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> # Max Mekhanikov - HW 13 - Okay to Post
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> # Question 2
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> #FP2(F,x,y): The list of fixed points of the transformation [x,y]->F. Try
#FP2([x-y,x=y],x,y);
FP2 := proc(F, x, y) local L, i :
L := [solve({F[1] = x, F[2] = y}, {x, y})] :

[seq(subs(L[i], [x, y]), i = 1 ..nops(L))] :
end:
```

```
> #SFP2(F,x,y): The list of Stable fixed points of the transformation [x,y]->F. Try
#SFP2([(1+x)/(1+y), (1+7*y)/(4+x)],x,y);
SFP2 := proc(F, x, y) local L, J, S, J0, i, pt, EV :

L := evalf(FP2(F, x, y)) :
    #F is the list of ALL fixed points of the transformation [x,y]->F using the previous procedure
    FP2(F,x,y), but since we are interested in numbers we take the floating point version using
    evalf

J := Matrix(normal([[diff(F[1], x), diff(F[2], x)], [diff(F[1], y), diff(F[2], y)]])) :
    #J is the Jacobian matrix in general (in terms of the variables x and y). Note that J is a
    SYMBOLIC matrix featuring variables x and y

S := [ ] :  #S is the list of stable fixed points that starts out empty

for i from 1 to nops(L) do   #we examime it case by case
pt := L[i] : #pt is the current fixed point to be examined

J0 := subs({x = pt[1], y = pt[2]}, J) :
    #J0 is the NUMERICAL matrix obtained by plugging-in the examined fixed pt

EV := Eigenvalues(J0) :
    # We used Maple's command Eigenavalues to find the eigenvalues of this 2 by 2 matrix

if abs(EV[1]) < 1 and abs(EV[2]) < 1  then
 S := [op(S), pt] :
    #If both eigenvalues have absolute value less than 1 it means that they are stable, so we
    append the examined fixed point, pt, to the list of fixed points
fi:

od:
S : #the output is S
end:
```

> $f := \frac{(x^2 + 8 \cdot x + 4)}{x}$

$$f := \frac{x^2 + 8\,x + 4}{x} \tag{1}$$

> $g := \frac{(4 \cdot x^2 + 8 \cdot x + 4)}{4 \cdot x^2 + 3}$

$$g := \frac{4\,x^2 + 8\,x + 4}{4\,x^2 + 3} \tag{2}$$

```
> SFPe := proc(f, x) local f1, L, i, M :
  f1 := normal(diff(f, x)) :
  L := [solve(numer(f-x), x)] :
  M := [ ] :

  for i from 1 to nops(L) do
   if subs(x = L[i], denom(f1) ) ≠ 0 then
    M := [op(M), [L[i], normal(subs(x = L[i], f1))]] :
   fi:
  od:
  M :

  end:
```

> $SFPe(f, x)$

$$\left[\left[-\frac{1}{2}, -15\right]\right] \tag{3}$$

