```
#OK to post
#Anne Somalwar, hw13, 10.18.21


read "C:/Users/aks238/OneDrive – Rutgers University/Documents/M13.txt"




#2


FP2([(x^2+9 ·x+3)/(x^2+x+1), (x^2+9·x+3)/(3·x^2+x+4)], x, y);
```

$$\left[\left[3, \frac{39}{34}\right], \left[RootOf\left(\_Z^2+3\,\_Z+1\right), \frac{22\,RootOf\left(\_Z^2+3\,\_Z+1\right)}{89}+\frac{2}{89}\right]\right] \quad (1)$$

```
# [3, 39/34] is a fixed point.


SFP2([(x^2+9 ·x+3)/(x^2+x+1), (x^2+9·x+3)/(3·x^2+x+4)], x, y);
```

$$[[3., 1.147058824]] \quad (2)$$

```
#[3, 39/4] is stable.


Orb2([(x^2+9 ·x+3)/(x^2+x+1), (x^2+9·x+3)/(3·x^2+x+4)], x, y, [9.5, 1.5], 1000, 1010);
```

$$[[3.000000001, 1.147058824], [2.999999998, 1.147058823], [3.000000001, 1.147058824], [2.999999998, 1.147058823], [3.000000001, 1.147058824], [2.999999998, 1.147058823], [3.000000001, 1.147058824], [2.999999998, 1.147058823], [3.000000001, 1.147058824], [2.999999998, 1.147058823]] \quad (3)$$

```
> #3
> for i from 1 to 20 by 1 do
  F := RT2(x, y, 1, 100);
  print('');
  print('Fixed points are:' );
  print(evalf(FP2drz(F, x, y)));
  print('');
  print('Stable fixed points are:');
  print(evalf(SFP2drz(F, x, y)));
  print('');
  print('Confirm with Orb');
  print(Orb2(F, x, y, [0.5, 0.5], 1000, 1010));
  print('');
  end do;
```

$$F := \left[\frac{93 + 85\,y + 73\,x}{48 + 14\,y + 83\,x}, \frac{74 + 96\,y + 40\,x}{88 + 73\,y + 25\,x} \right]$$

Fixed points are

$$[[-1.348698318, 0.8826706181], [1.575990006, 1.171839314], [-0.3894899252, -0.7813070079], [0.4077817284, -1.127400934]]$$

Stable fixed points are

$$[[1.575990006, 1.171839314]]$$

Confirm with Orb

$$[[1.575990005, 1.171839314], [1.575990005, 1.171839314], [1.575990005, 1.171839314], [1.575990005, 1.171839314], [1.575990005, 1.171839314], [1.575990005, 1.171839314], [1.575990005, 1.171839314], [1.575990005, 1.171839314], [1.575990005, 1.171839314], [1.575990005, 1.171839314]]$$

$$F := \left[\frac{95 + 76\,y + 57\,x}{32 + 5\,y + 4\,x}, \frac{51 + 6\,y + 79\,x}{56 + 47\,y + 45\,x} \right]$$

Fixed points are

$$[[9.726732862, 1.471479693], [-8.885680332, 3.678229236], [-0.3901010870, -1.085801266], [46.37337264, -47.14038840]]$$

Stable fixed points are

[[9.726732862, 1.471479693]]

Confirm with Orb

[[9.726732864, 1.471479693], [9.726732864, 1.471479693], [9.726732864, 1.471479693], [9.726732864, 1.471479693], [9.726732864, 1.471479693], [9.726732864, 1.471479693], [9.726732864, 1.471479693], [9.726732864, 1.471479693], [9.726732864, 1.471479693], [9.726732864, 1.471479693]]

$$F := \left[\frac{53 + 17\,y + 21\,x}{69 + 4\,y + 86\,x}, \frac{35 + 37\,y + 100\,x}{10 + 80\,y + 39\,x} \right]$$

Fixed points are

[[0.6606614880, 1.131718372], [−1.174614480 − 0.1343103149 I, 0.3792971405 + 0.9440550784 I], [0.4474423097, −0.9404966954], [−1.174614480 + 0.1343103149 I, 0.3792971405 − 0.9440550784 I]]

Stable fixed points are

[[0.6606614880, 1.131718372]]

