

```

> #OK to post homework
> #Shreya Ghosh, 10-18-2021, Assignment 13
> read "/Users/shreyaghosh/Documents/M13.txt"
> Help13( )
    RT2(x,y,d,K), Orb2(F,x,y,pt0,K1,K2), FP2(F,x,y), SFP2(F,x,y)          (1)
> Help9( )
    Orb(f,x,x0,K1,K2), Orb2D(f,x,x0,K) , FP(f,x) , SFP(f,x) , Comp(f,x)      (2)
> #2.
    
$$F := \left[ \frac{x^2 + 8x + 5}{x^2}, \frac{2x^2 + 8x + 5}{5x^2 + 5} \right]$$

    
$$F := \left[ \frac{x^2 + 8x + 5}{x^2}, \frac{2x^2 + 8x + 5}{5x^2 + 5} \right] \quad (3)$$

> evalf(FP2(F, x, y))
    [[3.604383935, 0.8550603063]]                                     (4)
> evalf(SFP2(F, x, y))
    [[3.604383935, 0.8550603063]]                                     (5)
> Orb2(F, x, y, [8.5, 5], 1000, 1010)
    [[3.604383934, 0.8550603060], [3.604383935, 0.8550603068], [3.604383936, 0.8550603063], [3.604383934, 0.8550603060], [3.604383935, 0.8550603068], [3.604383936, 0.8550603063], [3.604383934, 0.8550603060]]   (6)
>
> #3.
> n := 0 :
> do
    RT2(x, y, 1, 100);
    n := n + 1 :
    until n = 20;
    
$$\left[ \frac{74 + 90y + 74x}{27 + 98y + 72x}, \frac{2 + 73y + 85x}{41 + 4y + 44x} \right]$$

    n := 1
    
$$\left[ \frac{13 + 19y + 10x}{15 + 64y + 9x}, \frac{12 + 52y + 25x}{72 + 90y + 18x} \right]$$

    n := 2
    
$$\left[ \frac{43 + 55y + 40x}{17 + 70y + 52x}, \frac{81 + 87y + 34x}{85 + 9y + 68x} \right]$$

    n := 3
    
$$\left[ \frac{83 + 63y + 100x}{70 + 36y + 36x}, \frac{10 + 40y + 66x}{87 + 16y + 98x} \right]$$

    n := 4

```

$$\left[\frac{43 + 53y + 61x}{47 + 28y + 75x}, \frac{3 + 5y + 11x}{37 + 75y + 4x} \right]$$

$$n := 5$$

$$\left[\frac{91 + 22y + 40x}{58 + 93y + 98x}, \frac{11 + 30y + 6x}{32 + 40y + 24x} \right]$$

$$n := 6$$

$$\left[\frac{80 + 96y + 11x}{23 + 41y + 52x}, \frac{58 + 67y + 81x}{65 + 69y + 2x} \right]$$

$$n := 7$$

$$\left[\frac{36 + 61y + 84x}{96 + 94y + 31x}, \frac{81 + 31y + 54x}{67 + 59y + 66x} \right]$$

$$n := 8$$

$$\left[\frac{12 + 49y + 90x}{35 + 15y + 26x}, \frac{100 + 24y + 8x}{63 + 78y + 23x} \right]$$

$$n := 9$$

$$\left[\frac{73 + 22y + 32x}{98 + 9y + 53x}, \frac{3 + 98y + 69x}{3 + 73y + 88x} \right]$$

$$n := 10$$

$$\left[\frac{37 + 60y + 94x}{52 + 16y + 29x}, \frac{51 + 3y + 45x}{67 + 40y + 71x} \right]$$

$$n := 11$$

$$\left[\frac{74 + 49y + 60x}{69 + 33y + 30x}, \frac{1 + 83y + 9x}{64 + 43y + 57x} \right]$$

$$n := 12$$

$$\left[\frac{52 + 62y + 46x}{76 + 9y + 53x}, \frac{37 + 88y + 50x}{37 + 76y + 95x} \right]$$

$$n := 13$$

$$\left[\frac{8 + 92y + 92x}{2 + 97y + 44x}, \frac{9 + 30y + 14x}{79 + 73y + 21x} \right]$$

