Hw 14 - Alun Ho

ok to post

1)
$$x(n) = x(n-1) + y(n-1)^4 - \tfrac{1}{16}$$
$$y(n) = x(n-1)^2 + y(n-1) - \tfrac{1}{9}$$

$x = x + y^4 - \frac{1}{16} \Rightarrow y^4 = \frac{1}{16}$ $\quad y = \pm\frac{1}{2}$

$y = x^2 + y - \frac{1}{9} \Rightarrow x^2 = \frac{1}{9}$ $\quad x = \pm\frac{1}{3}$

$\boxed{\text{Fixed points: } \{(\tfrac{1}{2},\tfrac{1}{3}),\ (\tfrac{1}{2},-\tfrac{1}{3}),\ (-\tfrac{1}{2},\tfrac{1}{3}),\ (-\tfrac{1}{2},-\tfrac{1}{3})\}}$

$$J = \begin{bmatrix} 1 & 4y^3 \\ 2x & 1 \end{bmatrix} \qquad J_{(\frac{1}{2},\frac{1}{3})} = \begin{bmatrix} 1 & \frac{4}{27} \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1-\lambda & \frac{4}{27} \\ 1 & 1-\lambda \end{bmatrix}$$

$= 1 - 2\lambda + \lambda^2 - \frac{4}{27}$

$= \cancel{-2\lambda + \lambda^2 - \frac{4}{27} = -2\lambda = \frac{50}{27}}$

$= \lambda^2 - 2\lambda + \frac{23}{27}$ $\quad = \lambda = 0.61,\ 1.38$

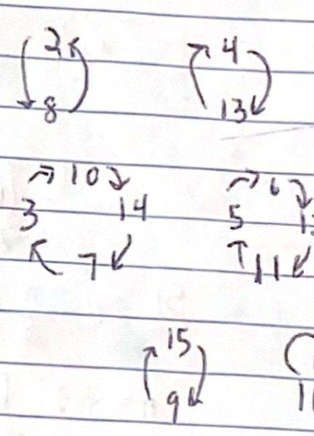
$\boxed{(\tfrac{1}{2},\tfrac{1}{3}) \text{ is not stable b/c not all of its eigenval } < 1}$

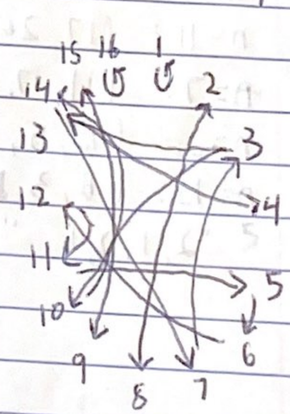
2)

$x \to x^3 \bmod 17$

$f(x) = x^3$

| x | f(x) | mod 17 | |
|---|---|---|---|
| 1 | 1 | | |
| 2 | 8 | | [2, 8, 2] |
| 3 | 27 | 10 | [3, 10, 14, 7, 3] |
| 4 | 64 | 13 | [4, 13, 4] |
| 5 | 125 | 6 | [5, 6, 12, 11, 5] |
| 6 | 216 | 12 | [6, 12, 11, 5, 6] |
| 7 | 343 | 3 | [7, 3, 10, 14, 7] |
| 8 | 512 | 2 | [8, 2, 8] |
| 9 | 729 | 15 | [9, 15, 9] |
| 10 | 1000 | 14 | [10, 14, 7, 3, 10] |
| 11 | 1,331 | 5 | [11, 5, 6, 12, 11] |
| 12 | 1,728 | 11 | [12, 11, 5, 6, 12] |
| 13 | 2,197 | 4 | [13, 4, 13] |
| 14 | 2,744 | 7 | [14, 7, 3, 10, 14] |
| 15 | 3,375 | 9 | [15, 9, 15] |
| 16 | 4,096 | 16 | [16, 16] |



3) i) $T_2(63) = 63-36 = [27, 45, 9, 81, 63, 27]$

$T_2(19) = 91-19 = (72, [45, 9, 81, 63, 27, 45])$

$T_2(74) = 74-47 = (27, [45, 9, 81, 63, 27, 45])$

$T_2(36) = 63-36 = (27, [45, 9, 81, 63, 27, 45])$

$T_2(95) = 95-59 = (36, [27, 45, 9, 81, 63, 27])$

$T_2(84) = 84-48 = (36, [27, 45, 9, 81, 63, 27])$

$T_2(72) = 72-27 = [45, 9, 81, 63, 27, 45]$

$T_2(26) = 62-26 = (36, [27, 45, 9, 81, 63, 27])$

$T_2(64) = 64-46 = (18, 63, [27, 45, 9, 81, 63, 27])$

$T_2(87) = 87-78 = [9, 81, 63, 27, 45, 9]$

ii) $T_3(346) = 643-346 = (297, 693, 594, [495, 495, 495])$

$T_3(759) = 975-579 = (396, 594, [495, 495])$

$T_3(472) = 742-247 = [495, 495, 495]$

$T_3(\cancel{8}294) = 942-249 = (693, 594, [495, 495])$

$T_3(827) = 872-278 = (594, [495, 495])$

iii) $T_4(7643) = 7643-3467 = (4176, [6174, 6174])$

$T_4(8793) = 9873-3789 = 6084, 7812, 7434, 3996, 6264, 4176, 6174$

$T_4(4527) = 7542-2457 = (5085, 7992, 7173, 6354, 3087, 8352$

→ $(6084, 8172, 7443, 3996, 6264, 4176, [6174, 6174])$

→ $8352, [6174, 6174])$

5) $n \to \frac{n}{2}$ (n=even) $\quad n \to \frac{3n+1}{2}$ (n=odd)

n=8: (4, [2, 1, 2, 1, 2, 1])

n=11: (17, 26, 13, 20, 10, 5, 8, 4 [2, 1, 2, 1])

n=7: (11, 17, 26, 13, 20, 10, 5, 8, 4, [2, 1, 2, 1])

n=20: (10, 5, 8, 4 [2, 1, 2, 1])

n=12: (6, 3, ~~2~~ 5, 4, [2, 1, 2, 1])

"2, 1, 2, 1..." is the ending cycle for any #