```
> FP := proc(f, x)
  evalf([solve(f = x)]) :
  end:
```

> $FP(f, x)$

$$[-0.5000000000] \tag{4}$$

> $SFPe(g, x)$

$$\left[\left[\frac{(161 + 3\sqrt{2118})^{1/3}}{6} + \frac{19}{6\,(161 + 3\sqrt{2118})^{1/3}} + \frac{1}{3}, -\left(36\left(2\,(161 + 3\sqrt{2118})^{4/3}\right.\right.\right.\right.$$

$$+ 2493 + 33\sqrt{2118} + 36\,(161 + 3\sqrt{2118})^{2/3} + 209\,(161 + 3\sqrt{2118})^{1/3}\Big)\Big(161$$

$$+ 3\sqrt{2118})^{2/3}\Big) \Big/ \Big(\big((161 + 3\sqrt{2118})^{4/3} + 1005 + 12\sqrt{2118} + 69\,(161$$

$$+ 3\sqrt{2118})^{2/3} + 76\,(161 + 3\sqrt{2118})^{1/3})^{2}\Big],\ \Bigg[-\frac{(161 + 3\sqrt{2118})^{1/3}}{12}$$

$$-\frac{19}{12\,(161 + 3\sqrt{2118})^{1/3}} + \frac{1}{3}$$

$$+\frac{I\sqrt{3}\left(\frac{(161 + 3\sqrt{2118})^{1/3}}{6} - \frac{19}{6\,(161 + 3\sqrt{2118})^{1/3}}\right)}{2},\ \Big(72\,\Big(2\,I\sqrt{3}\ (161 + 3\sqrt{2118})^{4/3}$$

$$- 2493\,I\sqrt{3} - 72\,(161 + 3\sqrt{2118})^{2/3} + 209\,(161 + 3\sqrt{2118})^{1/3} + 33\sqrt{2118}$$

$$+ 2493\Big)\ (161 + 3\sqrt{2118})^{2/3}\Big)\Big/\Big(I\sqrt{3}\ (161 + 3\sqrt{2118})^{4/3} + (161$$

$$+ 3\sqrt{2118})^{4/3} + 76\,I\sqrt{3}\ (161 + 3\sqrt{2118})^{1/3} - 12\,I\sqrt{3}\ \sqrt{2118} - 1005\,I\sqrt{3}$$

$$- 138\,(161 + 3\sqrt{2118})^{2/3} + 76\,(161 + 3\sqrt{2118})^{1/3} + 12\sqrt{2118} + 1005\Big)^{2}\Bigg],\ \Bigg[$$

$$-\frac{(161 + 3\sqrt{2118})^{1/3}}{12} - \frac{19}{12\,(161 + 3\sqrt{2118})^{1/3}} + \frac{1}{3}$$

$$-\frac{I\sqrt{3}\left(\frac{(161 + 3\sqrt{2118})^{1/3}}{6} - \frac{19}{6\,(161 + 3\sqrt{2118})^{1/3}}\right)}{2},$$

$$-\Big(72\,\Big(2\,I\sqrt{3}\,\big(161+3\sqrt{2118}\big)^{4/3}-2\,\big(161+3\sqrt{2118}\big)^{4/3}+209\,I\sqrt{3}\,\big(161+3\sqrt{2118}\big)^{1/3}-33\,I\sqrt{3}\sqrt{2118}-2493\,I\sqrt{3}+72\,\big(161+3\sqrt{2118}\big)^{2/3}-209\,\big(161+3\sqrt{2118}\big)^{1/3}-33\sqrt{2118}-2493\Big)\big(161+3\sqrt{2118}\big)^{2/3}\Big)\Big/\Big(I\sqrt{3}\,\big(161+3\sqrt{2118}\big)^{4/3}-\big(161+3\sqrt{2118}\big)^{4/3}+76\,I\sqrt{3}\,\big(161+3\sqrt{2118}\big)^{1/3}-12\,I\sqrt{3}\sqrt{2118}-1005\,I\sqrt{3}+138\,\big(161+3\sqrt{2118}\big)^{2/3}-76\,\big(161+3\sqrt{2118}\big)^{1/3}-12\sqrt{2118}-1005\Big)^2\Big]\Big]$$

> $FP(g, x)$

$$[1.921424369, -0.4607121842+0.5551499970\,I, -0.4607121842-0.5551499970\,I] \quad (6)$$

> $SFP2([f, g], x, y)$

Error, (in SFP2) cannot determine if this expression is true or false: abs(Eigenvalues(Matrix(2, 2, {(1, 1) = -15.00000000, (1, 2) = 1.250000000, (2, 1) = 0, (2, 2) = 0}))[1]) < 1 and abs(Eigenvalues(Matrix(2, 2, {(1, 1) = -15.00000000, (1, 2) = 1.250000000, (2, 1) = 0, (2, 2) = 0}))[2]) < 1

```
> #Orb2(F,x,y,pt,K1,K2): Inputs a mapping F=[f,g] from R^2  to R^2 where f and g describe
      functions of x and y, an initial point pt0=[x0,y0]
  #outputs the orbit starting at discrete time K1 and ending in discrete time K2. Try
  #F:=RT2(x,y,2,10);
  #Orb2(F,x,y,[1.1,1.2],1000,1010);
  Orb2 := proc(F, x, y, pt0, K1, K2)  local pt, L, i :
  pt := pt0 :

  for i from 1 to K1 do
  pt := subs({x = pt[1], y = pt[2]}, F) :
  od:

