```
> #Okay to Post
  #Nikita John, Assignment 13, October 17th, 2021
> #M13.txt: Maple code for Lecture 13 of Dynamical Modesl in Biology, Fall 2021 (taught by Dr. Z.)
Help13 :=proc( ) : print( `RT2(x,y,d,K), Orb2(F,x,y,pt0,K1,K2), FP2(F,x,y), SFP2(F,x,y) `) :
    end:

with(LinearAlgebra) :


    #RT2(x,y,d,K): A random rational transformation of degree d from R^2 to R^2 with postiive
    integer coefficients from 1 to K The inputs are variables x and y and
#the output is a pair of expressions  of (x,y) representing functions. It is for generating examples
#Try:
#RT2(x,y,2,10);
RT2 :=proc(x, y, d, K) local ra, i, j, f, g :
ra := rand(1 ..K) : #random integer from -K to K
f := add(add(ra( ) * x^i * y^j, j = 0 ..d - i), i = 0 ..d) / add(add(ra( ) * x^i * y^j, j = 0 ..d - i), i = 0
    ..d) :
g := add(add(ra( ) * x^i * y^j, j = 0 ..d - i), i = 0 ..d) / add(add(ra( ) * x^i * y^j, j = 0 ..d - i), i = 0
    ..d) :
[f, g] :
end:



    #Orb2(F,x,y,pt,K1,K2): Inputs a mapping F=[f,g] from R^2  to R^2 where f and g describe
    functions of x and y, an initial point pt0=[x0,y0]
#outputs the orbit starting at discrete time K1 and ending in discrete time K2. Try
#F:=RT2(x,y,2,10);
#Orb2(F,x,y,[1.1,1.2],1000,1010);
Orb2 :=proc(F, x, y, pt0, K1, K2)  local pt, L, i :
pt := pt0 :

for i from 1 to K1 do
pt := subs( {x = pt[1], y = pt[2]}, F) :
od:

L := [ ] :
for i from K1 + 1 to K2 do
pt := subs( {x = pt[1], y = pt[2]}, F) :
L := [op(L), pt] :
od:
L :
end:

#FP2(F,x,y): The list of fixed points of the transformation [x,y]->F. Try
#FP2([x-y,x=y],x,y);
FP2 :=proc(F, x, y) local L, i :
```

```
L := [solve({F[1]=x, F[2]=y}, {x, y})]:

[seq(subs(L[i], [x, y]), i=1..nops(L))]:
end:


#SFP2(F,x,y): The list of Stable fixed points of the transformation [x,y]->F. Try
#SFP2([(1+x)/(1+y), (1+7*y)/(4+x)],x,y);
SFP2 :=proc(F, x, y) local L, J, S, J0, i, pt, EV:

L := evalf(FP2(F, x, y)):
    #F is the list of ALL fixed points of the transformation [x,y]->F using the previous procedure
    FP2(F,x,y), but since we are interested in numbers we take the floating point version using
    evalf

J := Matrix(normal([[diff(F[1], x), diff(F[2], x)], [diff(F[1], y), diff(F[2], y)]])):
    #J is the Jacobian matrix in general (in terms of the variables x and y). Note that J is a
    SYMBOLIC matrix featuring variables x and y

S := [ ]:  #S is the list of stable fixed points that starts out empty

for i from 1 to nops(L) do  #we examime it case by case
pt := L[i]: #pt is the current fixed point to be examined

J0 := subs({x=pt[1], y=pt[2]}, J):
    #J0 is the NUMERICAL matrix obtained by plugging-in the examined fixed pt

EV := Eigenvalues(J0):
    # We used Maple's command Eigenavalues to find the eigenvalues of this 2 by 2 matrix

if abs(EV[1]) < 1 and abs(EV[2]) < 1  then
 S := [op(S), pt]:
    #If both eigenvalues have absolute value less than 1 it means that they are stable, so we
    append the examined fixed point, pt, to the list of fixed points
fi:

od:
S: #the output is S
end:




###old stuff
#M11.txt: Maple code for Lecture 11 of Dynamical Models in Biology taught by Dr. Z.
Help11 :=proc( ) : print(` SFPe(f,x), Orbk(k,z,f,INI,K1,K2) `) :end:


    #SFPe(f,x): The set of fixed points of x->f(x) done exactly (and allowing symbolic
    parameters), followed by the condition of stability (if it is netween -1 and 1 it is stable)
```

```
#Try: FPe(k*x*(1-x),x);
#VERSION OF Oct. 12, 2021 (avoiding division by 0)
SFPe := proc(f, x) local f1, L, i, M:
f1 := normal(diff(f, x)):
L := [solve(numer(f-x), x)]:
M := [ ]:

for i from 1 to nops(L) do
 if subs(x=L[i], denom(f1)) ≠ 0 then
  M := [op(M), [L[i], normal(subs(x=L[i], f1))]]:
 fi:
od:
M:

end:


#Added after class

    #Orbk(k,z,f,INI,K1,K2): Given a positive integer k, a letter (symbol), z, an expression f of z
    [1], ..., z[k] (representing a multi-variable function of the variables z[1],...,z[k]

    #a vector INI representing the initial values [x[1],..., x[k]], and (in applications) positive
    integres K1 and K2, outputs the

    #values of the sequence starting at n=K1 and ending at n=K2. of the sequence satisfying the
    difference equation
##x[n]=f(x[n-1],x[n-2],..., x[n-k+1]):

    #This is a generalization to  higher-order difference equation of procedure Orb(f,x,x0,K1,K2)
    . For example
#Orbk(1,z,5/2*z[1]*(1-z[1]),[0.5],1000,1010);  should be the same as
#Orb(5/2*z[1]*(1-z[1]),z[1],[0,5],1000,1010);
#Try:
#Orbk(2,z,(5/4)*z[1]-(3/8)*z[2],[1,2],1000,1010);
Orbk := proc(k, z, f, INI, K1, K2) local L, i, newguy:
L := INI:  #We start out with the list of initial values

if not (type(k, integer) and type(z, symbol) and type(INI, list) and nops(INI) = k and type(K1,
    integer) and type(K2, integer) and K1 > 0 and K2 > K1) then
    #checking that the input is OK
 print(`bad input`):
 RETURN(FAIL):
fi:

while nops(L) < K2 do
 newguy := subs({seq(z[i]=L[-i], i=1..k)}, f):
    #Using what we know about the value yesterday, the day before yesterday, ... up to k days
    before yesterday we find the value of the sequence today
```

```
L := [op(L), newguy] : #we append the new value to the running list of values of our sequence
od:

[op(K1 ..K2, L)] :

end:


####STAFT FROM M9.txt
#M9.txt: Maple Code for "Dynamical models in Biology" (Math 336) taught by Dr. Z., Lecture 9

Help9 :=proc( ) :
    print( ` Orb(f,x,x0,K1,K2), Orb2D(f,x,x0,K) , FP(f,x) , SFP(f,x) , Comp(f,x)  `) :end:


    #Orb(f,x,x0,K1,K2): Inputs an expression f in x (desccribing) a function of x, an initial point,
    x0, and a positive integer K, outputs
#the values of x[n] from n=K1 to n=K2. Try: where x[n]=f(x[n-1]), . Try:
#Orb(2*x*(1-x),x,0.4,1000,2000);
Orb :=proc( f, x, x0, K1, K2) local x1, i, L :
x1 := x0 :
for i from 1 to K1 do
 x1 := subs(x=x1, f) :
    #we don't record the first values of K1, since we are interested in the long-time behavior of
    the orbit
od:

L := [x1] :

for i from K1 to K2 do
 x1 := subs(x=x1, f) :   #we compute the next member of the orbit
 L := [op(L), x1] : #we append it to the list
od:

L : #that's the output

end:

#Orb2D(f,x,x0,K): 2D version of Orb(f,x,x0,0,K), just for illustration
Orb2D :=proc( f, x, x0, K) local L, L1, i :
L := Orb( f, x, x0, 0, K) :
L1 := [[L[1], 0], [L[1], L[2]], [L[2], L[2]] ] :
for i from 3 to nops(L) do
 L1 := [op(L1), [L[i-1], L[i]], [L[i], L[i]]] :
od:
L1 :
end:
```

```
#FP(f,x): The list of fixed points of the map x->f where f is an expression in x. Try:
#FP(2*x*(1-x),x);
FP := proc(f, x)
evalf([solve(f=x, x)]) :
end:


#SFP(f,x): The list of stable fixed points of the map x->f where f is an expression in x. Try:
#SFP(2*x*(1-x),x);
SFP := proc(f, x) local L, i, f1, pt, Ls :
L := FP(f, x) : #The list of fixed points (including complex ones)

Ls := [ ] :        #Ls is the list of stable fixed points, that starts out as the empty list

f1 := diff(f, x) :  #The derivative of the function f w.r.t. x

for i from 1 to nops(L) do
pt := L[i] :

if abs(subs(x=pt, f1)) < 1 then

 Ls := [op(Ls), pt] :  # if pt,  is stable we add it to the list of stable points

fi:

od:

Ls :   #The last line is the output

end:

#Comp(f,x): f(f(x))
Comp := proc(f, x) : normal(subs(x=f, f)) :end:
```

```
> #2
```

