

Jeton Hida

~~Ok~~ Ok to post Homeworks

Jeton Hida, Assignment 14, 10/23/21

$$1. \begin{aligned} x(n) &= x(n-1) + y(n-1)^4 - 1/6 & (x, y) \rightarrow (x + y^4 - 1/6, x^2 + y - 1/9) \\ y(n) &= x(n-1)^2 + y(n-1) - 1/9 \end{aligned}$$

$$x = x + y^4 - 1/6$$

$$y^4 - 1/6 = 0$$

$$y = \pm \sqrt[4]{1/6}$$

$$J(1/3, 1/2), (1/3, -1/2), (-1/3, 1/2), (-1/3, -1/2)$$

set of fixed points =

$$J = \begin{pmatrix} f_x & f_y \\ g_x & g_y \end{pmatrix} = \begin{pmatrix} 1 & 4y^3 \\ 2x & 1 \end{pmatrix}$$

$$J(1/3, 1/2) = \begin{pmatrix} 1 & 1/2 \\ 2/3 & 1 \end{pmatrix} \det \begin{pmatrix} 1-\lambda & 1/2 \\ 2/3 & 1-\lambda \end{pmatrix} = 0$$

$$(1-\lambda)^2 - 1/3 = 0$$

$$(1-\lambda) = \pm \sqrt{1/3}$$

$$\lambda = (1 + \sqrt{1/3}), (1 - \sqrt{1/3})$$

Since not all of $|\lambda| < 1$ $(1/3, 1/2)$ is unstable

$$J(1/3, -1/2) = \begin{pmatrix} 1 & -1/2 \\ 2/3 & 1 \end{pmatrix} \det \begin{pmatrix} 1-\lambda & -1/2 \\ 2/3 & 1-\lambda \end{pmatrix} = 0$$

$$(1-\lambda)^2 + 1/2 = 0$$

$$1-\lambda = \pm i\sqrt{1/2}$$

$$\lambda = 1 - i\sqrt{1/2}, 1 + i\sqrt{1/2}$$

$$a + Bi$$

iff $a^2 + B^2 < 1 = \text{stable}$

$1^2 + 1/2 = 3/2 > 1$ unstable at $(1/3, -1/2)$

$$J(-1/3, 1/2) = \begin{pmatrix} 1 & 1/2 \\ -2/3 & 1 \end{pmatrix}$$

$$\det \begin{pmatrix} 1-\lambda & 1/2 \\ -2/3 & 1-\lambda \end{pmatrix} = 0 \rightarrow (1-\lambda)^2 + 1/3 = 0$$

$$1^2 + \sqrt{1/3}^2 = 4/3 > 1 \text{ unstable at } (-1/3, 1/2)$$

$$J(-1/3, -1/2) = \begin{pmatrix} 1 & -1/2 \\ -2/3 & 1 \end{pmatrix}$$

$$\det \begin{pmatrix} 1-\lambda & -1/2 \\ -2/3 & 1-\lambda \end{pmatrix} = 0 \rightarrow (1-\lambda)^2 - 1/3 = 0$$

