Homework 13

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due 10/24

Please refrain from posting my answers.

1. $\tan(\pi/4) = 1$, $\arctan(1) = \pi/4$

$$\pi = 4\left(1 - \tfrac{1}{3} + \tfrac{1}{5} - \tfrac{1}{7} + \tfrac{1}{9} - \dots\right)$$

$$\begin{aligned}
\pi/4 &= \arctan(1) \\
&= \int_0^1 \frac{1}{x^2+1}\,dx \\
&= \int_0^1 \sum_{k=0}^{n} (-1)^k x^{2k} + \frac{(-1)^{n+1} x^{2n+2}}{1+x^2}\,dx \\
&= \sum_{k=0}^{n} \frac{(-1)^k}{2k+1} + (-1)^{n+1}\int_0^1 \frac{x^{2n+2}}{1+x^2}\,dx \\
&= \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} \\
&= 1 - \tfrac{1}{3} + \tfrac{1}{5} - \tfrac{1}{7} + \tfrac{1}{9} - \dots
\end{aligned}$$

$$\pi = 4\left(1 - \tfrac{1}{3} + \tfrac{1}{5} - \tfrac{1}{7} + \tfrac{1}{9} - \dots\right)$$

1' $4\left(1 - \tfrac{1}{3} + \tfrac{1}{5} - \tfrac{1}{7} + \tfrac{1}{9} - \tfrac{1}{11} + \tfrac{1}{13} - \tfrac{1}{15} + \tfrac{1}{17} - \tfrac{1}{19}\right)$

$= 3.0418396189\dots$

$$\begin{aligned}
|R_{10}| = |s - s_{10}| &\le b_{11} \\
&\le \frac{1}{2(11)+1}
\end{aligned}$$

error $\le \frac{1}{23}$ or $.043478\dots$

2. $\pi/4 = \arctan(1/2) + \arctan(1/3)$

$$\arctan(u) + \arctan(v) = \arctan\left(\frac{u+v}{1-uv}\right)$$

$$\begin{aligned}
\arctan(1/2) + \arctan(1/3) &= \arctan\left(\frac{1/2 + 1/3}{1-(1/2)(1/3)}\right) \\
&= \arctan\left(\frac{5/6}{1-1/6}\right) \\
&= \arctan(1) \\
&= \pi/4
\end{aligned}$$

2'. $\cancel{\pi/4 = 4\arctan(1/5) - \arctan(1/239)}$

$\pi/4 = \arctan(1/2) + \arctan(1/3)$; $\arctan(x) = \sum_0^\infty (-1)^n \frac{x^{2n+1}}{2n+1}$

$\pi/4 = \arctan(1)$ (from previous problem)

$\pi = 4\arctan(1)$

$= 4\sum_0^\infty (-1)^n \frac{1^{2n+1}}{2n+1} =$

$= 4(1 - 1/3 + 1/5 - 1/7 + 1/9 - 1/11 + \dots)$

$= 3.14\dots$

3. $\pi/4 = 4\arctan(1/5) - \arctan(1/239)$

$= 4(1/5 - 1/3\cdot5^3 + 1/5\cdot5^5 - 1/7\cdot5^7 + \dots) - (1/239 - 1/3\cdot239^3$
$+ 1/5\cdot239^5 - 1/7\cdot239^7 + \dots)$

$= 3.141\dots$

4. $13 - \left(\frac{1}{2}\right)^5\left(\frac{20\cdot50+13}{2^{10}}\right) + \left(\frac{1\cdot3}{2\cdot4}\right)^5\left(\frac{40\cdot91+13}{2^{20}}\right) - \left(\frac{1\cdot3\cdot5}{2\cdot4\cdot6}\right)\left(\frac{60\cdot132+13}{2^{30}}\right)$

$= 12.969\dots$

$128/12.969\dots = 9.8696\dots$

$\sqrt{9.8696\dots} = 3.1415926535\underline{9}$

first incorrect digit

This approximation is accurate for 11 digits (including the ones)