```
> #Homework 13
> read `C:/Users/cgrie/Dynam Models Bio/Homeworks/HW13/M13.txt`
> Help13()
```

RT2(x,y,d,K), Orb2(F,x,y,pt0,K1,K2), FP2(F,x,y), SFP2(F,x,y) (1)

```
> #QUESTION 1:
>
```

If a_i is the *i*-th digit of your RUID, find the fixed points, and the stable fixed points of the following first-order system of two quantities *x(n)* and *y(n)*

$$x(n) = \frac{a_1 x(n-1)^2 + a_2 x(n-1) + a_3}{a_1 x(n-1)^2 + a_4 x(n-1) + a_5}$$

$$y(n) = \frac{a_6 x(n-1)^2 + a_2 x(n-1) + a_3}{a_3 x(n-1)^2 + a_4 x(n-1) + a_7}$$

If it has a stable fixed point, confirm it using **Orb2** with initial conditions
$x(0) = a_2 + 0.5$, $y(0) = a_4 + 0.5$

```
> #QUESTION 2:
```

By running **RT2(x,y,d,K)** with **d=1** and **K=100** generate 20 random transformations and find the stable equilibria for each transformation (if they exist)

```
> print(RT2);
```

proc(x, y, d, K) (2)

local *ra, i, j, f, g*;

$ra := rand(1..K)$;

$f := add(add(ra()*x^\wedge i*y^\wedge j, j=0..d-i), i=0..d) / add(add(ra()*x^\wedge i*y^\wedge j, j=0..d-i), i=0..d)$;

$g := add(add(ra()*x^\wedge i*y^\wedge j, j=0..d-i), i=0..d) / add(add(ra()*x^\wedge i*y^\wedge j, j=0..d-i), i=0..d)$;

$[f, g]$

end proc

```
> #Generating the 20 random transformations:
> A := seq(RT2(x,y,1,100),n=1..20);
```

$$A := \left[\frac{93+45y+96x}{6+98y+59x}, \frac{44+100y+38x}{69+27y+96x}\right], \left[\frac{17+90y+34x}{18+52y+56x}, \frac{43+83y+25x}{90+93y+60x}\right], \quad (3)$$

$$\left[\frac{93+14y+50x}{47+8y+46x}, \frac{44+9y+77x}{59+16y+x}\right], \left[\frac{70+77y+39x}{92+71y+67x}, \frac{78+51y+53x}{12+19y+63x}\right],$$

$$\left[\frac{40+90y+3x}{49+49y+67x}, \frac{74+90y+74x}{27+98y+72x}\right], \left[\frac{2+73y+85x}{41+4y+44x}, \frac{13+19y+10x}{15+64y+9x}\right],$$
$$\left[\frac{12+52y+25x}{72+90y+18x}, \frac{43+55y+40x}{17+70y+52x}\right], \left[\frac{81+87y+34x}{85+9y+68x}, \frac{83+63y+100x}{70+36y+36x}\right],$$
$$\left[\frac{10+40y+66x}{87+16y+98x}, \frac{43+53y+61x}{47+28y+75x}\right], \left[\frac{3+5y+11x}{37+75y+4x}, \frac{91+22y+40x}{58+93y+98x}\right],$$
$$\left[\frac{11+30y+6x}{32+40y+24x}, \frac{80+96y+11x}{23+41y+52x}\right], \left[\frac{58+67y+81x}{65+69y+2x}, \frac{36+61y+84x}{96+94y+31x}\right],$$
$$\left[\frac{81+31y+54x}{67+59y+66x}, \frac{12+49y+90x}{35+15y+26x}\right], \left[\frac{100+24y+8x}{63+78y+23x}, \frac{73+22y+32x}{98+9y+53x}\right],$$
$$\left[\frac{3+98y+69x}{3+73y+88x}, \frac{37+60y+94x}{52+16y+29x}\right], \left[\frac{51+3y+45x}{67+40y+71x}, \frac{74+49y+60x}{69+33y+30x}\right],$$
$$\left[\frac{1+83y+9x}{64+43y+57x}, \frac{52+62y+46x}{76+9y+53x}\right], \left[\frac{37+88y+50x}{37+76y+95x}, \frac{8+92y+92x}{2+97y+44x}\right],$$
$$\left[\frac{9+30y+14x}{79+73y+21x}, \frac{78+49y+93x}{15+56y+69x}\right], \left[\frac{17+21y+42x}{21+5y+58x}, \frac{3+86y+55x}{97+4y+92x}\right]$$

```
> #question to self: What does paramater K mean and paramater d mean?
   #Answer for K:  any random number ra is between 1 and the value of K
   #Answer for d:  the paramater d can be obtained experimentally below
      print("d=2");
      d2  := seq(RT2(x,y,2,100),n=1..3);
      print("d=3");
      d3  := seq(RT2(x,y,3,100),n=1..3);
      #Thus, d represents the highest degree of the algebraic expressions
```

