at 6174
[6625, 4086, 8172, 7443, 3996, 6264, 4176, 6174, 6174] (12)

> #6 Brownie Points
RevOpTr(65235, 5);
[65235, 41976, 82962, 75933, 63954, 61974, 82962] (13)

> RevOpTr(59874, 5);
[59874, 52965, 70983, 94941, 84942, 73953, 63954, 61974, 82962, 75933, 63954] (14)

> RevOpTr(88888, 5);
[88888, 0, 0] (15)

> RevOpTr(65552, 5);
[65552, 39996, 62964, 71973, 83952, 74943, 62964] (16)

> RevOpTr(12345, 5);
[12345, 41976, 82962, 75933, 63954, 61974, 82962] (17)

> RevOpTr(20125, 5);
[20125, 50985, 92961, 86922, 75933, 63954, 61974, 82962, 75933] (18)

> RevOpTr(66678, 5);
[66678, 20988, 95931, 85932, 74943, 62964, 71973, 83952, 74943] (19)

> RevOpTr(77774, 5);
[77774, 29997, 71973, 83952, 74943, 62964, 71973] (20)

> #There seems to be one fixed point at 0, and two main cycles [63954, 61974, 82962, 75933, 63954]

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and its permutations (a.k.a starting and ending with different numbers, but its one of the 4 numbers in the list and the other three numbers occur once before the repeat), as well as [62964,71973,83952,74943,62964] and its permutations .