OK to post

Julian Herman, 10/22/21, Assignment 14

1.) I was absent, so I'll do it:

$$x(n) = x(n-1) + y(n-1)^4 - \frac{1}{16}$$

$$y(n) = x(n-1)^2 + y(n-1) - \frac{1}{9}$$

At equilibrium: $x(n) = x(n-1)$, $y(n) = y(n-1)$

$x = x + y^4 - \frac{1}{16}$ $\qquad$ $y = x^2 + y - \frac{1}{9}$

$y^4 = \frac{1}{16}$ $\qquad$ $x^2 = \frac{1}{9}$

$y = \pm \frac{1}{2}$ $\qquad$ $x = \pm \frac{1}{3}$

EQ points (x,y): $(\frac{1}{3}, \frac{1}{2})$, $(\frac{1}{3}, \frac{-1}{2})$, $(\frac{-1}{3}, \frac{1}{2})$, $(\frac{-1}{3}, \frac{-1}{2})$

Let $f(x,y) = x + y^4 - \frac{1}{16}$, $\quad g(x,y) = x^2 + y - \frac{1}{9}$

$$\text{Jacobian} = \begin{vmatrix} \frac{\partial f}{\partial x}, & \frac{\partial f}{\partial y} \\ \frac{\partial g}{\partial x}, & \frac{\partial g}{\partial y} \end{vmatrix} = \begin{vmatrix} 1, & 4y^3 \\ 2x, & 1 \end{vmatrix}$$

For $(\frac{1}{3}, \frac{1}{2})$: $\det\left(\begin{vmatrix} 1-\lambda, & \frac{1}{2} \\ 2/3, & 1-\lambda \end{vmatrix}\right) = 0 \quad (1-\lambda)^2 - \frac{1}{3} = 0$

$$1 - 2\lambda + \lambda^2 - \frac{1}{3} = 0$$

$$\lambda^2 - 2\lambda + \frac{2}{3} = 0$$

$$\lambda = \frac{2 \pm \sqrt{4 - 4(1)(\frac{2}{3})}}{2} = 1 \pm \frac{\sqrt{\frac{12}{3} - \frac{8}{3}}}{2} = 1 \pm \frac{\sqrt{\frac{4}{3}}}{2}$$

$$\lambda = 1 \pm \frac{1}{\sqrt{3}} \qquad \left|1 - \frac{1}{\sqrt{3}}\right| < 1 \checkmark$$

$$\left|1 + \frac{1}{\sqrt{3}}\right| \not< 1 \quad X$$

$(\frac{1}{3}, \frac{1}{2})$ is NOT stable

For $(\frac{1}{3}, -\frac{1}{2})$: $\det\left(\begin{vmatrix} 1-\lambda, & -\frac{1}{2} \\ \frac{2}{3}, & 1-\lambda \end{vmatrix}\right) = 0$

$$(1-\lambda)^2 + \frac{1}{3} = 0$$

$$1 - 2\lambda + \lambda^2 + \frac{1}{3} = 0$$

$$\lambda^2 - 2\lambda + \frac{4}{3} = 0$$

$\lambda = 4 \pm \sqrt{4 - \frac{16}{3}}$ -> $\sqrt{-}$ imaginary roots

$(\frac{1}{3}, -\frac{1}{2})$ indeterminate, cannot check if it is stable using this method.

Using numerics (refer to my maple code), this is NOT STABLE

For $\left(-\frac{1}{3}, \frac{1}{2}\right)$: $\det\left(\begin{vmatrix} 1-\lambda, & \frac{1}{2} \\ -\frac{2}{3}, & 1-\lambda \end{vmatrix}\right) = 0$ $\quad (1-\lambda)^2 + \frac{1}{3} = 0$

$\lambda^2 - 2\lambda + \frac{4}{3} = 0$ → SAME characteristic eq. as above pt.