$$n := 14$$

$$\left[\frac{78 + 49y + 93x}{15 + 56y + 69x}, \frac{17 + 21y + 42x}{21 + 5y + 58x} \right]$$

$$n := 15$$

$$\left[\frac{3 + 86y + 55x}{97 + 4y + 92x}, \frac{46 + 88y + 34x}{68 + 49y + 61x} \right]$$

$$n := 16$$

$$\left[\frac{21 + 86y + 42x}{5 + 33y + 77x}, \frac{98 + 58y + 98x}{29 + 65y + 29x} \right]$$

$$n := 17$$

$$\begin{aligned}
& \left[\frac{35 + 29y + 34x}{66 + 44y + 60x}, \frac{83 + 32y + 85x}{100 + 68y + 59x} \right] \\
& n := 18 \\
& \left[\frac{40 + 76y + 92x}{39 + 17y + 50x}, \frac{78 + 20y + 18x}{18 + 51y + 34x} \right] \\
& n := 19 \\
& \left[\frac{78 + 10y + 52x}{100 + 13y + 87x}, \frac{13 + 37y + 92x}{97 + 69y + 62x} \right] \\
& n := 20
\end{aligned} \tag{7}$$

$$\begin{aligned}
& > SFP2 \left(\left[\frac{74 + 90y + 74x}{27 + 98y + 72x}, \frac{2 + 73y + 85x}{41 + 4y + 44x} \right], x, y \right) \\
& > SFP2 \left(\left[\frac{13 + 19y + 10x}{15 + 64y + 9x}, \frac{12 + 52y + 25x}{72 + 90y + 18x} \right], x, y \right) \\
& \quad [[0.5763212698, 0.3986163086]]
\end{aligned} \tag{8}$$

$$\begin{aligned}
& > Orb2 \left(\left[\frac{13 + 19y + 10x}{15 + 64y + 9x}, \frac{12 + 52y + 25x}{72 + 90y + 18x} \right], x, y, [2.5, 1.5], 1000, 1010 \right) \\
& [[0.5763212700, 0.3986163084], [0.5763212700, 0.3986163084], [0.5763212700, \\
& 0.3986163084], [0.5763212700, 0.3986163084], [0.5763212700, 0.3986163084], \\
& [0.5763212700, 0.3986163084], [0.5763212700, 0.3986163084], [0.5763212700, \\
& 0.3986163084], [0.5763212700, 0.3986163084], [0.5763212700, 0.3986163084]]]
\end{aligned} \tag{9}$$

$$\begin{aligned}
& > SFP2 \left(\left[\frac{43 + 55y + 40x}{17 + 70y + 52x}, \frac{81 + 87y + 34x}{85 + 9y + 68x} \right], x, y \right) \\
& \quad [[0.9545434862, 1.487092984]]
\end{aligned} \tag{10}$$

$$\begin{aligned}
& > Orb2 \left(\left[\frac{43 + 55y + 40x}{17 + 70y + 52x}, \frac{81 + 87y + 34x}{85 + 9y + 68x} \right], x, y, [2.5, 1.5], 1000, 1010 \right) \\
& [[0.9545434857, 1.487092982], [0.9545434857, 1.487092982], [0.9545434857, 1.487092982], \\
& [0.9545434857, 1.487092982], [0.9545434857, 1.487092982], [0.9545434857, 1.487092982], \\
& [0.9545434857, 1.487092982], [0.9545434857, 1.487092982], [0.9545434857, 1.487092982], \\
& [0.9545434857, 1.487092982], [0.9545434857, 1.487092982], [0.9545434857, 1.487092982]]
\end{aligned} \tag{11}$$

$$\begin{aligned}
& > SFP2 \left(\left[\frac{83 + 63y + 100x}{70 + 36y + 36x}, \frac{10 + 40y + 66x}{87 + 16y + 98x} \right], x, y \right) \\
& \quad [[1.954335211, 0.5615386142]]
\end{aligned} \tag{12}$$