  L := [ ]:
  for i from K1 + 1 to K2 do
  L := [op(L), pt] :
  pt := subs({x = pt[1], y = pt[2]}, F) :

  od:
  L :
  end:
```

> $Orb2([f, g], x, y, [8.5, 0.5], 2000, 2010)$

[[16012.50600, 1.000124966], [16020.50625, 1.000124903], [16028.50650, 1.000124841], [16036.50675, 1.000124780], [16044.50700, 1.000124716], [16052.50725, 1.000124654], [16060.50750, 1.000124592], [16068.50775, 1.000124530], [16076.50800, 1.000124468], [16084.50825, 1.000124406]] (7)

> **# Appears that f(x) does not have a stable fixed point as the values continute to increase with each iteration. However, g(x) stabilizes at approx 1.000124 as its stable fixed point.**

Question 3

```
> #RT2(x,y,d,K): A random rational transformation of degree d from R^2 to R^2 with postiive
     integer coefficients from 1 to K The inputs are variables x and y and
#the output is a pair of expressions  of (x,y) representing functions. It is for generating examples
#Try:
#RT2(x,y,2,10);
RT2 := proc(x, y, d, K) local ra, i, j, f, g :
ra := rand(1 ..K) : #random integer from -K to K
f := add(add(ra( ) * x^i * y^j, j = 0 ..d - i), i = 0 ..d) / add(add(ra( ) * x^i * y^j, j = 0 ..d - i), i
     = 0 ..d) :
g := add(add(ra( ) * x^i * y^j, j = 0 ..d - i), i = 0 ..d) / add(add(ra( ) * x^i * y^j, j = 0 ..d - i), i
     = 0 ..d) :
[f, g] :
end:
```

```
> a1 := RT2(x, y, 1, 100)
```

$$a1 := \left[\frac{28 + 75\,y + 3\,x}{5 + 11\,y + 37\,x}, \frac{75 + 4\,y + 91\,x}{22 + 40\,y + 58\,x} \right] \quad (8)$$

(9)

```
> b2 := RT2(x, y, 1, 100)
```

$$b2 := \left[\frac{9 + 53\,y + 37\,x}{88 + 50\,y + 37\,x}, \frac{76 + 95\,y + 8\,x}{92 + 92\,y + 2\,x} \right] \quad (10)$$

```
> c3 := RT2(x, y, 1, 100)
```

$$c3 := \left[\frac{33 + 30\,y + x}{83 + 9\,y + 64\,x}, \frac{43 + 57\,y + 52\,x}{62 + 46\,y + 76\,x} \right] \quad (11)$$

```
> d4 := RT2(x, y, 1, 100)
```

$$d4 := \left[\frac{16 + 29\,y + 51\,x}{3 + 45\,y + 67\,x}, \frac{40 + 71\,y + 74\,x}{49 + 60\,y + 69\,x} \right] \quad (12)$$

```
> e5 := RT2(x, y, 1, 100)
```

$$e5 := \left[\frac{9 + 53\,y + 3\,x}{98 + 69\,y + 3\,x}, \frac{73 + 88\,y + 37\,x}{60 + 94\,y + 52\,x} \right] \quad (13)$$

```
> f6 := RT2(x, y, 1, 100)
```

$$f6 := \left[\frac{97 + 44\,y + 9\,x}{30 + 14\,y + 79\,x}, \frac{73 + 21\,y + 78\,x}{49 + 93\,y + 15\,x} \right] \quad (14)$$

```
> g7 := RT2(x, y, 1, 100)
```

$$g7 := \left[\frac{56 + 69\,y + 17\,x}{21 + 42\,y + 21\,x}, \frac{5 + 58\,y + 3\,x}{86 + 55\,y + 97\,x} \right] \quad (15)$$

```
> h8 := RT2(x, y, 1, 100)
```

$$h8 := \left[\frac{4 + 92\,y + 46\,x}{88 + 34\,y + 68\,x}, \frac{49 + 61\,y + 21\,x}{86 + 42\,y + 5\,x} \right] \quad (16)$$