→ NOT STABLE

For $\left(-\frac{1}{3}, -\frac{1}{2}\right)$: $\det\left(\begin{vmatrix} 1-\lambda, & -\frac{1}{2} \\ -\frac{2}{3}, & 1-\lambda \end{vmatrix}\right) = 0$ $\quad (1-\lambda)^2 - \frac{1}{3} = 0$

$\lambda^2 - 2\lambda + \frac{2}{3} = 0$ → SAME characteristic eq. as the first pt.

→ NOT STABLE

⇒ NO STABLE FIXED POINTS !

2) Mapping:

$x \to x^3 \bmod 17$
1 → 1
2 → 8
3 → 10
4 → 13
5 → 6
6 → 12
7 → 3
8 → 2
9 → 15
10 → 14
11 → 5
12 → 11
13 → 4
14 → 7
15 → 9
16 → 16

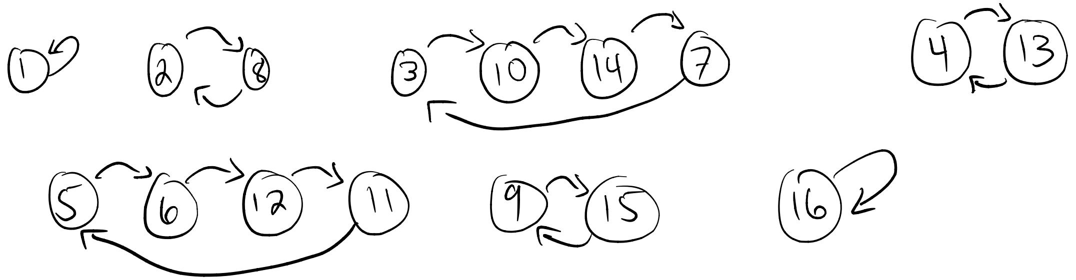
Periodic Cycles:

[1, 1], [2, 8, 2], [3, 10, 14, 7, 3]

[4, 13, 4], [5, 6, 12, 11, 5], [9, 15, 9]

[16, 16]

→ fixed points ([1, 1] and [16, 16])



All 16 trajectories:

[1,1], [2,8,2], [3,10,14,7,3], [4,13,4], [5,6,12,11,5]

[6,12,11,5,6], [7,3,10,14,7], [8,2,8], [9,15,9]

[10,14,7,3,10], [11,5,6,12,11], [12,11,5,6,12]

[13,4,13], [14,7,3,10,14], [15,9,15], [16,16]

3) i) random numbers: 68, 53, 47, 78, 36, 26, 99, 43, 27, 61

$T_2(68) = 86-68 = 18$ -> [68, 18, 63, 27, 45, 09, 81, 63]

$T_2(53) = 53-35 = 18$ -> [53, 18, 63, 27, 45, 09, 81, 63]

$T_2(47) = 74-47 = 27$ -> [47, 27, 45, 09, 81, 63, 27]

$T_2(78) = 87-78 = 9$ -> [78, 09, 81, 63, 27, 45, 09]

$T_2(36) = 63-36 = 27$ -> [36, 27, 45, 09, 81, 63, 27]

$T_2(26) = 62-26 = 36$ -> [26, 36, 27, 45, 09, 81, 63, 27]

$T_2(99) = 99-99 = 00$ -> [99, 00, 00] *Fixed pt: 00

$T_2(43) = 43-34 = 09$ -> [43, 09, 81, 63, 27, 45, 09]

$T_2(27) = 72-27 = 45$ -> [27, 45, 09, 81, 63, 27]

$T_2(61) = 61-16 = 45$ -> [61, 45, 09, 81, 63, 27, 48]

*ENDING CYCLE FOR ALL $T_2(n)$: [09, 81, 63, 27, 45, 09]
↳Also, fixed point 00

ii) random numbers: 411, 270, 694, 124, 701

*$T_3(411) = 411 - 114 = 297$
[411, 297, 693, 594, 495, 495]
ending cycle: [495, 495] ⇒ 495 is a FIXED POINT