$f := \frac{(x^2 + 8\cdot x + 4)}{x^2}$:

$g := \frac{(5\cdot y^2 + 8\cdot y + 4)}{4\cdot y^2 + 5}$:

```
FP2([f, g], x, y);
SFP2([f, g], x, y);
```

$[[-2, RootOf(4\_Z^3 - 5\_Z^2 - 3\_Z - 4)], [RootOf(\_Z^2 - 3\_Z - 2), RootOf(4\_Z^3 - 5\_Z^2 - 3\_Z - 4)]]$

$$[[3.561552813, 1.914550612]] \quad \textbf{(1)}$$

```
> Orb2([f, g], x, y, [8.5, 0.5, ], 1000, 1010);
```

[[3.561552809, 1.914550612], [3.561552817, 1.914550613], [3.561552809, 1.914550612], [3.561552817, 1.914550613], [3.561552809, 1.914550612], [3.561552817, **(2)** 1.914550613], [3.561552809, 1.914550612], [3.561552817, 1.914550613], [3.561552809, 1.914550612], [3.561552817, 1.914550613]]

> #3
F1 := RT2(x, y, 1, 100);
SFP2(F1, x, y);
Orb2(F1, x, y, [0.5, 1], 1000, 1010);

$$F1 := \left[\frac{92 + 71\,y + 67\,x}{78 + 51\,y + 53\,x}, \frac{12 + 19\,y + 63\,x}{40 + 90\,y + 3\,x} \right]$$

[[1.259593358, 0.8791974895]]