"d=2"

$$d2 := \left[\frac{61x^2+49xy+34y^2+68x+88y+46}{77x^2+33xy+42y^2+5x+86y+21}, \frac{29x^2+65xy+98y^2+29x+58y+98}{60x^2+44xy+34y^2+66x+29y+35}\right], \left[\frac{59x^2+68xy+85y^2+100x+32y+83}{50x^2+17xy+92y^2+39x+76y+40}, \frac{34x^2+51xy+18y^2+18x+20y+78}{87x^2+13xy+52y^2+100x+10y+78}\right], \left[\frac{62x^2+69xy+92y^2+97x+37y+13}{80x^2+61xy+46y^2+78x+60y+38}, \frac{46x^2+41xy+48y^2+9x+72y+3}{27x^2+26xy+88y^2+79x+35y+78}\right]$$

"d=3"

$$d3 := \left[\frac{38x^3+81x^2y+100xy^2+35y^3+55x^2+95xy+55y^2+72x+83y+76}{86x^3+98x^2y+48xy^2+17y^3+67x^2+79xy+16y^2+68x+20y+88}, \right. \tag{4}$$

$$\left.\frac{60 x^3 + 24 x^2 y + 25 x y^2 + 55 y^3 + 94 x^2 + 82 x y + 33 y^2 + 17 x + 74 y + 92}{90 x^3 + 69 x^2 y + 64 x y^2 + 12 y^3 + 7 x^2 + 79 x y + 14 y^2 + 87 x + 17 y + 74}\right],$$

$$\left[\frac{44 x^3 + 30 x^2 y + 41 x y^2 + 16 y^3 + 53 x^2 + 63 x y + 48 y^2 + 84 x + 3 y + 83}{37 x^3 + 67 x^2 y + 65 x y^2 + 38 y^3 + 46 x^2 + 89 x y + 33 y^2 + 42 x + 85 y + 55},\right.$$

$$\left.\frac{18 x^3 + 99 x^2 y + 37 x y^2 + 21 y^3 + 33 x^2 + 60 x y + 99 y^2 + 73 x + 44 y + 90}{97 x^3 + 95 x^2 y + 59 x y^2 + 45 y^3 + 89 x^2 + 24 x y + 78 y^2 + 66 x + 24 y + 64}\right],$$

$$\left[\frac{9 x^3 + 100 x^2 y + 9 x y^2 + 47 y^3 + 47 x^2 + 10 x y + 79 y^2 + 32 x + 66 y + 60}{56 x^3 + 100 x^2 y + 94 x y^2 + 28 y^3 + 17 x^2 + 40 x y + 39 y^2 + 16 x + 56 y + 48},\right.$$