```
> i9 := RT2(x, y, 1, 100)
```

$$i9 := \left[\frac{33 + 77\,y + 98\,x}{58 + 98\,y + 29\,x}, \frac{65 + 29\,y + 35\,x}{29 + 34\,y + 66\,x} \right] \quad (17)$$

> $j10 := RT2(x, y, 1, 100)$

$$j10 := \left[\frac{44 + 60\,y + 83\,x}{32 + 85\,y + 100\,x}, \frac{68 + 59\,y + 40\,x}{76 + 92\,y + 39\,x}\right] \quad (18)$$

> $k11 := RT2(x, y, 1, 100)$

$$k11 := \left[\frac{17 + 50\,y + 78\,x}{20 + 18\,y + 18\,x}, \frac{51 + 34\,y + 78\,x}{10 + 52\,y + 100\,x}\right] \quad (19)$$

> $l12 := RT2(x, y, 1, 100)$

$$l12 := \left[\frac{13 + 87\,y + 13\,x}{37 + 92\,y + 97\,x}, \frac{69 + 62\,y + 38\,x}{60 + 46\,y + 78\,x}\right] \quad (20)$$

> $m13 := RT2(x, y, 1, 100)$

$$m13 := \left[\frac{61 + 80\,y + 3\,x}{72 + 48\,y + 9\,x}, \frac{41 + 46\,y + 78\,x}{35 + 88\,y + 79\,x}\right] \quad (21)$$

> $n14 := RT2(x, y, 1, 100)$

$$n14 := \left[\frac{26 + 27\,y + 76\,x}{83 + 55\,y + 35\,x}, \frac{72 + 95\,y + 100\,x}{55 + 81\,y + 38\,x}\right] \quad (22)$$

> $o15 := RT2(x, y, 1, 100)$

$$o15 := \left[\frac{88 + 20\,y + 16\,x}{17 + 68\,y + 79\,x}, \frac{48 + 67\,y + 98\,x}{86 + 92\,y + 74\,x}\right] \quad (23)$$

> $p16 := RT2(x, y, 1, 100)$

$$p16 := \left[\frac{33 + 55\,y + 17\,x}{82 + 25\,y + 94\,x}, \frac{24 + 60\,y + 74\,x}{17 + 14\,y + 12\,x}\right] \quad (24)$$

> $q17 := RT2(x, y, 1, 100)$

$$q17 := \left[\frac{87 + 79\,y + 64\,x}{7 + 69\,y + 90\,x}, \frac{83 + 3\,y + 48\,x}{16 + 84\,y + 63\,x}\right] \quad (25)$$

> $r18 := RT2(x, y, 1, 100)$

$$r18 := \left[\frac{41 + 53\,y + 30\,x}{44 + 55\,y + 85\,x}, \frac{33 + 38\,y + 42\,x}{89 + 65\,y + 46\,x}\right] \quad (26)$$

> $s19 := RT2(x, y, 1, 100)$

$$s19 := \left[\frac{67 + 37\,y + 90\,x}{44 + 99\,y + 21\,x}, \frac{73 + 60\,y + 37\,x}{33 + 99\,y + 18\,x}\right] \quad (27)$$

> $t20 := RT2(x, y, 1, 100)$

$$t20 := \left[\frac{64 + 24\,y + 78\,x}{45 + 66\,y + 24\,x}, \frac{59 + 89\,y + 95\,x}{97 + 60\,y + 66\,x}\right] \quad (28)$$

> $Orb2(a1, x, y, [1.1, 1.2], 1000, 1010)$

[[1.652914952, 1.344550082], [1.652914953, 1.344550082], [1.652914952, 1.344550082], **(29)**
[1.652914953, 1.344550082], [1.652914952, 1.344550082], [1.652914953, 1.344550082],
[1.652914952, 1.344550082], [1.652914953, 1.344550082], [1.652914952, 1.344550082],
[1.652914953, 1.344550082]]

> $Orb2(b2, x, y, [0.8, 1.3], 1000, 1010)$

[[0.5049584880, 0.9436143438], [0.5049584876, 0.9436143443], [0.5049584880, **(30)** 0.9436143438], [0.5049584876, 0.9436143443], [0.5049584880, 0.9436143438], [0.5049584876, 0.9436143443], [0.5049584880, 0.9436143438], [0.5049584876, 0.9436143443], [0.5049584880, 0.9436143438], [0.5049584876, 0.9436143443]]

> *Orb2*(*c3*, *x*, *y*, [1.2, 1.0], 1000, 1010)

[[0.4839487184, 0.8451338337], [0.4839487184, 0.8451338337], [0.4839487184, 0.8451338337], [0.4839487184, 0.8451338337], [0.4839487184, 0.8451338337], [0.4839487184, 0.8451338337], [0.4839487184, 0.8451338337], [0.4839487184, 0.8451338337], [0.4839487184, 0.8451338337], [0.4839487184, 0.8451338337]] **(31)**

> *Orb2*(*d4*, *x*, *y*, [0.9, 1.4], 1000, 1010)