*$T_3(270) = 720 - 027 = 693$
[270, 693, 594, 495, 495]
same as previous: [495, 495]

*$T_3(694) = 964 - 469 = 495$
[694, 495, 495] SAME: [495, 495]

*$T_3(124) = 421 - 124 = 297$
[124, 297, 693, 594, 495, 495] SAME

*$T_3(701) = 710 - 017 = 693$
[701, 693, 594, 495, 495] SAME

ENDING CYCLE IS ALWAYS [495, 495]

495 is a FIXED POINT of $T_3(n)$

iii) random numbers: 4121, 3055, 7505

* $T_4(4121) = 4211 - 1124 = 3087$

[4121, 3087, 8352, 6174, 6174]

ending cycle is [6174, 6174] => 6174 is FIXED POINT

* $T_4(3055) = 5530 - 0355 = 5175$

[3055, 5175, 5994, 5355, 1998, 8082, 8532, 6174, 6174]

SAME AS PREVIOUS

* $T_4(7505) = 7550 - 0557 = 6993$

[7505, 6993, 6264, 4176, 6174, 6174]

SAME

ENDING CYCLE IS ALWAYS [6174, 6174]

6174 is a FIXED POINT of $T_4(n)$

5) random integers: 23, 6, 18, 30, 59

* [23, 35, 53, 80, 40, 20, 10, 5, 8, 4, 2, 1, 2]

* [6, 3, 5, 8, 4, 2, 1, 2]

* [18, 9, 14, 7, 11, 17, 26, 13, 20, 10, 5, 8, 4, 2, 1, 2]

* [30, 15, 23, 35, 53, 80, 40, 20, 10, 5, 8, 4, 2, 1, 2]

* [59, 89, 134, 67, 101, 152, 76, 38, 19, 29, 44, 22, 11, 17, 26, 13, 20, 10, 5, 8, 4, 2, 1, 2]

⇒ The trajectories end with the cycle: [2, 1, 2]

> #OK to post
#Julian Herman, October 25th, 2021, Assignment 14

>
read `/Users/julianherman/Documents/Rutgers/Fall 2021/Dynamical Models In Biology/HW/M13.txt`

> Help13( )

RT2(x,y,d,K), Orb2(F,x,y,pt0,K1,K2), FP2(F,x,y), SFP2(F,x,y), PlotOrb2(L), FP2drz(F,x,y), SFP2drz(F,x,y) **(1)**

> #1)

> $Orb2\left(\left[x+y^4-\frac{1}{16}, x^2+y-\frac{1}{9}\right], x, y, [0.34, -0.51], 1000, 1010\right)$

[[Float(∞), Float(∞)], [Float(∞), Float(∞)], [Float(∞), Float(∞)], [Float(∞), Float(∞)], [Float(∞), Float(∞)], [Float(∞), Float(∞)], [Float(∞), Float(∞)], [Float(∞), Float(∞)], [Float(∞), Float(∞)], [Float(∞), Float(∞)], [Float(∞), Float(∞)]] **(2)**

> #$\left[\frac{1}{3}, -\frac{1}{2}\right]$ *is* **not** *stable, a small deviation results inifinity*

> #2)

> $seq(i^3 \textbf{ mod } 17, i = 1..16)$

1, 8, 10, 13, 6, 12, 3, 2, 15, 14, 5, 11, 4, 7, 9, 16 **(3)**

> #3)

> #i)
ra := rand(10..99) :

> seq(ra( ), i = 1..10)

68, 53, 47, 78, 36, 26, 99, 43, 27, 61 **(4)**

> #ii)
ra := rand(100..999) :

> seq(ra( ), i = 1..5)

411, 270, 694, 124, 701 **(5)**

> #iii)

> ra := rand(1000..9999) :

> seq(ra( ), i = 1..3)

4121, 3055, 7505 **(6)**

> #4)