[[1.259593358, 0.8791974894], [1.259593358, 0.8791974894], [1.259593358, 0.8791974894], [1.259593358, 0.8791974894], [1.259593358, 0.8791974894], [1.259593358, 0.8791974894], [1.259593358, 0.8791974894], [1.259593358, 0.8791974894], [1.259593358, 0.8791974894], [1.259593358, 0.8791974894]] **(3)**

> F2 := RT2(x, y, 1, 100);
SFP2(F2, x, y);
Orb2(F2, x, y, [1.5, 1], 1000, 1010);

$$F2 := \left[\frac{49 + 49\,y + 67\,x}{74 + 90\,y + 74\,x}, \frac{27 + 98\,y + 72\,x}{2 + 73\,y + 85\,x} \right]$$

[[0.6528216031, 1.321846941]]

[[0.6528216034, 1.321846940], [0.6528216034, 1.321846940], [0.6528216034, 1.321846940], [0.6528216034, 1.321846940], [0.6528216034, 1.321846940], [0.6528216034, 1.321846940], [0.6528216034, 1.321846940], [0.6528216034, 1.321846940], [0.6528216034, 1.321846940], [0.6528216034, 1.321846940]] **(4)**

> F3 := RT2(x, y, 1, 100);
SFP2(F3, x, y);
Orb2(F3, x, y, [0.25, 5], 1000, 1010);

$$F3 := \left[\frac{41 + 4\,y + 44\,x}{13 + 19\,y + 10\,x}, \frac{15 + 64\,y + 9\,x}{12 + 52\,y + 25\,x} \right]$$

[[3.169979666, 0.6897016763]]

[[3.169979666, 0.6897016765], [3.169979666, 0.6897016765], [3.169979666, 0.6897016765], [3.169979666, 0.6897016765], [3.169979666, 0.6897016765], [3.169979666, 0.6897016765], [3.169979666, 0.6897016765], [3.169979666, 0.6897016765], [3.169979666, 0.6897016765], [3.169979666, 0.6897016765]] **(5)**

> F4 := RT2(x, y, 1, 100);
SFP2(F4, x, y);
Orb2(F4, x, y, [1.1, 1.1], 1000, 1010);

$$F4 := \left[\frac{72 + 90\,y + 18\,x}{43 + 55\,y + 40\,x}, \frac{17 + 70\,y + 52\,x}{81 + 87\,y + 34\,x} \right]$$

$$[[1.208133500, 0.7042432306]]$$

$$[[1.208133501, 0.7042432303], [1.208133499, 0.7042432312], [1.208133500, 0.7042432303], [1.208133499, 0.7042432307], [1.208133501, 0.7042432303], [1.208133499, 0.7042432312], [1.208133500, 0.7042432303], [1.208133499, 0.7042432307], [1.208133501, 0.7042432303], [1.208133499, 0.7042432312]] \quad (6)$$

> $F5 := RT2(x, y, 1, 100);$
> $SFP2(F5, x, y);$
> $Orb2(F5, x, y, [1.2, 1], 1000, 1010);$

$$F5 := \left[\frac{85 + 9\,y + 68\,x}{83 + 63\,y + 100\,x}, \frac{70 + 36\,y + 36\,x}{10 + 40\,y + 66\,x} \right]$$

$$[[0.6049974286, 1.350612004]]$$

$$[[0.6049974288, 1.350612003], [0.6049974288, 1.350612004], [0.6049974288, 1.350612003], [0.6049974288, 1.350612004], [0.6049974288, 1.350612003], [0.6049974288, 1.350612004], [0.6049974288, 1.350612003], [0.6049974288, 1.350612004], [0.6049974288, 1.350612003], [0.6049974288, 1.350612004]] \quad (7)$$

> $F6 := RT2(x, y, 1, 100);$
> $SFP2(F6, x, y);$
> $Orb2(F6, x, y, [5, 8.5], 1000, 1010);$

$$F6 := \left[\frac{87 + 16\,y + 98\,x}{43 + 53\,y + 61\,x}, \frac{47 + 28\,y + 75\,x}{3 + 5\,y + 11\,x} \right]$$

$$[[0.586850559, 6.506538819]]$$

$$[[0.5868505549, 6.506538820], [0.5868505549, 6.506538820], [0.5868505549, 6.506538820], [0.5868505549, 6.506538820], [0.5868505549, 6.506538820], [0.5868505549, 6.506538820], [0.5868505549, 6.506538820], [0.5868505549, 6.506538820], [0.5868505549, 6.506538820], [0.5868505549, 6.506538820]] \quad (8)$$

> $F7 := RT2(x, y, 1, 100);$
> $SFP2(F7, x, y);$
> $Orb2(F7, x, y, [1.1, 1.3], 1000, 1010);$

$$F7 := \left[\frac{37 + 75\,y + 4\,x}{91 + 22\,y + 40\,x}, \frac{58 + 93\,y + 98\,x}{11 + 30\,y + 6\,x} \right]$$

$$[[1.459248295, 4.082633122]]$$

$$[[1.459248294, 4.082633122], [1.459248294, 4.082633120], [1.459248294, 4.082633122], [1.459248294, 4.082633120], [1.459248294, 4.082633122], [1.459248294, 4.082633120], [1.459248294, 4.082633122], [1.459248294, 4.082633120], [1.459248294, 4.082633122], [1.459248294, 4.082633120]] \quad (9)$$