[[0.8391036651, 1.039150886], [0.8391036651, 1.039150886], [0.8391036651, 1.039150886], [0.8391036651, 1.039150886], [0.8391036651, 1.039150886], [0.8391036651, 1.039150886], [0.8391036651, 1.039150886], [0.8391036651, 1.039150886], [0.8391036651, 1.039150886], [0.8391036651, 1.039150886]] **(32)**

> *Orb2*(*e5*, *x*, *y*, [1.0, 0.7], 1000, 1010)

[[0.3766860779, 1.007486264], [0.3766860779, 1.007486264], [0.3766860779, 1.007486264], [0.3766860779, 1.007486264], [0.3766860779, 1.007486264], [0.3766860779, 1.007486264], [0.3766860779, 1.007486264], [0.3766860779, 1.007486264], [0.3766860779, 1.007486264], [0.3766860779, 1.007486264]] **(33)**

> *Orb2*(*f6*, *x*, *y*, [0.7, 1.3], 1000, 1010)

[[1.145764021, 1.100538302], [1.145764021, 1.100538301], [1.145764021, 1.100538302], [1.145764021, 1.100538301], [1.145764021, 1.100538302], [1.145764021, 1.100538301], [1.145764021, 1.100538302], [1.145764021, 1.100538301], [1.145764021, 1.100538302], [1.145764021, 1.100538301]] **(34)**

> *Orb2*(*g7*, *x*, *y*, [1.4, 0.9], 1000, 1010)

[[1.543756249, 0.05330678412], [1.543756249, 0.05330678412], [1.543756249, 0.05330678412], [1.543756249, 0.05330678412], [1.543756249, 0.05330678412], [1.543756249, 0.05330678412], [1.543756249, 0.05330678412], [1.543756249, 0.05330678412], [1.543756249, 0.05330678412], [1.543756249, 0.05330678412]] **(35)**

> *Orb2*(*h8*, *x*, *y*, [1.4, 1.2], 1000, 1010)

[[0.7326409777, 0.9430704525], [0.7326409777, 0.9430704525], [0.7326409777, 0.9430704525], [0.7326409777, 0.9430704525], [0.7326409777, 0.9430704525], [0.7326409777, 0.9430704525], [0.7326409777, 0.9430704525], [0.7326409777, 0.9430704525], [0.7326409777, 0.9430704525], [0.7326409777, 0.9430704525]] **(36)**

> *Orb2*(*i9*, *x*, *y*, [1.1, 0.6], 1000, 1010)

[[1.203176552, 0.9568427187], [1.203176552, 0.9568427187], [1.203176552, 0.9568427187], [1.203176552, 0.9568427187], [1.203176552, 0.9568427187], [1.203176552, 0.9568427187], [1.203176552, 0.9568427187], [1.203176552, 0.9568427187], [1.203176552, 0.9568427187], [1.203176552, 0.9568427187]] **(37)**

> $Orb2(j10, x, y, [0.9, 0.8], 1000, 1010)$

[[0.8767042216, 0.8161846602], [0.8767042216, 0.8161846606], [0.8767042212, 0.8161846602], [0.8767042221, 0.8161846601], [0.8767042216, 0.8161846602], [0.8767042216, 0.8161846606], [0.8767042212, 0.8161846602], [0.8767042221, 0.8161846601], [0.8767042216, 0.8161846602], [0.8767042216, 0.8161846606]] **(38)**

> $Orb2(k11, x, y, [1, 1.4], 1000, 1010)$

[[3.353494571, 0.8758183604], [3.353494570, 0.8758183602], [3.353494571, 0.8758183604], [3.353494570, 0.8758183602], [3.353494571, 0.8758183604], [3.353494570, 0.8758183602], [3.353494571, 0.8758183604], [3.353494570, 0.8758183602], [3.353494571, 0.8758183604], [3.353494570, 0.8758183602]] **(39)**

> $Orb2(l12, x, y, [1.2, 1.5], 1000, 1010)$

[[0.5822288819, 1.012580430], [0.5822288819, 1.012580430], [0.5822288819, 1.012580430], [0.5822288819, 1.012580430], [0.5822288819, 1.012580430], [0.5822288819, 1.012580430], [0.5822288819, 1.012580430], [0.5822288819, 1.012580430], [0.5822288819, 1.012580430], [0.5822288819, 1.012580430]] **(40)**

