Hw 14

1)

$x(n) = x(n-1) + y(n-1)^4 - 1/16$

$y(n) = x(n-1)^2 + y(n-1) - 1/4$

$$(x,y) \to (x + y^4 - 1/16,\ x^2 + y - 1/4)$$

$$\left.\begin{aligned} x &= x + y^4 - 1/16 \\ y &= x^2 + y - 1/4 \end{aligned}\right\} \quad \begin{aligned} y^4 - 1/16 &= 0 \\ x^2 - 1/4 &= 0 \end{aligned}$$

$$\begin{aligned} y &= \pm 1/2 \\ x &= \pm 1/3 \end{aligned} \Rightarrow \{(1/3, 1/2), (1/3, -1/2), (-1/3, 1/2), (-1/3, -1/2)\}$$

$$J = \begin{vmatrix} f_x & f_y \\ g_x & g_y \end{vmatrix} = \begin{vmatrix} 1 & 4y^3 \\ 2x & 1 \end{vmatrix}$$

$$J(1/3, 1/2) = \begin{vmatrix} 1 & 4(1/2)^3 \\ 2(1/3) & 1 \end{vmatrix} = \begin{vmatrix} 1 & 1/2 \\ 2/3 & 1 \end{vmatrix}$$

Given Eigenvalues $\Rightarrow$ $\frac{3+\sqrt{3}}{3}$, $\frac{3-\sqrt{3}}{3}$ Since both are not less that 1 these are not stable

$$J(1/3, -1/2) = \begin{vmatrix} 1 & -1/2 \\ 2/3 & 1 \end{vmatrix} = \boxed{\begin{aligned} \lambda &= 1 + \frac{i\sqrt{3}}{3} \\ \lambda &= 1 - \frac{i\sqrt{3}}{3} \end{aligned}}$$

$$J(-1/3, 1/2) = \begin{vmatrix} 1 & 1/2 \\ 1/3 & 1 \end{vmatrix} = \boxed{\lambda = \frac{3+\sqrt{3}}{3}, \frac{3-\sqrt{3}}{3}}$$

$$J(-1/3, 1/2) = \begin{vmatrix} 1 & -1/2 \\ 1/3 & 1 \end{vmatrix} = \boxed{\begin{matrix} \lambda = 1 + \frac{i\sqrt{3}}{3} \\ \lambda = 1 - \frac{i\sqrt{3}}{3} \end{matrix}}$$

Since not all pts are in absolute value < 1
none are stable

2) $z \to z^3 \mod 17$

$S = \{1 \ldots 16\}$ $\qquad f(z) = z^3 \pmod{17}$

z	$f(z)$	Trajectory
1	1	
2	8	
3	10	$\leftarrow (27)$
4	13	$\leftarrow (64)$
5	6	
6	12	
7	3	
8	2	
9	15	
10	14	
11	5	
12	11	
13	4	
14	7	
15	9	
16	16	

$1 \leftarrow 16 \leftarrow 9 \leftarrow 7 \leftarrow 4 \leftarrow 11 \leftarrow 5 \leftarrow 14$

$1 \to 8 \to 10 \to 13 \to 6 \to 12 \to 3 \to 2 \to 15 \to 14$

3) i) {10, 12, 41, 22, 67, 47, 57, 18, 80, 97}

1) 10 - 01 = 09

90 - 09 = 81

81 - 18 = 63

63 - 36 = 27

72 - 27 = 45

54 - 45 = 9

[9, 81, 63, 27, 45, 9]

2) 21 - 12 = 9

will repeat to same as above

[9, 81, 63, 27, 45, 9]

3) 41 - 14 = 27

72 - 27 = 45

54 - 45 = 09

90 - 09 = 81

81 - 18 = 63

63 - 36 = 27

[27, 45, 09, 81, 63, 27]

4) 22 - 22 = 0

[0]

5) 74 - 47 = 27

Same as (3)

[27, 45, 09, 81, 63, 27]

6) 75 - 57 = 18

81 - 18 = 63

63 - 36 = 27

72 - 27 = 45

54 - 45 = 09

90 - 09 = 81

81 - 18 = 63

[18, 63, 27, 45, 09, 81, 63]

7) 76 - 67 = $\boxed{9}$

[9, 81, 63, 27, 45, 9]

8) 86 - 08 = 72

72 - 27 = 45

54 - 45 = 09

90 - 09 = 81

81 - 18 = 63

63 - 36 = 27

72 - 27 = 45

[72, 45, 09, 81, 63, 27, 45]

9) 81 - 18 = 63

[63, 27, 45, 09, 81, 63]

10) 97 - 79 = 18

81 - 18 = 63

[18, 63, 27, 45, 09, 81, 63]

ii) [108, 784, 991, 411, 338]

1) 810 - 018 = 792
972 - 279 = 693
963 - 369 = 594
954 - 459 = 495 [792, 693, 594, 495, 495]
954 - 459 = 495

2) 874 - 478 = 396
963 - 369 = 594 [396, 594, 495, 495]
954 - 459 = 495

3) 991 - 199 = 792
972 - 279 = 693 [792, 693, 594, 495, 495]

4) 411 - 114 = 297
972 - 279 = 693 [297, 693, 594, 495, 495]

5) 833 - 338 = 495
[495, 495]

ii) [1027, 7872, 4485]

1) 7210 - 0127 = 7083
8730 - 0378 = 8352
8532 - 2358 = 6174
7641 - 1467 = 6174

[7083, 8352, 6174, 6174]

2) 8772 - 2778 = 5994
9954 - 4599 = 5355
5553 - 3555 = 1998
9981 - 1899 = 8082
8820 - 0288 = 8532
8532 - 2358 = 6174

[5994, 5355, 1998, 8082, 8532, 6174, 6174]

3) 8544 - 4458 = 4086
8640 - 0468 = 8172
8721 - 1278 = 7443
7443 - 3447 = 3996
9963 - 3699 = 6264
6642 - 2466 = 4176
7641 - 1467 = 6174

[4086, 8172, 7443, 3996, 6264, 4176, 6174, 6174]

5)

$n \to n/2$ if n is even

$n \to \frac{3n+1}{2}$ if n is odd

[8, 21, 5, 32, 40]

i) $\frac{8}{2} = 4$, $\frac{4}{2} = 2$ $\frac{2}{2} = 1$ $\frac{3(1)+1}{2} = 2$

So, [8, 4, 2, 1, 2 ....]

ii) $\frac{3(21)+1}{2} = 32$, $\frac{32}{2} = 16$, $\frac{16}{2} = 8$, $\frac{8}{2} = 4$ $\frac{4}{2} = 2$

So, [21, 32, 16, 8, 4, 2, 1, 2 ...]

iii) $\frac{3(5)+1}{2} = 8$, $\frac{8}{2} = 4$ $\frac{4}{2} = 2$ $\frac{2}{2} = 1$

So [5, 8, 4, 2, 1, 2 ..]

iv) $\frac{32}{2} = 16$, $\frac{16}{2} = 8$ $\frac{8}{2} = 4$, $\frac{4}{2} = 2$

So, [32, 16, 8, 4, 2, 1, 2 ..]

v) $\frac{40}{2} = 20$ $\frac{20}{2} = 10$ $\frac{10}{2} = 5$, repeats from iii

SO, [40, 20, 10, 5, 8, 4, 2, 1, 2 ...]