

Max Mekharikov - HW 14 - okay to post

$$1. \quad x(n) = x(n-1) + y(n-1)^4 - 1/16$$

$$y(n) = x(n-1)^2 + y(n-1) - 1/9$$

$$F: \mathbb{R}^2 \rightarrow \mathbb{R}^2$$

$$(x, y) \rightarrow (x + y^4 - 1/16, x^2 + y - 1/9)$$

$$x = x + y^4 - 1/16 \quad y = x^2 + y - 1/9$$

$$y^4 = 1/16 \quad x^2 = 1/9$$

$$y = \pm 1/2 \quad x = \pm 1/3$$

$$\left\{ (1/3, 1/2), (-1/3, 1/2), (1/3, -1/2), (-1/3, -1/2) \right\}$$

$$f(x, y) = x + y^4 - 1/16$$

$$f_x = 1, \quad f_y = 4y^3$$

$$g(x, y) = x^2 + y - 1/9$$

$$g_x = 2x, \quad g_y = 1$$

$$J(x, y) = \begin{pmatrix} 1 & 4y^3 \\ 2x & 1 \end{pmatrix}$$

$$J(x,y) = \begin{pmatrix} 1 & 4y^3 \\ 2x & 1 \end{pmatrix}$$

$$J(1/3, 1/2) = \begin{pmatrix} 1 & 4(1/2)^3 \\ 2/3 & 1 \end{pmatrix}$$

$$\det(J - \lambda I)$$

$$\lambda = \frac{3 + \sqrt{3}}{3}, \frac{3 - \sqrt{3}}{3} \rightarrow \text{not stable } (\lambda \geq 1)$$

$$J(-1/3, 1/2) = \begin{pmatrix} 1 & 4(1/2)^3 \\ -2/3 & 1 \end{pmatrix}, \lambda = 1 \pm i \frac{\sqrt{3}}{3}$$

$\hookrightarrow$  not stable ( $\lambda \geq 1$ )

$$J(1/3, -1/2) = \begin{pmatrix} 1 & 4(-1/2)^3 \\ 2/3 & 1 \end{pmatrix}, \lambda = 1 \pm i \frac{\sqrt{3}}{3}$$

$\hookrightarrow$  not stable ( $\lambda \geq 1$ )

$$J(-1/3, -1/2) = \begin{pmatrix} 1 & 4(-1/2)^3 \\ -2/3 & 1 \end{pmatrix}, \lambda = \frac{3 + \sqrt{3}}{3}, \frac{3 - \sqrt{3}}{3}$$

$\hookrightarrow$  not stable ( $\lambda \geq 1$ )

$$2) x \rightarrow x^3 \pmod{17}$$

1.  $[1, 1]$

2.  $[2, 8, 2]$

3.  $[3, 10, 14, 7, 3]$

4.  $[4, 13, 4]$

5.  $[5, 6, 12, 11, 5]$

6.  $[6, 12, 11, 5, 6]$

7.  $[7, 3, 10, 14, 7]$

8.  $[8, 2, 8]$

9.  $[9, 15, 9]$

10.  $[10, 14, 7, 3, 10]$

11.  $[11, 5, 6, 12, 11]$

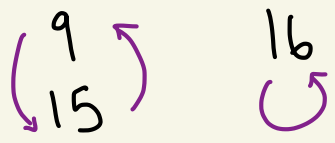
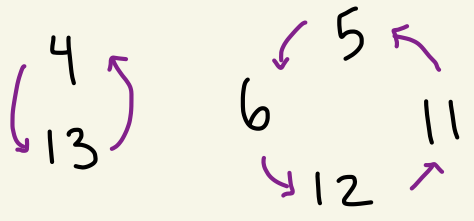
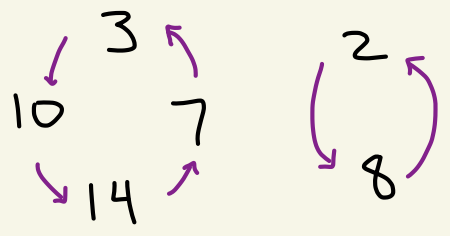
12.  $[12, 11, 5, 6, 12]$