> $Orb2(m13, x, y, [0.8, 1.3], 1000, 1010)$

[[1.077863827, 0.8431202951], [1.077863827, 0.8431202951], [1.077863827, 0.8431202951], [1.077863827, 0.8431202951], [1.077863827, 0.8431202951], [1.077863827, 0.8431202951], [1.077863827, 0.8431202951], [1.077863827, 0.8431202951], [1.077863827, 0.8431202951], [1.077863827, 0.8431202951]] **(41)**

> $Orb2(n14, x, y, [1.5, 1.2], 1000, 1010)$

[[0.6069070593, 1.388720927], [0.6069070589, 1.388720926], [0.6069070593, 1.388720927], [0.6069070589, 1.388720926], [0.6069070593, 1.388720927], [0.6069070589, 1.388720926], [0.6069070593, 1.388720927], [0.6069070589, 1.388720926], [0.6069070593, 1.388720927], [0.6069070589, 1.388720926]] **(42)**

> $Orb2(o15, x, y, [1.3, 0.7], 1000, 1010)$

[[0.8434406341, 0.8287362461], [0.8434406341, 0.8287362461], [0.8434406341, 0.8287362461], [0.8434406341, 0.8287362461], [0.8434406341, 0.8287362461], [0.8434406341, 0.8287362461], [0.8434406341, 0.8287362461], [0.8434406341, 0.8287362461], [0.8434406341, 0.8287362461], [0.8434406341, 0.8287362461]] **(43)**

> $Orb2(p16, x, y, [1.4, 0.9], 1000, 1010)$

[[0.9801935164, 3.968689691], [0.9801935160, 3.968689691], [0.9801935160, 3.968689692], [0.9801935164, 3.968689691], [0.9801935160, 3.968689691], [0.9801935160, 3.968689692], [0.9801935164, 3.968689691], [0.9801935160, 3.968689691], [0.9801935160, 3.968689692], [0.9801935164, 3.968689691]] **(44)**

> $Orb2(q17, x, y, [1.3, 1.2], 1000, 1010)$

[[1.298832575, 0.8669482427], [1.298832575, 0.8669482427], [1.298832575, 0.8669482427], [1.298832575, 0.8669482427], [1.298832575, 0.8669482427], [1.298832575, 0.8669482427], [1.298832575, 0.8669482427], [1.298832575, 0.8669482427], **(45)** [1.298832575, 0.8669482427], [1.298832575, 0.8669482427]]

> *Orb2*(*r18*, *x*, *y*, [1.0, 1.4], 1000, 1010)

[[0.6826253192, 0.5282760494], [0.6826253197, 0.5282760493], [0.6826253192, 0.5282760494], [0.6826253197, 0.5282760493], [0.6826253192, 0.5282760494], [0.6826253197, 0.5282760493], [0.6826253192, 0.5282760494], [0.6826253197, 0.5282760493], [0.6826253192, 0.5282760494], [0.6826253197, 0.5282760493]] **(46)**

> *Orb2*(*s19*, *x*, *y*, [1.6, 1.2], 1000, 1010)

[[1.205130256, 1.116987304], [1.205130256, 1.116987304], [1.205130256, 1.116987304], [1.205130256, 1.116987304], [1.205130256, 1.116987304], [1.205130256, 1.116987304], [1.205130256, 1.116987304], [1.205130256, 1.116987304], [1.205130256, 1.116987304], [1.205130256, 1.116987304]] **(47)**

> *Orb2*(*t20*, *x*, *y*, [0.8, 1.3], 1000, 1010)

[[1.268082882, 1.126654461], [1.268082882, 1.126654461], [1.268082882, 1.126654461], [1.268082882, 1.126654461], [1.268082882, 1.126654461], [1.268082882, 1.126654461], [1.268082882, 1.126654461], [1.268082882, 1.126654461], [1.268082882, 1.126654461], [1.268082882, 1.126654461]] **(48)**

>

2) RUID: 184004391

$$x(n) = \frac{x(n-1)^2 + 8x(n-1) + 4}{x(n-1)^2}$$

$$y(n) = \frac{4x(n-1)^2 + 8x(n-1) + 4}{4x(n-1)^2 + 3}$$

$$x(n) = f(x_{n-1}) \qquad y(n) = g(x_{n-1})$$

$$f(x) = \frac{x^2 + 8x + 4}{x} \qquad g(x) = \frac{4x^2 + 8x + 4}{4x^2 + 3}$$