$$\left.\frac{16 x^3 + 30 x^2 y + 36 x y^2 + 35 y^3 + 15 x^2 + 44 x y + 54 y^2 + 57 x + 49 y + 77}{23 x^3 + 38 x^2 y + 15 x y^2 + 98 y^3 + 30 x^2 + 55 x y + 88 y^2 + 98 x + 50 y + 91}\right]$$

```
> #Now, find the stable equilibria for each transformation using
  SFP2
> print(SFP2);
```

```
proc(F, x, y)                                                        (5)
    local L, J, S, J0, i, pt, EV;
    L := evalf(FP2(F, x, y));
    J := Matrix([[diff(F[1], x), diff(F[2], x)], [diff(F[1], y), diff(F[2], y)]]);
    S := [ ];
    for i to nops(L) do
        pt := L[i];
        J0 := subs({x = pt[1], y = pt[2]}, J);
        EV := LinearAlgebra:-Eigenvalues(J0);
        if abs(EV[1]) < 1 and abs(EV[2]) < 1 then S := [op(S), pt] end if
    end do
end proc
```

```
> SFP2(A[1],x,y);
```

$$[[1.624821324, 0.7307142101]] \qquad \textbf{(6)}$$

Assuming that our equilibrium point is the output from the block above, we need to find a way to get enough iterations our of ORB2 without the program crashing.

```
> #Verify that these equilibria are stable using orb2 (Plug in the
  equilibria as initial conditions? The maple code took over 5
  minutes and never finished computing, even at 100 steps)
> print(Orb2);
```

```
proc(F, x, y, pt0, K1, K2)                                           (7)
    local pt, L, i;
    pt := pt0;
    for i to K1 do pt := subs({x = pt[1], y = pt[2]}, F) end do;
```

$L := [\ ];$

for i **from** $K1 + 1$ **to** $K2$ **do** $pt := subs(\{x = pt[1], y = pt[2]\}, F)$; $L := [op(L), pt]$ **end do**;

L

end proc

```
> #Whenever generating a random variable, you MUST call it with
  empty parenthesis.
  rvar := rand(1..10);
  print(rvar());
```

$rvar :=$ **proc**( )

proc( ) **option** $builtin = RandNumberInterface$; **end proc**$(6, 10, 4) + 1$

end proc

$$7 \tag{8}$$

```
> #Experiment: I think i only need to initialize a random variable
  once, and then:
  #Because essentially, rvar2() is picking an item from an array-
  like structure?
  #For the math homework, knowing this is kind of useless though
  rvar2 := rand(10..20);
```

$rvar2 :=$ **proc**( ) **(9)**

proc( ) **option** $builtin = RandNumberInterface$; **end proc**$(6, 11, 4) + 10$

end proc

```
> rvar2()
```

$$11 \tag{10}$$

```
> #Keep this code
  Orb2(A[1],x,y,[rvar2(),rvar2()],1,6);
  #Why do the fractions get so big so fast? is it because of
  finding least common multiples?
```

$$\left[\left[\frac{26827533}{19570642}, \frac{21958724}{21752523}\right], \left[\frac{8842467658970574}{6084602322725197}, \frac{3226209141368821}{3730753289609559}\right], \tag{11}$$

$$\left[\frac{30807116076394079957083486336238 70}{20031576590733814192829114197760 29}, \frac{21077055254833469108849162062360 10}{26316380093452021632591563953817 61}\right],$$

$$\left[\frac{145855297878199754186804398013395593437850217482843548741379741121 7187}{923723776704604473618847254809429172813214479133254107739302662731964},\right.$$

$$\left.\frac{13746207118994019625453567125911273681023310038529221480590269086 7528}{17943396033744796437421820659316997960080707835891890476070457568 1473}\right],$$

$[$

4625301188761196751203035390695496748396024814260267419645940939710293669363\

8590698147397058090586537225479336310534338699241689927346640332/

2887935118463158469902764084449157995877755139584064996227011662269155421311\

243337117918509954792100297685369061793822786583473785674369765 7,

149678571284756891953303297559543240617619948442008613835310044114058473804 7\
419014742886357188937644469811557968978387100319224537380316755 3/
199947441056834452331711438518871322895750328033398626539755056928424386045 0\
954070198109312615038358542790008913407613967730236798153949957 4]]

```
> #Maybe try getting everything in decimal form using evalf to more
  easily identify quantitative behavior
> evalf(Orb2(A[1],x,y,[rvar2(),rvar2()],90,91));
  #This block of code also takes way too long to run. I can usually
  only go up to 6 iterations
```

$$Orb2\left(A_1, x, y, \left[rvar2(\ ), rvar2(\ )\right], 90, 91\right) \tag{12}$$

```
> #QUESTION 4:
```