> $F8 := RT2(x, y, 1, 100);$
> $SFP2(F8, x, y);$
> $Orb2(F8, x, y, [0.55, 0.55], 1000, 1010);$

$$F8 := \left[\frac{32 + 40\,y + 24\,x}{80 + 96\,y + 11\,x}, \frac{23 + 41\,y + 52\,x}{58 + 67\,y + 81\,x} \right]$$

$$[[0.47477235, 0.5258468959]]$$

$[[0.4747723604, 0.5258468959], [0.4747723604, 0.5258468959], [0.4747723604, 0.5258468959], [0.4747723604, 0.5258468959], [0.4747723604, 0.5258468959], [0.4747723604, 0.5258468959], [0.4747723604, 0.5258468959], [0.4747723604, 0.5258468959], [0.4747723604, 0.5258468959], [0.4747723604, 0.5258468959]]$ **(10)**

```
> F9 := RT2(x, y, 1, 100);
  SFP2(F9, x, y);
  Orb2(F9, x, y, [0.77, 1], 1000, 1010);
```

$$F9 := \left[\frac{65 + 69\,y + 2\,x}{36 + 61\,y + 84\,x}, \frac{96 + 94\,y + 31\,x}{81 + 31\,y + 54\,x}\right]$$

$$[[0.8556835833, 1.523365570]]$$

$[[0.8556835824, 1.523365569], [0.8556835833, 1.523365571], [0.8556835829, 1.523365570], [0.8556835824, 1.523365569], [0.8556835833, 1.523365571], [0.8556835829, 1.523365570], [0.8556835824, 1.523365569], [0.8556835833, 1.523365571], [0.8556835829, 1.523365570], [0.8556835824, 1.523365569]]$ **(11)**

```
> F10 := RT2(x, y, 1, 100);
  SFP2(F10, x, y);
  Orb2(F10, x, y, [0.67, 0.7], 1000, 1010);
```

$$F10 := \left[\frac{94 + 52\,y + 16\,x}{29 + 51\,y + 3\,x}, \frac{45 + 67\,y + 40\,x}{71 + 74\,y + 49\,x}\right]$$

$$[[2.2557065, 0.7837939465]]$$

$[[2.255706518, 0.7837939465], [2.255706518, 0.7837939465], [2.255706518, 0.7837939465], [2.255706518, 0.7837939465], [2.255706518, 0.7837939465], [2.255706518, 0.7837939465], [2.255706518, 0.7837939465], [2.255706518, 0.7837939465], [2.255706518, 0.7837939465], [2.255706518, 0.7837939465]]$ **(12)**

```
> F11 := RT2(x, y, 1, 100);
  SFP2(F11, x, y);
  Orb2(F11, x, y, [8, 4.2], 1000, 1010);
```

$$F11 := \left[\frac{46 + 76\,y + 9\,x}{53 + 37\,y + 88\,x}, \frac{50 + 37\,y + 76\,x}{95 + 8\,y + 92\,x}\right]$$

$$[[0.760670490, 0.8021705343]]$$

$[[0.7606704850, 0.8021705342], [0.7606704855, 0.8021705341], [0.7606704850, 0.8021705342], [0.7606704855, 0.8021705341], [0.7606704850, 0.8021705342], [0.7606704855, 0.8021705341], [0.7606704850, 0.8021705342], [0.7606704855, 0.8021705341], [0.7606704850, 0.8021705342], [0.7606704855, 0.8021705341]]$ **(13)**

```
> F12 := RT2(x, y, 1, 100);
  SFP2(F12, x, y);
  Orb2(F12, x, y, [0.9, 0.9], 1000, 1010);
```

$$F12 := \left[\frac{92 + 2\,y + 97\,x}{44 + 9\,y + 30\,x}, \frac{14 + 79\,y + 73\,x}{21 + 78\,y + 49\,x}\right]$$

[[2.605574293, 1.231921109]]

[[2.605574293, 1.231921112], [2.605574293, 1.231921112], [2.605574293, 1.231921112], [2.605574293, 1.231921112], [2.605574293, 1.231921112], [2.605574293, 1.231921112], [2.605574293, 1.231921112], [2.605574293, 1.231921112], [2.605574293, 1.231921112], [2.605574293, 1.231921112]] **(14)**

```
> F13 := RT2(x, y, 1, 100);
SFP2(F13, x, y);
Orb2(F13, x, y, [1.15, 4], 1000, 1010);
```

$$F13 := \left[\frac{93 + 15\,y + 56\,x}{69 + 17\,y + 21\,x}, \frac{42 + 21\,y + 5\,x}{58 + 3\,y + 86\,x} \right]$$

[[1.771779264, 0.2674316445]]

[[1.771779263, 0.2674316446], [1.771779263, 0.2674316446], [1.771779263, 0.2674316446], [1.771779263, 0.2674316446], [1.771779263, 0.2674316446], [1.771779263, 0.2674316446], [1.771779263, 0.2674316446], [1.771779263, 0.2674316446], [1.771779263, 0.2674316446], [1.771779263, 0.2674316446]] **(15)**

```
> F14 := RT2(x, y, 1, 100);
SFP2(F14, x, y);
Orb2(F14, x, y, [5.2, 2.5], 1000, 1010);
```

$$F14 := \left[\frac{42 + 5\,y + 33\,x}{77 + 98\,y + 58\,x}, \frac{98 + 29\,y + 65\,x}{29 + 35\,y + 29\,x} \right]$$

[[0.2255303705, 1.703110459]]