$$\lambda = 1 \pm \sqrt{1/3}$$

Since not all $|\lambda| < 1$, $(-1/3, -1/2)$ is unstable

2. $x \rightarrow x^3 \pmod{17}$

$$1 \rightarrow 1$$

$$2 \rightarrow 8$$

$$3 \rightarrow 10$$

$$4 \rightarrow 13$$

$$1 \rightarrow 1$$

$$8 \rightarrow 2$$

$$10 \rightarrow 14$$

$$13 \rightarrow 4$$

$$[1, 13]$$

$$[2, 8, 27]$$

$$[14 \rightarrow 7]$$

$$[4, 13, 4]$$

$$[7 \rightarrow 3]$$

$$[3, 10, 14, 7, 3]$$

$$5 \rightarrow 6$$

$$6 \rightarrow 12$$

$$7 \rightarrow 3$$

$$8 \rightarrow 2$$

$$6 \rightarrow 12$$

$$12 \rightarrow 11$$

$$3 \rightarrow 10$$

$$2 \rightarrow 8$$

$$12 \rightarrow 11$$

$$11 \rightarrow 5$$

$$10 \rightarrow 14$$

$$[8, 2, 8]$$

$$11 \rightarrow 5$$

$$5 \rightarrow 6$$

$$14 \rightarrow 7$$

$$[5, 6, 12, 11, 5]$$

$$[6, 12, 11, 5, 6]$$

$$[7, 3, 10, 14, 7]$$

$$9 \rightarrow 15$$

$$10 \rightarrow 14$$

$$11 \rightarrow 5$$

$$12 \rightarrow 11$$

$$15 \rightarrow 9$$

$$14 \rightarrow 7$$

$$5 \rightarrow 6$$

$$11 \rightarrow 5$$

$$[9, 15, 9]$$

$$7 \rightarrow 3$$

$$6 \rightarrow 12$$

$$5 \rightarrow 6$$

$$3 \rightarrow 10$$

$$12 \rightarrow 11$$

$$6 \rightarrow 12$$

$$[16, 14, 7, 3, 10]$$

$$[11, 5, 6, 12, 11]$$

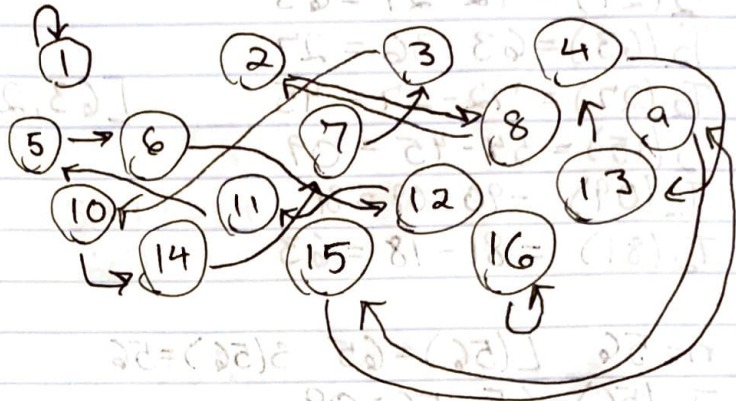
$$[12, 11, 5, 6, 12]$$

13 → 4
 4 → 13
 [13, 4, 13]

14 → 7
 7 → 3
 3 → 10
 10 → 14
 [14, 7, 3, 10, 14]

15 → 9
 9 → 15
 [5, 9, 15]

16 → 16
 [6, 16]



3. $T_k(n)$, ϵ_2 , 18, P_0

i. $k=2$

$n=21$ $L(21)=21$ $S(21)=12$
 $T_2(21) = 21 - 12 = 09$
 $T_2(09) = 90 - 09 = 81$ [09, 81, 63, 27, 45, 09]
 $T_2(81) = 81 - 18 = 63$
 $T_2(63) = 63 - 36 = 27$
 $T_2(27) = 72 - 27 = 45$
 $T_2(45) = 54 - 45 = 09$

$n=17$ $L(17)=71$ $S(17)=17$
 $T_2(17) = 71 - 17 = 54$
 $T_2(54) = 54 - 45 = 09$
 $T_2(09) = 90 - 09 = 81$ [09, 81, 63, 27, 45, 09]
 $T_2(81) = 81 - 18 = 63$
 $T_2(63) = 63 - 36 = 27$
 $T_2(27) = 72 - 27 = 45$
 $T_2(45) = 54 - 45 = 09$

$$n=44 \quad L(44)=44 \quad S(44)=44$$

$$T_2(44) = 44 - 44 = 00$$

$$T_2(00) = 00 - 00 = 00$$

[00,00]

$$n=29 \quad L(29)=92 \quad S(29)=29$$

$$T_2(29) = 92 - 29 = 63$$

$$T_2(63) = 63 - 36 = 27$$

$$T_2(27) = 72 - 27 = 45$$

$$T_2(45) = 54 - 45 = 09$$

$$T_2(09) = 90 - 09 = 81$$

$$T_2(81) = 81 - 18 = 63$$

[63,27,45,09,81,63]

$$n=56 \quad L(56)=65 \quad S(56)=56$$

$$T_2(56) = 65 - 56 = 09$$

$$T_2(09) = 90 - 09 = 81$$

$$T_2(81) = 81 - 18 = 63$$

$$T_2(63) = 63 - 36 = 27$$

$$T_2(27) = 72 - 27 = 45$$

$$T_2(45) = 54 - 45 = 09$$

[09,81,63,27,45,09]

$$n=82 \quad L(82)=82 \quad S(82)=28$$

$$T_2(82) = 82 - 28 = 54$$

$$T_2(54) = 54 - 45 = 09$$

$$T_2(09) = 81$$

$$T_2(81) = 63$$

$$T_2(63) = 27$$

$$T_2(27) = 45$$

$$T_2(45) = 09$$

[09,81,63,27,45,09]