$$\begin{aligned}
& > Orb2 \left(\left[\frac{83 + 63y + 100x}{70 + 36y + 36x}, \frac{10 + 40y + 66x}{87 + 16y + 98x} \right], x, y, [2.5, 1.5], 1000, 1010 \right) \\
& [[1.954335212, 0.5615386143], [1.954335212, 0.5615386143], [1.954335212, 0.5615386143], \\
& [1.954335212, 0.5615386143], [1.954335212, 0.5615386143], [1.954335212, 0.5615386143], \\
& [1.954335212, 0.5615386143], [1.954335212, 0.5615386143], [1.954335212, 0.5615386143], \\
& [1.954335212, 0.5615386143], [1.954335212, 0.5615386143], [1.954335212, 0.5615386143]]
\end{aligned} \tag{13}$$

$$> SFP2 \left(\left[\frac{43 + 53y + 61x}{47 + 28y + 75x}, \frac{3 + 5y + 11x}{37 + 75y + 4x} \right], x, y \right)$$

$$[[0.9123053419, 0.2422215939]] \quad (14)$$

$$> Orb2\left(\left[\frac{43 + 53 y + 61 x}{47 + 28 y + 75 x}, \frac{3 + 5 y + 11 x}{37 + 75 y + 4 x}\right], x, y, [2.5, 1.5], 1000, 1010\right) \\ [[0.9123053421, 0.2422215937], [0.9123053413, 0.2422215939], [0.9123053421, 0.2422215937], \quad (15)$$

$$[0.9123053413, 0.2422215939], [0.9123053421, 0.2422215937], [0.9123053413, 0.2422215939], [0.9123053413, 0.2422215937], [0.9123053413, 0.2422215939], [0.9123053421, 0.2422215937], [0.9123053413, 0.2422215939]]$$

$$> SFP2\left(\left[\frac{91 + 22 y + 40 x}{58 + 93 y + 98 x}, \frac{11 + 30 y + 6 x}{32 + 40 y + 24 x}\right], x, y\right) \\ [[0.7613173482, 0.4199596291]] \quad (16)$$

$$> Orb2\left(\left[\frac{91 + 22 y + 40 x}{58 + 93 y + 98 x}, \frac{11 + 30 y + 6 x}{32 + 40 y + 24 x}\right], x, y, [2.5, 1.5], 1000, 1010\right) \\ [[0.7613173483, 0.4199596292], [0.7613173483, 0.4199596292], [0.7613173483, 0.4199596292], \quad (17)$$

$$[0.7613173483, 0.4199596292], [0.7613173483, 0.4199596292], [0.7613173483, 0.4199596292], [0.7613173483, 0.4199596292], [0.7613173483, 0.4199596292], [0.7613173483, 0.4199596292]]$$

$$> SFP2\left(\left[\frac{80 + 96 y + 11 x}{23 + 41 y + 52 x}, \frac{58 + 67 y + 81 x}{65 + 69 y + 2 x}\right], x, y\right) \\ [[1.501783006, 1.606293788]] \quad (18)$$

$$> Orb2\left(\left[\frac{80 + 96 y + 11 x}{23 + 41 y + 52 x}, \frac{58 + 67 y + 81 x}{65 + 69 y + 2 x}\right], x, y, [2.5, 1.5], 1000, 1010\right) \\ [[1.501783006, 1.606293789], [1.501783006, 1.606293789], [1.501783006, 1.606293789], \quad (19)$$

$$[1.501783006, 1.606293789], [1.501783006, 1.606293789], [1.501783006, 1.606293789], [1.501783006, 1.606293789], [1.501783006, 1.606293789]]$$

$$> SFP2\left(\left[\frac{36 + 61 y + 84 x}{96 + 94 y + 31 x}, \frac{81 + 31 y + 54 x}{67 + 59 y + 66 x}\right], x, y\right)$$

$$> SFP2\left(\left[\frac{12 + 49 y + 90 x}{35 + 15 y + 26 x}, \frac{100 + 24 y + 8 x}{63 + 78 y + 23 x}\right], x, y\right) \\ [[2.451345827, 0.7696344406]] \quad (20)$$

$$> Orb2\left(\left[\frac{12 + 49 y + 90 x}{35 + 15 y + 26 x}, \frac{100 + 24 y + 8 x}{63 + 78 y + 23 x}\right], x, y, [2.5, 1.5], 1000, 1010\right) \\ [[2.451345827, 0.7696344406], [2.451345827, 0.7696344406], [2.451345827, 0.7696344406], \quad (21)$$