13.  $[13, 4, 13]$

14.  $[14, 7, 3, 10, 14]$

15.  $[15, 9, 15]$

16.  $[16, 16]$



$$3) T_k(n) = L(n) - S(n)$$

$$k=2$$

$$i) n=71$$

$$71-17=54$$

$$54-45=9$$

$$9-9=0$$

$$[54, 9, 0]$$

$$n=35$$

$$2) [18, 63, 27, 45, 9, 0]$$

$$3) n=65$$

$$[9, 0]$$

$$4) n=99$$

$$[0]$$

$$5) n=93$$

$$[54, 9, 0]$$

$$6) n=81$$

$$[63, 27, 45, 9, 0]$$

$$7) n=13$$

$$[18, 63, 27, 45, 9, 0]$$

$$8) n=50$$

$$[45, 9, 0]$$

$$9) n=42$$

$$[18, 63, 27, 45, 9, 0]$$

$$10) n=27$$

$$[45, 9, 0]$$

$$ii) 1) n=123$$

$$[198, 792, 693, 594, 495, 495]$$

$$2) n=808$$

$$[720, 693, 594, 495, 495]$$

3)  $n = 495$   
[495]

4)  $n = 609$   
[891, 792, 693, 594, 495, 495]

5)  $n = 442$   
[198, 792, 693, 594, 495, 495]

iii)

1)  $n = 9012$   
[9081, 9621, 8352, 6174, 6174]

2)  $n = 6310$   
[6174]

3)  $n = 1760$   
[7443, 3996, 6264, 4176, 6174]

5)  $n \rightarrow \frac{n}{2}$  if  $n$  is even

$n \rightarrow \frac{3n+1}{2}$  if  $n$  is odd

1)  $n = 10$

[10, 5, 8, 4, 2, 1, 2]

2)  $n = 8$

[8, 4, 2, 1, 2, 1]

$$3) n = 100$$

[100, 50, 25, 38, 19, 29, 44, 22, 11, 17, 26, 13,  
40, 20, 10, 5, 8, 4, 2, 1]

$$4) n = 66$$

[66, 33, 100, 50, 25, 38, 19, 29, 44, 22, 11  
17, 26, 13, 40, 20, 10, 5, 8, 4, 2, 1]

$$5) n = 123$$

[123, 185, 278, 139, 209, 314, 157, 236, 118, 59, 89,  
134, 67, 101, 152, 76, 38, 19, 29, 44, 22, 11  
17, 26, 13, 40, 20, 10, 5, 8, 4, 2, 1]

> # Max Mekhanikov - HW 14 - Okay to post

> #RevOp(n,k): The operation of taking a k-digit number, sorting its digits from large to small, and subtracting it from the revers. For example

#RevOp(39,2) should give 93-39=54

RevOp := **proc**(n, k) **local** L, L1, L2, i :

**if not** (type(n, integer) **and** n ≥ 0 **and** n < 10^k) **then**

print( `Bad input` ) :

RETURN (FAIL) :

**fi**:

L := convert(n, base, 10) :

L1 := sort([op(L), 0\$(k-nops(L))]) :

L2 := [seq(L1[k + 1 - i], i = 1 ..k)]:

add(L1[i] \* 10^(i-1), i = 1 ..k) - add(L2[i] \* 10^(i-1), i = 1 ..k) :

**end**:

#RevOpTr(n,k): The trajectory of the dynamical system RevOp(n,k) until it hits the first repetition (and then it keeps cycling for ever)

RevOpTr := **proc**(n, k) **local** L, n1 :

**if not** (type(n, integer) **and** n ≥ 0 **and** n < 10^k) **then**

RETURN (FAIL) :

**fi**:

L := [ ]:

n1 := n :

**while not** member(n1, L) **do**

L := [op(L), n1] :

n1 := RevOp(n1, k) :

**od**:

[op(L), n1] :

**end**:

> RevOpTr(71, 2)

[71, 54, 9, 81, 63, 27, 45, 9]

(1)

> RevOpTr(35, 2)

[35, 18, 63, 27, 45, 9, 81, 63]

(2)

> RevOpTr(99, 2)

[99, 0, 0]

(3)

> RevOpTr(123, 3)

[123, 198, 792, 693, 594, 495, 495]

(4)

> RevOpTr(808, 3)

(5)

	[808, 792, 693, 594, 495, 495]	(5)
> <i>RevOpTr</i> (495, 3)	[495, 495]	(6)
> <i>RevOpTr</i> (9012, 4)	[9012, 9081, 9621, 8352, 6174, 6174]	(7)
> <i>RevOpTr</i> (6310, 4)	[6310, 6174, 6174]	(8)
> <i>RevOpTr</i> (1760, 4)	[1760, 7443, 3996, 6264, 4176, 6174, 6174]	(9)
>		