[[0.2255303705, 1.703110459], [0.2255303705, 1.703110459], [0.2255303705, 1.703110459], [0.2255303705, 1.703110459], [0.2255303705, 1.703110459], [0.2255303705, 1.703110459], [0.2255303705, 1.703110459], [0.2255303705, 1.703110459], [0.2255303705, 1.703110459], [0.2255303705, 1.703110459]] **(16)**

```
> F15 := RT2(x, y, 1, 100);
SFP2(F15, x, y);
Orb2(F15, x, y, [3.2, 1], 1000, 1010);
```

$$F15 := \left[\frac{92 + 39\,y + 17\,x}{50 + 78\,y + 20\,x}, \frac{18 + 18\,y + 51\,x}{34 + 78\,y + 10\,x} \right]$$

[[1.055721848, 0.8044562822]]

[[1.055721848, 0.8044562819], [1.055721847, 0.8044562823], [1.055721848, 0.8044562819], [1.055721847, 0.8044562823], [1.055721848, 0.8044562819], [1.055721847, 0.8044562823], [1.055721848, 0.8044562819], [1.055721847, 0.8044562823], [1.055721848, 0.8044562819], [1.055721847, 0.8044562823]] **(17)**

```
> F16 := RT2(x, y, 1, 100);
SFP2(F16, x, y);
Orb2(F16, x, y, [2.2, 2.2], 1000, 1010);
```

$$F16 := \left[\frac{46 + 78\,y + 61\,x}{80 + 3\,y + 72\,x}, \frac{48 + 9\,y + 41\,x}{46 + 78\,y + 35\,x} \right]$$

$$[[1.047321025, 0.7063310413]]$$

$$[[1.047321025, 0.7063310417], [1.047321025, 0.7063310422], [1.047321025, 0.7063310417], [1.047321025, 0.7063310422], [1.047321025, 0.7063310417], [1.047321025, 0.7063310422], [1.047321025, 0.7063310417], [1.047321025, 0.7063310422], [1.047321025, 0.7063310417], [1.047321025, 0.7063310422]] \quad \textbf{(18)}$$

> $F17 := RT2(x, y, 1, 100);$
> $SFP2(F17, x, y);$
> $Orb2(F17, x, y, [3.6, 1], 1000, 1010);$

$$F17 := \left[\frac{81 + 38\,y + 88\,x}{20 + 16\,y + 17\,x}, \frac{68 + 79\,y + 48\,x}{67 + 98\,y + 86\,x}\right]$$

$$[[4.710672055, 0.6445781102]]$$

$$[[4.710672054, 0.6445781103], [4.710672054, 0.6445781103], [4.710672054, 0.6445781103], [4.710672054, 0.6445781103], [4.710672054, 0.6445781103], [4.710672054, 0.6445781103], [4.710672054, 0.6445781103], [4.710672054, 0.6445781103], [4.710672054, 0.6445781103], [4.710672054, 0.6445781103]] \quad \textbf{(19)}$$

> $F18 := RT2(x, y, 1, 100);$
> $SFP2(F18, x, y);$
> $Orb2(F18, x, y, [7.7, 0.77], 1000, 1010);$

$$F18 := \left[\frac{53 + 30\,y + 44\,x}{55 + 85\,y + 33\,x}, \frac{38 + 42\,y + 89\,x}{65 + 46\,y + 67\,x}\right]$$

$$[[0.7182305877, 0.9042384702]]$$

$$[[0.7182305866, 0.9042384706], [0.7182305866, 0.9042384706], [0.7182305866, 0.9042384706], [0.7182305866, 0.9042384706], [0.7182305866, 0.9042384706], [0.7182305866, 0.9042384706], [0.7182305866, 0.9042384706], [0.7182305866, 0.9042384706], [0.7182305866, 0.9042384706], [0.7182305866, 0.9042384706]] \quad \textbf{(20)}$$

> $F19 := RT2(x, y, 1, 100);$
> $SFP2(F19, x, y);$
> $Orb2(F19, x, y, [0.33, 0.33], 1000, 1010);$

$$F19 := \left[\frac{37 + 90\,y + 44\,x}{99 + 21\,y + 73\,x}, \frac{60 + 37\,y + 33\,x}{99 + 18\,y + 64\,x}\right]$$

$$[[0.7824018820, 0.6894053820]]$$

$$[[0.7824018819, 0.6894053818], [0.7824018819, 0.6894053818], [0.7824018819, 0.6894053818], [0.7824018819, 0.6894053818], [0.7824018819, 0.6894053818], [0.7824018819, 0.6894053818], [0.7824018819, 0.6894053818], [0.7824018819, 0.6894053818], [0.7824018819, 0.6894053818], [0.7824018819, 0.6894053818]] \quad \textbf{(21)}$$

> $F20 := RT2(x, y, 1, 100);$
> $SFP2(F20, x, y);$
> $Orb2(F20, x, y, [4.4, 5], 1000, 1010);$

$$F20 := \left[\frac{47 + 32\,y + 10\,x}{9 + 47\,y + 100\,x}, \frac{9 + 48\,y + 56\,x}{39 + 28\,y + 16\,x}\right]$$

[[0.6876317888, 1.267309974]]