> **read** `/Users/julianherman/Documents/Rutgers/Fall 2021/Dynamical Models In Biology/HW/M14.txt`

> Help14( )

RevOp(n,k), RevOpTr(n,k) **(7)**

> T3 := [seq(RevOpTr(i, 3), i = 100..999)] :

> T3[100..110] #just to display a few of the above

[[199, 792, 693, 594, 495, 495], [200, 198, 792, 693, 594, 495, 495], [201, 198, 792, 693, 594, 495, 495], [202, 198, 792, 693, 594, 495, 495], [203, 297, 693, 594, 495, 495], [204, 396, **(8)** 594, 495, 495], [205, 495, 495], [206, 594, 495, 495], [207, 693, 594, 495, 495], [208, 792, 693, 594, 495, 495], [209, 891, 792, 693, 594, 495, 495]]

```
> #For T_3(n): Yes, they are all fixed points! That fixed point being 495 (there is also the fixed point of 000).
> T4 := [seq(RevOpTr(i, 4), i = 1000..9999)]:
> T4[100..110] #just to display a few of the above
```

[[1099, 9711, 8532, 6174, 6174], [1100, 1089, 9621, 8352, 6174, 6174], [1101, 999, 8991, 8082, 8532, 6174, 6174], [1102, 1998, 8082, 8532, 6174, 6174], [1103, 2997, 7173, 6354, 3087, 8352, 6174, 6174], [1104, 3996, 6264, 4176, 6174, 6174], [1105, 4995, 5355, 1998, 8082, 8532, 6174, 6174], [1106, 5994, 5355, 1998, 8082, 8532, 6174, 6174], [1107, 6993, 6264, 4176, 6174, 6174], [1108, 7992, 7173, 6354, 3087, 8352, 6174, 6174], [1109, 8991, 8082, 8532, 6174, 6174]] **(9)**

```
> #For T_4(n): Yes, they are all fixed points! That fixed point being 6174 (there is also the fixed point of 0000).
>
> lastTwo := proc(list) :
    #Using recursion: tells you if the last two values of every nested list are equal. In other words, it tells you if all of the trajectories end in a fixed point!
  if list = [ ] then return true fi:
  return evalb(evalb(list[1][-1] = list[1][-2]) and lastTwo(list[2..])) :
  end proc:
> lastTwo(T3)
```

true **(10)**

```
> lastTwo(T4)
```

true **(11)**

```
> #the above proves that for both T_3(n) and T_4(n), the trajectories always end at a fixed point. there are no cycles before reaching the first fixed point.
```

```
> #6)
> brownie := proc(n) local L, n1 :
  if not (type(n, integer) and n > 0) then
    print(`Bad input`) :
    RETURN(FAIL) :
  fi:
  n1 := n :
  L := [ ] :
  while not member(n1, L) do
    L := [op(L), n1] :
    if (n1 mod 2 = 0) then n1 := n1/2 :
    else n1 := (3·n1 + 1)/2 :
    fi:
```

```
    od:
   [op(L), n1]:
    end proc:
> manyTraj := [seq(brownie(i), i = 2 ..1000)]:
> twoOneTwo := proc(list) :
    if list = [ ] then return true fi:
    return evalb(evalb(list[1][-1] = 2) and twoOneTwo(list[2 ..])) :
       #if the last number is a 2, the sequence must be [2,1] before it!
    end proc:
> #The above checks if the last sequence of each nested list is a [2,1,2]... If this proves to be true for
       many different iterations of brownie, then it would seen as if this is the only cycle or periodic
       orbit.
> twoOneTwo(manyTraj)
                                    true                                    (12)
> brownie(1)
       #this would mess up twoOneTwo() because the last digit is 1 so we don't include it in
       manyTraj (even though it is indeed the same cycle)
                                  [1, 2, 1]                                 (13)
> #The only periodic orbit appears to be [2,1,2] or [1,2,1] which is the same, just offset.
```