$$[2.451345827, 0.7696344406], [2.451345827, 0.7696344406], [2.451345827, 0.7696344406], [2.451345827, 0.7696344406], [2.451345827, 0.7696344406], [2.451345827, 0.7696344406]]$$

$$> SFP2\left(\left[\frac{73 + 22 y + 32 x}{98 + 9 y + 53 x}, \frac{3 + 98 y + 69 x}{3 + 73 y + 88 x}\right], x, y\right)$$

$$> SFP2\left(\left[\frac{37 + 60 y + 94 x}{52 + 16 y + 29 x}, \frac{51 + 3 y + 45 x}{67 + 40 y + 71 x}\right], x, y\right)$$

$$\begin{aligned}
& > SFP2\left(\left[\frac{74+49y+60x}{69+33y+30x}, \frac{1+83y+9x}{64+43y+57x}\right], x, y\right) \\
& > SFP2\left(\left[\frac{52+62y+46x}{76+9y+53x}, \frac{37+88y+50x}{37+76y+95x}\right], x, y\right) \\
& > SFP2\left(\left[\frac{8+92y+92x}{2+97y+44x}, \frac{9+30y+14x}{79+73y+21x}\right], x, y\right) \\
& > SFP2\left(\left[\frac{78+49y+93x}{15+56y+69x}, \frac{17+21y+42x}{21+5y+58x}\right], x, y\right) \\
& > SFP2\left(\left[\frac{3+86y+55x}{97+4y+92x}, \frac{46+88y+34x}{68+49y+61x}\right], x, y\right) \\
& \quad [[0.7527147520, 0.9728903376]] \tag{22}
\end{aligned}$$

$$\begin{aligned}
& > Orb2\left(\left[\frac{3+86y+55x}{97+4y+92x}, \frac{46+88y+34x}{68+49y+61x}\right], x, y, [2.5, 1.5], 1000, 1010\right) \\
& [[0.7527147518, 0.9728903373], [0.7527147522, 0.9728903379], [0.7527147518, \\
& 0.9728903373], [0.7527147522, 0.9728903379], [0.7527147518, 0.9728903373], \\
& [0.7527147522, 0.9728903379], [0.7527147518, 0.9728903373], [0.7527147522, \\
& 0.9728903379], [0.7527147518, 0.9728903373], [0.7527147522, 0.9728903379]] \tag{23}
\end{aligned}$$

$$\begin{aligned}
& > SFP2\left(\left[\frac{21+86y+42x}{5+33y+77x}, \frac{98+58y+98x}{29+65y+29x}\right], x, y\right) \\
& > SFP2\left(\left[\frac{35+29y+34x}{66+44y+60x}, \frac{83+32y+85x}{100+68y+59x}\right], x, y\right) \\
& \quad [[0.5738382141, 0.8318160241]] \tag{24}
\end{aligned}$$

$$\begin{aligned}
& > Orb2\left(\left[\frac{35+29y+34x}{66+44y+60x}, \frac{83+32y+85x}{100+68y+59x}\right], x, y, [2.5, 1.5], 1000, 1010\right) \\
& [[0.5738382136, 0.8318160247], [0.5738382137, 0.8318160242], [0.5738382136, \\
& 0.8318160247], [0.5738382137, 0.8318160242], [0.5738382136, 0.8318160247], \\
& [0.5738382137, 0.8318160242], [0.5738382136, 0.8318160247], [0.5738382137, \\
& 0.8318160242], [0.5738382136, 0.8318160247], [0.5738382137, 0.8318160242]] \tag{25}
\end{aligned}$$

$$\begin{aligned}
& > SFP2\left(\left[\frac{40+76y+92x}{39+17y+50x}, \frac{78+20y+18x}{18+51y+34x}\right], x, y\right) \\
& \quad [[1.922169930, 0.9893531309]] \tag{26}
\end{aligned}$$

