Okay to post!

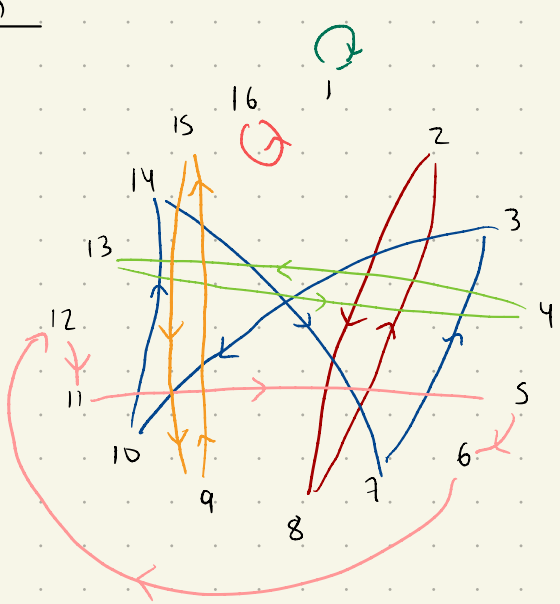
Anusha Nagar, Homework 14, 10/25/2021

(1) I got the attendenu question correct.

(2) $x \to x^3 \mod 17$

$F(z) = z^3 \mod 17$

z	F(z)
1	1
2	8
3	10
4	13
5	6
6	12
7	3
8	2
9	15
10	14
11	5
12	11
13	4
14	7
15	9
16	16



[1,1]

[2,8,2], [8,2,8]

[3,10,14,7,3], [10,14,7,3,10], [14,7,3,10,14], [7,3,10,14,9]

[4,13,4], [13,4,13]

[5,6,12,11,5], [6,12,11,5,6], [12,11,5,6,12], [11,5,6,12,11]

[9,15,9], [15,9,15]

[16,16]

(3) a) From RNG: 72, 78, 28, 48, 10, 38, 31, 57, 33, 16

$n = 72$:

$L(72) = 72$, $S(72) = 27$ $\Rightarrow T_2(72) = 45$

$T_2(45) = 54 - 45 = 09$

$T_2(09) = 90 - 9 = 81$

$T_2(81) = 81 - 18 = 63$

$T_2(63) = 63 - 36 = 27$

$T_2(27) = 72 - 27 = 45$

[72, 45, 09, 81, 63, 27, 45, ...]

Length 5 $\Rightarrow$ Cycle: [45, 09, 81, 63, 27, 45]

$n = 78$: $T_2(78) = 09$

$T_2(09) = 81$

$T_2(81) = 63$

$T_2(63) = 27$

$T_2(27) = 45$

$T_2(45) = 09$

[78, 09, 81, 63, 27, 45, 09]

Length 5

Cycle: [09, 81, 63, 27, 45, 09]

$n = 28$: $T_2(28) = 82 - 28 = 54$

$T_2(54) = 09$

$T_2(09) = 81$

$T_2(81) = 63$

$T_2(63) = 27$

$T_2(27) = 45$

$T_2(45) = 09$

[28, 54, 09, 81, 63, 27, 45, 09, ...]

Length: 5

Cycle: [09, 81, 63, 27, 45, 09]

$n = 48$: $T_2(48) = 36$

$T_2(36) = 27$

$T_2(27) = 45$

$T_2(45) = 09$

$T_2(09) = 81$

$T_2(81) = 63$

$T_2(63) = 27$

[48, 36, 27, 45, 09, 81, 63, 27, ...]