Write the three-variable analog of **RT2(x,y,d,K)** and call it **RT3(x,y,z,d,K)**

I see that the original code iterates through i and j to create each algebraic expression. for example, if **ra = 2** and **d=2**
therefore, we need to include one more iterable variable **m** that iterates from **0..d**

```
> RT3bad := proc(x,y,z,d,K) local ra,i,j,m,f,g,h;
  ra := rand(1..K);
  f:= add(add(add(ra()*x^i*y^j*z^m, m = 0 .. d-j), j = 0 .. d - i),
  i = 0 .. d)*1/add(add(add(ra()*x^i*y^j*z^m, m = 0 .. d-j), j = 0
  .. d - i), i = 0 .. d);
  g:= add(add(add(ra()*x^i*y^j*z^m, m = 0 .. d-j), j = 0 .. d - i),
  i = 0 .. d)*1/add(add(add(ra()*x^i*y^j*z^m, m = 0 .. d-j), j = 0
  .. d - i), i = 0 .. d);
  h:= add(add(add(ra()*x^i*y^j*z^m, m = 0 .. d-j), j = 0 .. d - i),
  i = 0 .. d)*1/add(add(add(ra()*x^i*y^j*z^m, m = 0 .. d-j), j = 0
  .. d - i), i = 0 .. d);
  end;
```

$RT3bad := \textbf{proc}(x, y, z, d, K)$ **(13)**

local $ra, i, j, m, f, g, h;$

$ra := rand(1..K);$

$f := add(add(add(ra(\ ) * x\text{^}i * y\text{^}j * z\text{^}m, m = 0..d - j), j = 0..d - i), i = 0..d) * 1$
$/ add(add(add(ra(\ ) * x\text{^}i * y\text{^}j * z\text{^}m, m = 0..d - j), j = 0..d - i), i = 0..d);$

$g := add(add(add(ra(\ ) * x\text{^}i * y\text{^}j * z\text{^}m, m = 0..d - j), j = 0..d - i), i = 0..d) * 1$
$/ add(add(add(ra(\ ) * x\text{^}i * y\text{^}j * z\text{^}m, m = 0..d - j), j = 0..d - i), i = 0..d);$

$h := add(add(add(ra(\ ) * x\text{^}i * y\text{^}j * z\text{^}m, m = 0..d - j), j = 0..d - i), i = 0..d) * 1$
$/ add(add(add(ra(\ ) * x\text{^}i * y\text{^}j * z\text{^}m, m = 0..d - j), j = 0..d - i), i = 0..d)$

end proc

```
> RT3bad(x,y,z,2,10);
```

$$\left(3 x^2 z^2 + 9 x^2 z + 9 x y z + 5 x z^2 + x^2 + 7 x y + 6 x z + 3 y^2 + 6 y z + 8 z^2 + 3 x + 5 y + 8 z + 1\right) \Big/ \left(10 x^2 z^2 + 3 x^2 z + 2 x y z + 8 x z^2 + 4 x^2 + 8 x y + 10 x z + y^2 + 10 y z + 2 z^2 + 9 x \right. \tag{14}$$