Confirm with Orb

[[0.6606614892, 1.131718371], [0.6606614887, 1.131718372], [0.6606614892, 1.131718371], [0.6606614887, 1.131718372], [0.6606614892, 1.131718371], [0.6606614887, 1.131718372], [0.6606614892, 1.131718371], [0.6606614887, 1.131718372], [0.6606614892, 1.131718371], [0.6606614887, 1.131718372]]

$$F := \left[\frac{74 + 24\,y + 87\,x}{61 + 94\,y + 31\,x}, \frac{83 + 74\,y + 70\,x}{8 + 36\,y + 86\,x} \right]$$

Fixed points are

[[0.6630908848, 2.024755766], [−0.2297210197 − 1.236723607 I, 0.03828188286 + 0.9939680150 I], [−1.176112825, −0.004017579530], [−0.2297210197 + 1.236723607 I, 0.03828188286 − 0.9939680150 I]]

Stable fixed points are

[[0.6630908848, 2.024755766]]

Confirm with Orb

[[0.6630908828, 2.024755765], [0.6630908828, 2.024755765], [0.6630908828, 2.024755765], [0.6630908828, 2.024755765], [0.6630908828, 2.024755765], [0.6630908828, 2.024755765], [0.6630908828, 2.024755765], [0.6630908828, 2.024755765], [0.6630908828, 2.024755765]]

$$F := \left[\frac{3 + 39\,y + 42\,x}{58 + 5\,y + 5\,x}, \frac{52 + 43\,y + 60\,x}{47 + 33\,y + 50\,x} \right]$$

Fixed points are

[[1.645527500, 1.198065444], [−6.049703500, 1.201459608], [−0.6867691250, −0.2740747484], [27.27917438, −42.65307954]]

Stable fixed points are

[[1.645527500, 1.198065444]]

Confirm with Orb

[[1.645529071, 1.198065443], [1.645529071, 1.198065443], [1.645529071, 1.198065443], [1.645529071, 1.198065443], [1.645529071, 1.198065443], [1.645529071, 1.198065443], [1.645529071, 1.198065443], [1.645529071, 1.198065443], [1.645529071, 1.198065443]]

$$F := \left[\frac{82 + 57\,y + 61\,x}{85 + 30\,y + 41\,x}, \frac{79 + 43\,y + 5\,x}{17 + 100\,y + 6\,x} \right]$$

Fixed points are

[[1.299208341, 1.020133937], [−2.653675202, 1.047018579], [−1.036659335, −0.7130398952], [0.9352899748, −0.8184697638]]

Stable fixed points are

[[1.299208341, 1.020133937]]

Confirm with Orb

[[1.299208364, 1.020133937], [1.299208365, 1.020133937], [1.299208364, 1.020133937], [1.299208365, 1.020133937], [1.299208364, 1.020133937], [1.299208365, 1.020133937], [1.299208364, 1.020133937], [1.299208365, 1.020133937], [1.299208364, 1.020133937], [1.299208365, 1.020133937]]

$$F := \left[\frac{41 + 18\,y + 41\,x}{99 + 40\,y + 16\,x}, \frac{89 + 59\,y + 88\,x}{34 + 92\,y + 31\,x} \right]$$

Fixed points are

[[0.5436654156, 1.264675031], [1.563147521 − 8.672504860 I, −2.251014713 + 3.502069146 I], [0.9205567815, −1.378743806], [1.563147521 + 8.672504860 I, −2.251014713 − 3.502069146 I]]

Stable fixed points are

[[0.5436654156, 1.264675031]]

Confirm with Orb

[[0.5436654160, 1.264675031], [0.5436654160, 1.264675031], [0.5436654160, 1.264675031], [0.5436654160, 1.264675031], [0.5436654160, 1.264675031], [0.5436654160, 1.264675031], [0.5436654160, 1.264675031], [0.5436654160, 1.264675031], [0.5436654160, 1.264675031]]

$$F := \left[\frac{14 + 87\,y + 33\,x}{43 + 88\,y + 84\,x}, \frac{9 + 17\,y + 99\,x}{68 + 44\,y + 45\,x} \right]$$

Fixed points are

[[0.5984229630, 0.6425817595], [−37.34908217, 34.61717827], [−0.1442764020, −0.1373595978], [1.700671541, −3.925344646]]

Stable fixed points are

[[0.5984229630, 0.6425817595]]

Confirm with Orb

[[0.5984229627, 0.6425817598], [0.5984229631, 0.6425817591], [0.5984229627, 0.6425817598], [0.5984229631, 0.6425817591], [0.5984229627, 0.6425817598], [0.5984229631, 0.6425817591], [0.5984229627, 0.6425817598], [0.5984229631, 0.6425817591], [0.5984229627, 0.6425817598], [0.5984229631, 0.6425817591]]

$$F := \left[\frac{100 + 10\,y + 82\,x}{87 + 18\,y + 27\,x}, \frac{51 + 76\,y + 95\,x}{59 + 5\,y + 7\,x} \right]$$