Length 5

Cycle: [27, 45, 09, 81, 63, 27]

$n = 10$: $T_2(10) = 09$
$T_2(09) = 81$
$T_2(81) = 63$
$T_2(63) = 27$
$T_2(27) = 45$
$T_2(45) = 09$

$[10, 09, 81, 63, 27, 45, 09, \ldots)$

Length 5

Cycle: $[09, 81, 63, 27, 45, 09)$

$n = 38$: $T_2(38) = 45$
$T_2(45) = 54 - 45 = 09$
$T_2(09) = 90 - 9 = 81$
$T_2(81) = 81 - 18 = 63$
$T_2(63) = 63 - 36 = 27$
$T_2(27) = 72 - 27 = 45$

$[38, 45, 09, 81, 63, 27, 45, \ldots]$

Length 5

Cycle: $[45, 09, 81, 63, 27, 45]$

$n = 31$: $T_2(31) = 18$
$T_2(18) = 63$
$T_2(63) = 27$
$T_2(27) = 45$
$T_2(45) = 09$
$T_2(09) = 81$
$T_2(81) = 63$

$[31, 18, 63, 27, 45, 09, 81, 63, \ldots]$

Length 5

Cycle: $[63, 27, 45, 09, 81, 63]$

$n = 57$: $T_2(57) = 18$
$T_2(18) = 63$
$T_2(63) = 27$
$T_2(27) = 45$
$T_2(45) = 09$
$T_2(09) = 81$
$T_2(81) = 63$

$[57, 18, 63, 27, 45, 09, 81, 63, \ldots]$

Length 5

Cycle: $[63, 27, 45, 09, 81, 63]$

$n = 33$: $T_2(33) = 00$
$T_2(00) = 00$

Length 1 $\Rightarrow$ cycle: $[00, 00)$

$[33, 00, 00, \ldots]$

$n = 16$: $T_2(16) = 45$
$T_2(45) = 09$
$T_2(09) = 81$
$T_2(81) = 63$
$T_2(63) = 27$
$T_2(27) = 45$

$[16, 45, 09, 81, 63, 27, 45, \ldots)$

Length 5 $\Rightarrow$ cycle: $[45, 09, 81, 63, 27, 45]$

* Interesting to note:
lengths of 1 & 5 only

(ii) $n = 922, 539, 782, 579, 583$

$n = 922$: $T_3(922) = 922 - 229 = 693$

$T_3(693) = 963 - 369 = 594$

$T_3(594) = 954 - 459 = 495$

$T_3(495) = 954 - 459 = 495$

$[922, 693, 594, 495, 495, \ldots]$

Length 1

Cycle: $[495, 495, \ldots]$

$n = 539$: $T_3(539) = 594$

$T_3(594) = 495$

$T_3(495) = 495$

$[539, 594, 495, 495, \ldots]$

Length 1

Cycle: $[495, \ldots]$

$n = 782$: $T_3(782) = 594$

$T_3(594) = 495$

$T_3(495) = 495$

$[782, 594, 495, 495, \ldots]$

Length 1

Cycle: $[495, 495, \ldots]$

$n = 579$: $T_3(579) = 396$

$T_3(396) = 594$

$T_3(594) = 495$

$T_3(495) = 495$

$[579, 396, 594, 495, 495, \ldots]$

Length 1

Cycle: $[495, 495, \ldots]$

$n = 583$: $T_3(583) = 495$

$T_3(495) = 495$

$[583, 495, 495, \ldots]$

Length 1

Cycle: $[495, 495, \ldots]$

(iii) $n = 2193, 4890, 6890$

$n = 2193$: $T_4(2193) = 8082$

$T_4(8082) = 8532$

$T_4(8532) = 6174$

$T_4(6174) = 6174$

$[2193, 8082, 8532, 6174, 6174, \ldots]$

Length 1

Cycle: $[6174, \ldots]$

$n = 4890$: $T_4(4890) = 9351$

$T_4(9351) = 8172$

$T_4(8172) = 7443$

$T_4(7443) = 3996$

$T_4(3996) = 6264$

$T_4(6264) = 4176$

$T_4(4176) = 6174$

$T_4(6174) = 6174$

$[4890, 9351, 8172, 7443, 3996, 6264, 4176, 6174, 6174, \ldots]$

Length 1

Cycle: $[6174, \ldots]$

n=6840: $T_4(6840) = 9171$

$T_4(9171) = 8532$

$T_4(8532) = 6174$

$T_4(6174) = 6174$

[6840, 9171, 8532, 6174, 6174, ...]