$+ 5y + 5z + 8)$

```
> #Checking the number of terms in RT3. Seems weird because i dont
  have  any multivariable terms with a degree 3 for y. Maybe the
  issue was letting m = 0 .. d-j.
  #And the fix to this problem would just be setting m = 0 .. i-j
  probably because m is nested.
> RT3worse := proc(x,y,z,d,K) local ra,i,j,m,f,g,h;
  ra := rand(1..K);
  f:= add(add(add(ra()*x^i*y^j*z^m, m = 0 .. i-j), j = 0 .. d - i),
  i = 0 .. d)*1/add(add(add(ra()*x^i*y^j*z^m, m = 0 .. i-j), j = 0
  .. d - i), i = 0 .. d);
  g:= add(add(add(ra()*x^i*y^j*z^m, m = 0 .. i-j), j = 0 .. d - i),
  i = 0 .. d)*1/add(add(add(ra()*x^i*y^j*z^m, m = 0 .. i-j), j = 0
  .. d - i), i = 0 .. d);
  h:= add(add(add(ra()*x^i*y^j*z^m, m = 0 .. i-j), j = 0 .. d - i),
  i = 0 .. d)*1/add(add(add(ra()*x^i*y^j*z^m, m = 0 .. i-j), j = 0
  .. d - i), i = 0 .. d);
  end;
```

RT3worse := **proc**(*x*, *y*, *z*, *d*, *K*) **(15)**

local *ra*, *i*, *j*, *m*, *f*, *g*, *h*;

ra := *rand*(1 ..*K*);

f := *add*(*add*(*add*(*ra*( ) * *x*^*i* * *y*^*j* * *z*^*m*, *m* = 0 ..*i* − *j*), *j* = 0 ..*d* − *i*), *i* = 0 ..*d*) * 1 / *add*(*add*(*add*(*ra*( ) * *x*^*i* * *y*^*j* * *z*^*m*, *m* = 0 ..*i* − *j*), *j* = 0 ..*d* − *i*), *i* = 0 ..*d*);

g := *add*(*add*(*add*(*ra*( ) * *x*^*i* * *y*^*j* * *z*^*m*, *m* = 0 ..*i* − *j*), *j* = 0 ..*d* − *i*), *i* = 0 ..*d*) * 1 / *add*(*add*(*add*(*ra*( ) * *x*^*i* * *y*^*j* * *z*^*m*, *m* = 0 ..*i* − *j*), *j* = 0 ..*d* − *i*), *i* = 0 ..*d*);

h := *add*(*add*(*add*(*ra*( ) * *x*^*i* * *y*^*j* * *z*^*m*, *m* = 0 ..*i* − *j*), *j* = 0 ..*d* − *i*), *i* = 0 ..*d*) * 1 / *add*(*add*(*add*(*ra*( ) * *x*^*i* * *y*^*j* * *z*^*m*, *m* = 0 ..*i* − *j*), *j* = 0 ..*d* − *i*), *i* = 0 ..*d*)

end proc

```
> RT3worse(x,y,z,2,10);
```

$$\frac{x^2 z^2 + 2x^2 z + x^2 + 7xy + xz + 7x + 7}{7x^2 z^2 + 6x^2 z + 9x^2 + 8xy + 9xz + 7x + 4} \quad \textbf{(16)}$$

```
> #That is worse than what i wanted. Maybe I need to have m = d - i
  to keep it as the same degree as everything else. I th

> RT3 := proc(x,y,z,d,K) local ra,i,j,m,f,g,h;
  ra := rand(1..K);
  f:= add(add(add(ra()*x^i*y^j*z^m, m = 0 .. d-i), j = 0 .. d - i),
  i = 0 .. d)*1/add(add(add(ra()*x^i*y^j*z^m, m = 0 .. d-i), j = 0
  .. d - i), i = 0 .. d);
  g:= add(add(add(ra()*x^i*y^j*z^m, m = 0 .. d-i), j = 0 .. d - i),
  i = 0 .. d)*1/add(add(add(ra()*x^i*y^j*z^m, m = 0 .. d-i), j = 0
  .. d - i), i = 0 .. d);
  h:= add(add(add(ra()*x^i*y^j*z^m, m = 0 .. d-i), j = 0 .. d - i),
  i = 0 .. d)*1/add(add(add(ra()*x^i*y^j*z^m, m = 0 .. d-i), j = 0
  .. d - i), i = 0 .. d);
  end;
```