[[0.6876317884, 1.267309974], [0.6876317884, 1.267309974], [0.6876317884, 1.267309974], [0.6876317884, 1.267309974], [0.6876317884, 1.267309974], [0.6876317884, 1.267309974], [0.6876317884, 1.267309974], [0.6876317884, 1.267309974], [0.6876317884, 1.267309974], [0.6876317884, 1.267309974]] **(22)**

```
> #4(i)
RT3 := proc(x, y, z, d, K) local ra, i, j, k, f, g, h :
ra := rand(1 ..K) : #random integer from -K to K
f := add(add(add(ra( ) * x^i * y^j·z^k, k = 0 ..d − j) , j = 0 ..d − i), i = 0 ..d)
     / add(add(add(ra( ) * x^i * y^j·z^k, k = 0 ..d − j), j = 0 ..d − i), i = 0 ..d) :
 g := add(add(add(ra( ) * x^i * y^j·z^k, k = 0 ..d − j) , j = 0 ..d − i), i = 0 ..d)
     / add(add(add(ra( ) * x^i * y^j·z^k, k = 0 ..d − j), j = 0 ..d − i), i = 0 ..d) :
 h := add(add(add(ra( ) * x^i * y^j·z^k, k = 0 ..d − j) , j = 0 ..d − i), i = 0 ..d)
     / add(add(add(ra( ) * x^i * y^j·z^k, k = 0 ..d − j), j = 0 ..d − i), i = 0 ..d) :
[f, g, h] :
 end:
```

```
> #4(ii) and (iii) (they have the same thing written)
Orb3 := proc(F, x, y, z, pt0, K1, K2)  local pt, L, i :
pt := pt0 :

for i from 1 to K1 do
pt := subs( {x = pt[1], y = pt[2], z = pt[3]}, F) :
od:

L := [ ] :
for i from K1 + 1 to K2 do
pt := subs( {x = pt[1], y = pt[2], z = pt[3]}, F) :
L := [op(L), pt] :
od:
L :
 end:
```

```
> #4(iv)
FP3 := proc(F, x, y, z) local L, i :
L := [solve( {F[1] = x, F[2] = y, F[3] = z}, {x, y, z}) ] :

[seq(subs(L[i], [x, y, z]), i = 1 ..nops(L)) ] :
 end:
```

```
> #4(v)
SFP3 := proc(F, x, y, z) local L, J, S, J0, i, pt, EV :

L := evalf (FP3(F, x, y, z)) :
    #F is the list of ALL fixed points of the transformation [x,y]->F using the previous procedure
    FP2(F,x,y), but since we are interested in numbers we take the floating point version using
    evalf
```

```
J := Matrix(normal([[diff(F[1], x), diff(F[2], x), diff(F[3], x)], [diff(F[1], y), diff(F[2],
    y), diff(F[3], y)], [diff(F[1], z), diff(F[2], z), diff(F[3], z)]])) :
    #J is the Jacobian matrix in general (in terms of the variables x and y). Note that J is a
    SYMBOLIC matrix featuring variables x and y

S := [ ] :  #S is the list of stable fixed points that starts out empty

for i from 1 to nops(L) do  #we examime it case by case
pt := L[i] : #pt is the current fixed point to be examined

J0 := subs({x=pt[1], y=pt[2], z=pt[3]}, J) :
    #J0 is the NUMERICAL matrix obtained by plugging-in the examined fixed pt

EV := Eigenvalues(J0) :
    # We used Maple's command Eigenavalues to find the eigenvalues of this 3 by 3 matrix

if abs(EV[1]) < 1 and abs(EV[2]) < 1  and abs(EV[3]) < 1 then
 S := [op(S), pt] :
    #If both eigenvalues have absolute value less than 1 it means that they are stable, so we
    append the examined fixed point, pt, to the list of fixed points
fi:

od:
S : #the output is S
 end:
```

```
> R1 := RT3(x, y, z, 1, 10);
  SFP3(R1, x, y, z);
  Orb3(R1, x, y, z, [0.5, 1, 2], 1000, 1010);
```

$$R1 := \left[\frac{7\,x\,z + 10\,x + y + 3\,z + 2}{4\,x\,z + 10\,x + 8\,y + 6\,z + 6}, \frac{10\,x\,z + x + 8\,y + 3\,z + 5}{7\,x\,z + 9\,x + 10\,y + 8\,z + 6}, \frac{7\,x\,z + 3\,x + 10\,y + 2\,z + 8}{2\,x\,z + 5\,x + 7\,y + 7\,z + 4} \right]$$

[[0.6391972893, 0.663607060, 1.126608191]]

[[0.6391972894, 0.6636070029, 1.126608207], [0.6391972894, 0.6636070029, 1.126608207], [0.6391972894, 0.6636070029, 1.126608207], [0.6391972894, 0.6636070029, 1.126608207], [0.6391972894, 0.6636070029, 1.126608207], [0.6391972894, 0.6636070029, 1.126608207], [0.6391972894, 0.6636070029, 1.126608207], [0.6391972894, 0.6636070029, 1.126608207], [0.6391972894, 0.6636070029, 1.126608207], [0.6391972894, 0.6636070029, 1.126608207], [0.6391972894, 0.6636070029, 1.126608207]] **(23)**

```
> R2 := RT3(x, y, z, 1, 10);
  SFP3(R2, x, y, z);
  Orb3(R2, x, y, z, [0.5, 1, 2], 1000, 1010);
```

$$R2 := \left[\frac{2\,x\,z + 10\,x + 3\,y + z + 9}{7\,x\,z + 8\,x + 2\,y + z + 9}, \frac{2\,x\,z + 2\,x + 9\,y + z + 1}{x\,z + 4\,x + y + 3\,z + 5}, \right.$$

$$\left.\frac{4xz+10x+9y+4z+6}{4xz+6x+4y+5z+8}\right]$$

[[0.9068131926, 1.132241635, 1.213603282]]