Length 1

Cycle: [6174, ...]

(4) See maple code (next page)

```
> #OK to post
> #Anusha Nagar, Homework 14, 10.25.2021
>
> #Problem 4
>
> read "C://Users/an646/Documents/M14.txt"
> #k =3, n=922, 539, 782, 579, 583
> RevOp(922, 3)
```

693 **(1)**

```
> RevOp(693, 3)
```

594 **(2)**

```
> RevOp(594, 3)
```

495 **(3)**

```
> RevOp(495, 3)
```

495 **(4)**

```
> #Fixed point at 495 -> cycle: [495,495]
> RevOpTr(922, 3)
```

[922, 693, 594, 495, 495] **(5)**

```
> RevOp(539, 3)
```

594 **(6)**

```
> RevOp(594, 3)
```

495 **(7)**

```
> RevOp(495, 3)
```

495 **(8)**

```
> #Fixed point at 495 -> cycle: [495,495]
> #Demonstrate using RevOpTr for remainder:
> RevOpTr(782, 3)
```

[782, 594, 495, 495] **(9)**

```
> RevOpTr(579, 3)
```

[579, 396, 594, 495, 495] **(10)**

```
> RevOpTr(583, 3)
```

[583, 495, 495] **(11)**

```
> RevOpTr(2193, 4)
```

[2193, 8082, 8532, 6174, 6174] **(12)**

```
> RevOpTr(4890, 4)
```

[4890, 9351, 8172, 7443, 3996, 6264, 4176, 6174, 6174] **(13)**

```
> RevOpTr(6890, 4)
```

[6890, 9171, 8532, 6174, 6174] **(14)**

```
> #We see that there is only one fixed point for both k =3 and k=4. For k=3, it is at 495, and for k=
      4, it is at 6174
>
```

(5) $n \to \frac{n}{2}$ if even, $n \to \frac{3n+1}{2}$ if odd

$n = 5 \Rightarrow 8$
$n = 8 \Rightarrow 4$ [5, 8, 4, 2, 1, 2]
$n = 4 \Rightarrow 2$
$n = 2 \Rightarrow 1$
$n = 1 \Rightarrow 2$

These all end up in an orbit between 2 + 1 [2, 1, 2]

$n = 12 \Rightarrow 6$
$n = 6 \Rightarrow 3$ [12, 6, 3, 5, 8, 4, 2, 1, 2]
$n = 3 \Rightarrow 5$
$n = 5 \Rightarrow 8$
$n = 8 \Rightarrow 4$
$n = 4 \Rightarrow 2$
$n = 2 \Rightarrow 1$
$n = 1 \Rightarrow 2$

$n = 75 \Rightarrow 113$
$n = 113 \Rightarrow 170$
$n = 170 \Rightarrow 85$
$n = 85 \Rightarrow 128$ [75, 113, 170, 85, 128, 64, 32, 16, 8, 4, 2, 1, 2]
$n = 128 \Rightarrow 64$
$n = 64 \Rightarrow 32$
$n = 32 \Rightarrow 16$
8
4
2
1
2

$n = 1 \Rightarrow 2$
$n = 2 \Rightarrow 1$ [1, 2, 1, 2]
$n = 1 \Rightarrow 2$

$n = 7 \Rightarrow 11$
$n = 11 \Rightarrow 17$
$n = 17 \Rightarrow 26$
$n = 26 \Rightarrow 13$ [7, 11, 17, 26, 13, 20, 10, 5, 8, 4, 2, 1, 2]
$n = 13 \Rightarrow 20$
$n = 20 \Rightarrow 10$
$n = 10 \Rightarrow 5$

$n = 5 \Rightarrow 8$
$n = 8 \Rightarrow 4$
$n = 4 \Rightarrow 2$
$n = 2 \Rightarrow 1$
$n = 1 \Rightarrow 2$