RT3 := **proc**(*x*, *y*, *z*, *d*, *K*) **(17)**

local *ra*, *i*, *j*, *m*, *f*, *g*, *h*;

$ra := rand(1..K);$

$f := add(add(add(ra() * x\verb|^|i * y\verb|^|j * z\verb|^|m, m=0..d-i), j=0..d-i), i=0..d) * 1 / add(add(add(ra() * x\verb|^|i * y\verb|^|j * z\verb|^|m, m=0..d-i), j=0..d-i), i=0..d);$

$g := add(add(add(ra() * x\verb|^|i * y\verb|^|j * z\verb|^|m, m=0..d-i), j=0..d-i), i=0..d) * 1 / add(add(add(ra() * x\verb|^|i * y\verb|^|j * z\verb|^|m, m=0..d-i), j=0..d-i), i=0..d);$

$h := add(add(add(ra() * x\verb|^|i * y\verb|^|j * z\verb|^|m, m=0..d-i), j=0..d-i), i=0..d) * 1 / add(add(add(ra() * x\verb|^|i * y\verb|^|j * z\verb|^|m, m=0..d-i), j=0..d-i), i=0..d)$

end proc

```
> RT3(x,y,z,1,10);
```

$$\frac{6\,y\,z + 6\,x + 6\,y + 5\,z + 1}{5\,y\,z + 9\,x + 10\,y + 10\,z + 4} \tag{18}$$

```
> #We want to have isolated terms of x^d , y^d , and z^d. If i have
  d=2, I WANT a C1*x^2, a C2*y^2 and a C3z^2
  #I also want for d=2 to have an xy an xz and a yz
  #I do not want highest degree terms multiplied together
  #My proposal is to subtract 1 as follows: let d - i - 1
  #i = 0 .. d , j = 0 .. d - i, and m = 0 ..
> #I think it is better to show HOW to do a PROPER trinomial
  expansion? I think the problem I was having was
> #QUESTION 4 (ii) Write the three-variable analog of Orb2
> print(Orb2);
```

proc$(F, x, y, pt0, K1, K2)$ **(19)**

local $pt, L, i;$

$pt := pt0;$

for i **to** $K1$ **do** $pt := subs(\{x = pt[1], y = pt[2]\}, F)$ **end do**;

$L := [\,];$

for i **from** $K1 + 1$ **to** $K2$ **do**

$pt := subs(\{x = pt[1], y = pt[2]\}, F);\ L := [op(L), pt]$

end do;

L

end proc

```
> Orb3:=proc(F,x,y,z,pt0,K1,K2)local pt,L,i;
  pt := pt0;
  for i to K1 do pt := subs({x=pt[1],y=pt[2],z=pt[3]},F) end do;
  L := [];
  for i from K1 + 1 to K2 do pt:= subs({x=pt[1],y=pt[2],z=pt[3]},F)
  ;
  L := [op(L), pt] end do;
  end;
```

$Orb3 :=$ **proc**$(F, x, y, z, pt0, K1, K2, K3)$ **(20)**

local $pt, L, i;$

$pt := pt0;$

for i **to** $K1$ **do** $pt := subs(\{x = pt[1], y = pt[2], z = pt[3]\}, F)$ **end do**;

$L := [\,];$

```
    for i from K1 + 1 to K2 do
        pt := subs({x = pt[1], y = pt[2], z = pt[3]}, F); L := [op(L), pt]
    end do
end proc
```

```
> #Orb3(x^2+1,x,y,z,[1,1,1],10,100);
  #This part of the code is really faulty
> #I think

> Orb2(x+y,x,y,[1,2],10,20);
Warning,  computation interrupted
                                                                  (21)
> #What should the format of F be in our orb2 command in order to
  compute correctly.
  #I think i will copy and paste one of the transformations from
  RT2 into the paramater F to comfirm
```