$$n=95 \quad L(95)=95 \quad S(95)=59$$

$$T_2(95) = 95 - 59 = 36$$

$$T_2(36) = 63 - 36 = 27$$

$$T_2(27) = 45$$

$$T_2(45) = 09 \quad [27, 45, 09, 81, 63, 27]$$

$$T_2(09) = 81$$

$$T_2(81) = 63$$

$$T_2(63) = 27$$

$$n=41 \quad L(41)=41 \quad S(41)=14$$

$$T_2(41) = 41 - 14 = 27$$

$$T_2(27) = 45$$

$$T_2(45) = 09 \quad [27, 45, 09, 81, 63, 27]$$

$$T_2(09) = 81$$

$$T_2(81) = 63$$

$$T_2(63) = 27$$

$$n=37 \quad L(37)=73 \quad S(37)=37$$

$$T_2(37) = 73 - 37 = 36$$

$$T_2(36) = 27$$

$$T_2(27) = 45 \quad [27, 45, 09, 81, 63, 27]$$

$$T_2(45) = 09$$

$$T_2(09) = 81$$

$$T_2(81) = 63$$

$$T_2(63) = 27$$

$$n=90 \quad L(90)=90 \quad S(90)=09$$

$$T_2(90) = 81$$

$$T_2(81) = 63 \quad [81, 63, 27, 45, 09, 81]$$

$$T_2(63) = 27$$

$$T_2(27) = 45$$

$$T_2(45) = 09$$

$$T_2(09) = 81$$

$$Li. k=3$$

$$n=112 \quad L(112)=211, B(112)=712$$

$$T_3(112) = 211 - 112 = 99$$

$$T_3(099) = 990 - 099 = 891$$

$$T_3(891) = 981 - 189 = 792$$

$$T_3(792) = 972 - 279 = 693$$

$$T_3(693) = 963 - 369 = 594$$

$$T_3(594) = 954 - 459 = 495$$

$$T_3(495) = 954 - 459 = 495$$

$$n=973$$

$$T_3(973) = 973 - 379 = 594$$

$$T_3(594) = 495 \quad [495, 495]$$

$$T_3(495) = 495$$

$$n=718$$

$$T_3(718) = 871 - 178 = 693$$

$$T_3(693) = 594$$

$$T_3(594) = 495$$

$$T_3(495) = 495$$

$$n=360$$

$$T_3(360) = 630 - 036 = 594$$

$$T_3(594) = 495$$

$$T_3(495) = 495$$

$$n=257$$

$$T_3(257) = 752 - 257 = 495$$

$$T_3(495) = 495 \quad [495, 495]$$

iii. $k=4$

$n=2021$

$T_4(2021) = 2210 - 0122 = 2088$

$T_4(2088) = 8820 - 0288 = 8532$

$T_4(8532) = 8532 - 2358 = 6174$

[6174, 6174]

$T_4(6174) = 7641 - 1467 = 6174$

$n=8641$

$T_4(8641) = 8641 - 1468 = 7173$

$T_4(7173) = 7731 - 1773 = 5958$

$T_4(5958) = 9855 - 5589 = 4266$

$T_4(4266) = 6642 - 2466 = 4176$

$T_4(4176) = 7641 - 1467 = 6174$

[6174, 6174]

$T_4(6174) = 6174$

$n=9991$

$T_4(9991) = 9991 - 1999 = 7992$

$T_4(7992) = 9972 - 2799 = 7173$

$T_4(7173) = 5958$

$T_4(5958) = 4266$

$T_4(4266) = 4176$

[6174, 6174]

$T_4(4176) = 6174$

$T_4(6174) = 6174$

5. $n \rightarrow \frac{n}{2}$ if n is even, $n \rightarrow \frac{3n+1}{2}$ if n is odd

$n=26$

$26 \rightarrow 13$ $2 \rightarrow 1$

$13 \rightarrow 20$ $1 \rightarrow 2$

$20 \rightarrow 10$ [2, 1, 2]