[[0.9068131925, 1.132241637, 1.213603282], [0.9068131925, 1.132241637, 1.213603281], [0.9068131925, 1.132241637, 1.213603282], [0.9068131925, 1.132241637, 1.213603281], [0.9068131925, 1.132241637, 1.213603282], [0.9068131925, 1.132241637, 1.213603281], [0.9068131925, 1.132241637, 1.213603282], [0.9068131925, 1.132241637, 1.213603281], [0.9068131925, 1.132241637, 1.213603282], [0.9068131925, 1.132241637, 1.213603281]] **(24)**

> $R3 := RT3(x, y, z, 1, 10);$
> $SFP3(R3, x, y, z);$
> $Orb3(R3, x, y, z, [0.5, 1, 2], 1000, 1010);$

$$R3 := \left[\frac{3xz+7x+9y+9z+8}{3xz+2x+9y+3z+5}, \frac{7xz+6x+9y+4z+3}{4xz+5x+7y+z+6}, \frac{2xz+10x+2y+5z+5}{xz+4x+2y+5z+7}\right]$$

[[1.618890838, 1.360844092, 1.392220870]]

[[1.618890838, 1.360844101, 1.392220859], [1.618890838, 1.360844100, 1.392220860], [1.618890838, 1.360844101, 1.392220859], [1.618890838, 1.360844100, 1.392220860], [1.618890838, 1.360844101, 1.392220859], [1.618890838, 1.360844100, 1.392220860], [1.618890838, 1.360844101, 1.392220859], [1.618890838, 1.360844100, 1.392220860], [1.618890838, 1.360844101, 1.392220859], [1.618890838, 1.360844100, 1.392220860]] **(25)**

> $R4 := RT3(x, y, z, 1, 10);$
> $SFP3(R4, x, y, z);$
> $Orb3(R4, x, y, z, [0.5, 1, 2], 1000, 1010);$

$$R4 := \left[\frac{9xz+2x+9y+6z+3}{9xz+7x+5y+z+5}, \frac{8xz+3x+8y+z+7}{4xz+6x+3y+2z+10}, \frac{4xz+x+8y+3z+4}{9xz+7x+10y+3z+10}\right]$$

[[0.9868151735, 1.03086122, 0.5040014]]

[[0.9868151735, 1.030861294, 0.5040015651], [0.9868151730, 1.030861294, 0.5040015651], [0.9868151735, 1.030861294, 0.5040015653], [0.9868151735, 1.030861294, 0.5040015651], [0.9868151730, 1.030861294, 0.5040015651], [0.9868151735, 1.030861294, 0.5040015653], [0.9868151735, 1.030861294, 0.5040015651], [0.9868151730, 1.030861294, 0.5040015651], [0.9868151735, 1.030861294, 0.5040015653], [0.9868151735, 1.030861294, 0.5040015651]] **(26)**

> $R5 := RT3(x, y, z, 1, 10);$
> $SFP3(R5, x, y, z);$
> $Orb3(R5, x, y, z, [0.5, 1, 2], 1000, 1010);$

$$R5 := \left[\frac{6xz+9x+8y+5z+1}{3xz+4x+y+3z+10}, \frac{6xz+x+y+9z+3}{9xz+3x+4y+5z+4}, \frac{3xz+9x+5y+4z+7}{9xz+7x+10y+2z+2} \right]$$

[[1.398718148, 0.773216063, 0.937850464]]

[[1.398718148, 0.7732160700, 0.9378504767], [1.398718147, 0.7732160700, 0.9378504770], [1.398718147, 0.7732160703, 0.9378504770], [1.398718148, 0.7732160700, 0.9378504767], [1.398718147, 0.7732160700, 0.9378504770], [1.398718147, 0.7732160703, 0.9378504770], [1.398718148, 0.7732160700, 0.9378504767], [1.398718147, 0.7732160700, 0.9378504770], [1.398718147, 0.7732160703, 0.9378504770], [1.398718148, 0.7732160700, 0.9378504767]] **(27)**

```
> R6 := RT3(x, y, z, 1, 10);
  SFP3(R6, x, y, z);
  Orb3(R6, x, y, z, [0.5, 1, 2], 1000, 1010);
```

$$R6 := \left[\frac{4xz+5x+8y+7z+7}{7xz+6x+5y+z+8}, \frac{9xz+9x+4y+5z+4}{8xz+4x+9y+2z+1}, \frac{7xz+8x+6y+6z+4}{xz+6x+10y+3z+10} \right]$$

[[1.140657383, 1.240910326, 1.048905563]]

[[1.140657383, 1.240910326, 1.048905564], [1.140657383, 1.240910326, 1.048905564], [1.140657383, 1.240910326, 1.048905564], [1.140657383, 1.240910326, 1.048905564], [1.140657383, 1.240910326, 1.048905564], [1.140657383, 1.240910326, 1.048905564], [1.140657383, 1.240910326, 1.048905564], [1.140657383, 1.240910326, 1.048905564], [1.140657383, 1.240910326, 1.048905564], [1.140657383, 1.240910326, 1.048905564]] **(28)**

```
> R7 := RT3(x, y, z, 1, 10);
  SFP3(R7, x, y, z);
  Orb3(R7, x, y, z, [0.5, 1, 2], 1000, 1010);
```

$$R7 := \left[\frac{8xz+x+2y+4z+2}{4xz+8x+6y+3z+8}, \frac{4xz+9x+5y+4z+8}{3xz+4x+9y+8z+7}, \frac{6xz+9x+10y+3z+5}{xz+6x+4y+5z+3} \right]$$