$$\begin{aligned}
& > Orb2\left(\left[\frac{40+76y+92x}{39+17y+50x}, \frac{78+20y+18x}{18+51y+34x}\right], x, y, [2.5, 1.5], 1000, 1010\right) \\
& [[1.922169930, 0.9893531312], [1.922169931, 0.9893531312], [1.922169930, 0.9893531312], \\
& [1.922169931, 0.9893531312], [1.922169930, 0.9893531312], [1.922169931, 0.9893531312], \\
& 0.9893531312], [1.922169930, 0.9893531312], [1.922169931, 0.9893531312], \\
& [1.922169930, 0.9893531312], [1.922169931, 0.9893531312]] \tag{27}
\end{aligned}$$

$$\begin{aligned}
& > SFP2\left(\left[\frac{78+10y+52x}{100+13y+87x}, \frac{13+37y+92x}{97+69y+62x}\right], x, y\right) \\
& \quad [[0.7127296415, 0.5521890015]] \tag{28}
\end{aligned}$$

> $Orb2\left(\left[\frac{78 + 10y + 52x}{100 + 13y + 87x}, \frac{13 + 37y + 92x}{97 + 69y + 62x}\right], x, y, [2.5, 1.5], 1000, 1010\right)$
 $[[0.7127296417, 0.5521890016], [0.7127296417, 0.5521890016], [0.7127296417,$
 $0.5521890016], [0.7127296417, 0.5521890016], [0.7127296417, 0.5521890016],$
 $[0.7127296417, 0.5521890016], [0.7127296417, 0.5521890016], [0.7127296417,$
 $0.5521890016], [0.7127296417, 0.5521890016], [0.7127296417, 0.5521890016]]$ (29)

=>

> #4i.

```
RT3 := proc(x, y, z, d, K) local ra, i, j, k, f, g, h :
ra := rand(1 .. K) : #random integer from -K to K
f := add(add(add(ra() · x^i · y^j · z^k, k = 0 .. d - j - i), j = 0 .. d - i), i = 0 .. d)
  / add(add(add(ra() · x^i · y^j · z^k, k = 0 .. d - j - i), j = 0 .. d - i), i = 0 .. d) :
g := add(add(add(ra() · x^i · y^j · z^k, k = 0 .. d - j - i), j = 0 .. d - i), i = 0 .. d)
  / add(add(add(ra() · x^i · y^j · z^k, k = 0 .. d - j - i), j = 0 .. d - i), i = 0 .. d) :
h := add(add(add(ra() · x^i · y^j · z^k, k = 0 .. d - j - i), j = 0 .. d - i), i = 0 .. d) :
  / add(add(add(ra() · x^i · y^j · z^k, k = 0 .. d - j - i), j = 0 .. d - i), i = 0 .. d) :
[f, g, h] :
end:
```

> $RT3(x, y, z, 1, 100)$

$$\left[\frac{2 + 74z + 16y + 13x}{82 + 35z + 80y + 45x}, \frac{83 + 36z + 38y + 14x}{39 + 85z + 50y + 10x}, \frac{37 + 61z + 2y + 58x}{3 + 77z + 22y + 57x} \right] \quad (30)$$

> #4ii.

```
Orb3 := proc(F, x, y, z, pt0, K1, K2) local pt, L, i :
pt := pt0 :
```

```
for i from 1 to K1 do
```

```
pt := subs({x = pt[1], y = pt[2], z = pt[3]}, F) :
```

```
od:
```

```
L := [] :
```

```
for i from K1 + 1 to K2 do
```

```
L := [op(L), pt] :
```

```
pt := subs({x = pt[1], y = pt[2], z = pt[3]}, F) :
```

```
od:
```

```
L :
```

```
end:
```

> $Orb3\left(\left[\frac{2 + 74z + 16y + 13x}{82 + 35z + 80y + 45x}, \frac{83 + 36z + 38y + 14x}{39 + 85z + 50y + 10x}, \frac{37 + 61z + 2y + 58x}{3 + 77z + 22y + 57x}\right], x, y,$
 $z, [1.5, 1.3, 1.2], 1000, 1010\right)$