$10 \rightarrow 5$

$5 \rightarrow 8$

$8 \rightarrow 4$

$4 \rightarrow 2$

$n=17$

$17 \rightarrow 26$

$26 \rightarrow 13$

$13 \rightarrow 20$

$20 \rightarrow 10$

$10 \rightarrow 5$

$5 \rightarrow 8$

$8 \rightarrow 4$

$4 \rightarrow 20$

$2 \rightarrow 1$

$1 \rightarrow 2$

[2, 1, 2]

```
[> read "/Users/jeton/Desktop/Math 336/M14.txt"
```

```
> #Number 4
```

```
> RevOpTr(134,3)
```

```
[134, 297, 693, 594, 495, 495]
```

```
(1)
```

```
> RevOpTr(590,3)
```

```
[590, 891, 792, 693, 594, 495, 495]
```

```
(2)
```

```
> RevOpTr(619,3)
```

```
[619, 792, 693, 594, 495, 495]
```

```
(3)
```

```
> RevOpTr(495,3)
```

```
[495, 495]
```

```
(4)
```

```
> RevOpTr(999,3)
```

```
[999, 0, 0]
```

```
(5)
```

```
> RevOpTr(210,3)
```

```
[210, 198, 792, 693, 594, 495, 495]
```

```
(6)
```

```
> RevOpTr(1234,4)
```

```
[1234, 3087, 8352, 6174, 6174]
```

```
(7)
```

```
> RevOpTr(3902,4)
```

```
[3902, 9081, 9621, 8352, 6174, 6174]
```

```
(8)
```

```
> RevOpTr(2001,4)
```

```
[2001, 2088, 8532, 6174, 6174]
```

```
(9)
```

```
> RevOpTr(4444,4)
```

```
[4444, 0, 0]
```

```
(10)
```

```
> #For T3, 495 and 0 are fixed points. For T4, 6174 and 0 are fixed points.
```


n = 250

250 → 125

125 → 188

188 → 94

94 → 47

47 → 71

71 → 107

107 → 161

161 → 242

242 → 121

121 → 182

182 → 91

91 → 137

137 → 206

206 → 103

103 → 155

155 → 233

233 → 350

350 → 175

175 → 263

263 → 395

395 → 593

593 → 890

890 → 445

445 → 668

668 → 334

334 → 167

167 → 251

251 → 377

377 → 566

566 → 283

283 → 425

425 → 638

638 → 319

319 → 479

479 → 719

719 → 1079

1079 → 1619

1619 → 2429

2429 → 3644

n = 330
3644 → 1822
1822 → 911
911 → 1367
1367 → 2051
2051 → 3077
3077 → 4616
4616 → 2308
2308 → 1154
1154 → 577
577 → 866
866 → 433
433 → 650
650 → 325

325 → 488

488 → 244

244 → 122

122 → 61

61 → 92

92 → 46

46 → 23

23 → 35

35 → 53

53 → 80

80 → 40

40 → 20

20 → 10

10 → 5

5 → 8

8 → 4

4 → 2

2 → 1

1 → 2

[2, 1, 2]

1 PPP = 1

5 PPP = PPP1 - 1 PPP = (1 PPP) PT

8 PPP = PPP8 - 6 PPP = (8 PPP) PT

[2, 1, 2] = (8 PPP) PT

2 PPP = (8 PPP) PT

2 PPP = (2 PPP) PT

4 PPP = (2 PPP) PT

4 PPP = (4 PPP) PT

1000 at n = 1000, reverse n = 1000

11 = 11

2-0 = 0

25 = 25

1 = 1

81 = 28

81 = 28

5 = 1

02 = 81

02 = 81

[6, 1, 8]

80 = 10

80 = 10

7 = 10

7 = 10

8 = 7

8 = 7

14 = 8

14 = 8

5 = 14

$$n = 21$$

$$21 \rightarrow 32$$

$$32 \rightarrow 16$$

$$16 \rightarrow 8$$

$$8 \rightarrow 4$$

$$4 \rightarrow 2$$

$$2 \rightarrow 1$$

$$1 \rightarrow 2$$

[2, 1, 2]

$$n = 45$$

$$45 \rightarrow 68$$

$$68 \rightarrow 34$$

$$34 \rightarrow 17$$

$$17 \rightarrow 26$$

$$26 \rightarrow 13$$

$$13 \rightarrow 20$$

$$20 \rightarrow 10$$

$$10 \rightarrow 5$$

$$5 \rightarrow 8$$

$$8 \rightarrow 4$$

$$4 \rightarrow 2$$

$$2 \rightarrow 1$$

$$1 \rightarrow 2$$

[2, 1, 2]