[[0.703645740, 0.874690403, 1.594491647]]

[[0.7036457417, 0.8746904032, 1.594491647], [0.7036457417, 0.8746904032, 1.594491647], [0.7036457417, 0.8746904032, 1.594491647], [0.7036457417, 0.8746904032, 1.594491647], [0.7036457417, 0.8746904032, 1.594491647], [0.7036457417, 0.8746904032, 1.594491647], [0.7036457417, 0.8746904032, 1.594491647], [0.7036457417, 0.8746904032, 1.594491647], [0.7036457417, 0.8746904032, 1.594491647], [0.7036457417, 0.8746904032, 1.594491647]] **(29)**

```
> R8 := RT3(x, y, z, 1, 10);
  SFP3(R8, x, y, z);
```

$Orb3(R8, x, y, z, [0.5, 1, 2], 1000, 1010);$

$$R8 := \left[\frac{3xz + 4x + 7y + 3z + 9}{8xz + 3x + 10y + z + 1}, \frac{6xz + 7x + 5y + 9z + 7}{8xz + 4x + 5y + 5z + 2}, \frac{8xz + 10x + 4y + 6z + 4}{3xz + x + 4y + z + 4} \right]$$

[[0.9431840530, 1.293653716, 2.696736463]]

[[0.9431840530, 1.293653723, 2.696736464], [0.9431840530, 1.293653724, 2.696736463], [0.9431840532, 1.293653724, 2.696736465], [0.9431840533, 1.293653723, 2.696736465], [0.9431840530, 1.293653723, 2.696736465], [0.9431840530, 1.293653723, 2.696736464], [0.9431840530, 1.293653724, 2.696736463], [0.9431840532, 1.293653724, 2.696736465], [0.9431840533, 1.293653723, 2.696736465], [0.9431840530, 1.293653723, 2.696736465]] **(30)**

> $R9 := RT3(x, y, z, 1, 10);$
$SFP3(R9, x, y, z);$
$Orb3(R9, x, y, z, [0.5, 1, 2], 1000, 1010);$

$$R9 := \left[\frac{2xz + 8x + 6y + 7z + 3}{3xz + 10x + 6y + 4z + 5}, \frac{8xz + 8x + 2y + 4z + 7}{10xz + x + 7y + 2z + 6}, \frac{2xz + 4x + 6y + 5z + 2}{8xz + 3x + 6y + 8z + 5} \right]$$

[[0.9053792656, 1.091088680, 0.6818563962]]

[[0.9053792658, 1.091088675, 0.6818564013], [0.9053792654, 1.091088675, 0.6818564013], [0.9053792658, 1.091088675, 0.6818564013], [0.9053792654, 1.091088675, 0.6818564013], [0.9053792658, 1.091088675, 0.6818564013], [0.9053792654, 1.091088675, 0.6818564013], [0.9053792658, 1.091088675, 0.6818564013], [0.9053792654, 1.091088675, 0.6818564013], [0.9053792658, 1.091088675, 0.6818564013], [0.9053792654, 1.091088675, 0.6818564013]] **(31)**

> $R10 := RT3(x, y, z, 1, 10);$
$SFP3(R10, x, y, z);$
$Orb3(R10, x, y, z, [0.5, 1, 2], 1000, 1010);$

$$R10 := \left[\frac{3xz + 6x + 3y + 8z + 8}{5xz + 2x + 5y + 5z + 4}, \frac{10xz + 6x + 5y + 10z + 6}{8xz + 5x + 2y + 2z + 9}, \frac{6xz + x + 2y + 3z + 10}{4xz + 4x + 8y + 8z + 4} \right]$$

[[1.3080878, 1.3728489, 0.7357241903]]

[[1.308089928, 1.372848921, 0.7357241900], [1.308089928, 1.372848921, 0.7357241900], [1.308089928, 1.372848921, 0.7357241900], [1.308089928, 1.372848921, 0.7357241900], [1.308089928, 1.372848921, 0.7357241900], [1.308089928, 1.372848921, 0.7357241900], [1.308089928, 1.372848921, 0.7357241900], [1.308089928, 1.372848921, 0.7357241900], [1.308089928, 1.372848921, 0.7357241900], [1.308089928, 1.372848921, 0.7357241900]] **(32)**

```
> #5 Extra Credit
  NB := proc(l, a, c) local F, L, i, S :
   #here, x = N and y = P
   F := [l·x·exp(-a·y), c·x·(1 − exp(-a·y))] :
  L := [solve({F[1]=x, F[2]=y}, {x, y})] :
   S := SFP2(F, x, y) :
       #calculates if there is a stable fixed point and is outputted as last entry of list
  [seq(subs(L[i], [x, y]), i=1 ..nops(L)), S] :
   end:
> Q1 := NB(0.2, 0.5, 0.5); #example of when there is a stable fixed point
```

$$Q1 := [[0., 0.], [1.609437912, -3.218875825], [[0., 0.]]] \quad \textbf{(33)}$$

```
> Q2 := NB(2, 4.1, 0.2); #example of when there is no stable fixed point
```

$$Q2 := [[0., 0.], [1.690602879, 0.1690602879], [\ ]] \quad \textbf{(34)}$$

```
>
```