$[[0.4580256140, 0.9185354898, 1.000613335], [0.4580256145, 0.9185354898, 1.000613334],$ (31)
 $[0.4580256140, 0.9185354898, 1.000613335], [0.4580256145, 0.9185354898,$
 $1.000613334], [0.4580256140, 0.9185354898, 1.000613335], [0.4580256145,$

```

0.9185354898, 1.000613334], [0.4580256140, 0.9185354898, 1.000613335],
[0.4580256145, 0.9185354898, 1.000613334], [0.4580256140, 0.9185354898,
1.000613335], [0.4580256145, 0.9185354898, 1.000613334]]

```

> #4iv.

```

FP3 := proc(F, x, y, z) local L, i :
L := [solve({F[1]=x, F[2]=y, F[3]=z}, {x, y, z})] :
[seq(subs(L[i], [x, y, z]), i = 1 .. nops(L))] :
end:

```

> $\text{evalf}\left(\text{FP3}\left(\left[\frac{2+74z+16y+13x}{82+35z+80y+45x}, \frac{83+36z+38y+14x}{39+85z+50y+10x}, \frac{37+61z+2y+58x}{3+77z+22y+57x}\right], x, y, z\right)\right)$ [[0.4580256143, 0.918535488, 1.000613336]] (32)

> #4v.

```
SFP3 := proc(F, x, y, z) local L, J, S, J0, i, pt, EV :
```

```
L := evalf(FP3(F, x, y, z)) :
```

```
J := Matrix(normal([[diff(F[1], x), diff(F[2], x), diff(F[3], x)], [diff(F[1], y), diff(F[2], y), diff(F[3], y)], [diff(F[1], z), diff(F[2], z), diff(F[3], z)]])) :
```

```
S := [] :
```

```
for i from 1 to nops(L) do #we examine it case by case
pt := L[i] :
```

```
J0 := subs({x=pt[1], y=pt[2], z=pt[3]}, J) :
```

```
EV := Eigenvalues(J0) :
```

```
if abs(EV[1]) < 1 and abs(EV[2]) < 1 and abs(EV[3]) < 1 then
```

```
  S := [op(S), pt] :
```

```
  fi:
```

```
od:
```

```
S :
```

```
end:
```

> $\text{SFP3}\left(\left[\frac{2+74z+16y+13x}{82+35z+80y+45x}, \frac{83+36z+38y+14x}{39+85z+50y+10x}, \frac{37+61z+2y+58x}{3+77z+22y+57x}\right], x, y, z\right)$ [[0.4580256143, 0.918535488, 1.000613336]] (33)

> #5.

```
RT3(x, y, z, 1, 100)
```

$$\left[\frac{99 + 51z + 47y + 33x}{77 + 20z + 83y + 10x}, \frac{40 + 16z + 19y + 40x}{40 + 78z + 91y + 71x}, \frac{75 + 5z + 78y + 29x}{23 + 81z + 21y + 6x} \right] \quad (34)$$

> $SFP3\left(\left[\frac{99 + 51z + 47y + 33x}{77 + 20z + 83y + 10x}, \frac{40 + 16z + 19y + 40x}{40 + 78z + 91y + 71x}, \frac{75 + 5z + 78y + 29x}{23 + 81z + 21y + 6x} \right], x, y, z\right)$

$$[[1.49824862, 0.4559974964, 1.17156721]] \quad (35)$$

> $Orb3\left(\left[\frac{99 + 51z + 47y + 33x}{77 + 20z + 83y + 10x}, \frac{40 + 16z + 19y + 40x}{40 + 78z + 91y + 71x}, \frac{75 + 5z + 78y + 29x}{23 + 81z + 21y + 6x} \right], x, y, z, [1.5, 1.2, 1.3], 1000, 1010\right)$

$$[[1.498248617, 0.4559974965, 1.171567198], [1.498248617, 0.4559974965, 1.171567198], \quad (36)$$

$$[1.498248617, 0.4559974965, 1.171567198], [1.498248617, 0.4559974965, 1.171567198],$$

$$[1.498248617, 0.4559974965, 1.171567198], [1.498248617, 0.4559974965, 1.171567198],$$

$$[1.498248617, 0.4559974965, 1.171567198], [1.498248617, 0.4559974965, 1.171567198],$$

$$[1.498248617, 0.4559974965, 1.171567198], [1.498248617, 0.4559974965, 1.171567198]]$$

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