Fixed points are

[[1.085898161, 6.571516082], [−0.5359672048 − 1.605170977 I, −1.737691583

$+ 4.511466416\,I], [81.91196229, -123.9183551], [-0.5359672048 + 1.605170977\,I, -1.737691583 - 4.511466416\,I]]$

Stable fixed points are

$[[1.085898161, 6.571516082]]$

Confirm with Orb

$[[1.085898160, 6.571516081], [1.085898160, 6.571516082], [1.085898160, 6.571516081], [1.085898160, 6.571516082], [1.085898160, 6.571516081], [1.085898160, 6.571516082], [1.085898160, 6.571516081], [1.085898160, 6.571516082], [1.085898160, 6.571516081], [1.085898160, 6.571516082]]$

$$F := \left[\frac{47 + 88\,y + 76\,x}{98 + 89\,y + 68\,x}, \frac{32 + 75\,y + 39\,x}{75 + 48\,y + 52\,x}\right]$$

Fixed points are

$[[-1.886535055, 0.5998759050], [0.7960595747, 0.7932999790], [-0.4941872330, -0.3126553787], [2.059574283, -3.008917378]]$

Stable fixed points are

$[[0.7960595747, 0.7932999790]]$

Confirm with Orb

$[[0.7960595748, 0.7932999787], [0.7960595748, 0.7932999787], [0.7960595748, 0.7932999787], [0.7960595748, 0.7932999787], [0.7960595748, 0.7932999787], [0.7960595748, 0.7932999787], [0.7960595748, 0.7932999787], [0.7960595748, 0.7932999787], [0.7960595748, 0.7932999787], [0.7960595748, 0.7932999787]]$

$$F := \left[\frac{57 + 38\,y + 64\,x}{21 + 35\,y + 88\,x}, \frac{52 + 51\,y + 66\,x}{76 + 40\,y + 41\,x}\right]$$

Fixed points are

$[[-0.7390610858, 0.3577147933], [1.085464517, 1.092278443], [-0.004521860424, -1.488635831], [1.084764387, -2.829067237]]$

Stable fixed points are

$[[1.085464517, 1.092278443]]$

Confirm with Orb

[[1.085464515, 1.092278442], [1.085464515, 1.092278442], [1.085464515, 1.092278442], [1.085464515, 1.092278442], [1.085464515, 1.092278442], [1.085464515, 1.092278442], [1.085464515, 1.092278442], [1.085464515, 1.092278442], [1.085464515, 1.092278442], [1.085464515, 1.092278442]]

$$F := \left[\frac{47 + 36\,y + 35\,x}{74 + 65\,y + 11\,x}, \frac{45 + 77\,y + 27\,x}{45 + y + 63\,x} \right]$$

Fixed points are

[[−6.100506100, 0.2877427002], [0.6097631158, 5.263111557], [−0.02337449812, −1.276823965], [0.5035663257, −7.518398558]]

Stable fixed points are

[[0.6097631158, 5.263111557]]

Confirm with Orb

[[0.6097631115, 5.263111555], [0.6097631115, 5.263111555], [0.6097631115, 5.263111555], [0.6097631115, 5.263111555], [0.6097631115, 5.263111555], [0.6097631115, 5.263111555], [0.6097631115, 5.263111555], [0.6097631115, 5.263111555], [0.6097631115, 5.263111555], [0.6097631115, 5.263111555]]

$$F := \left[\frac{15 + 5\,y + 10\,x}{89 + 57\,y + 51\,x}, \frac{18 + 57\,y + 71\,x}{59 + 39\,y + 63\,x} \right]$$

Fixed points are

[[0.1458738656, 0.7212256460], [−3.298939025, 1.447458759], [0.4146577191 − 0.1896901192 I, −1.527226098 + 0.3485605502 I], [0.4146577191 + 0.1896901192 I, −1.527226098 − 0.3485605502 I]]

Stable fixed points are

[[0.1458738656, 0.7212256460]]

Confirm with Orb

[[0.1458738654, 0.7212256460], [0.1458738654, 0.7212256460], [0.1458738654, 0.7212256460], [0.1458738654, 0.7212256460], [0.1458738654, 0.7212256460], [0.1458738654, 0.7212256460], [0.1458738654, 0.7212256460], [0.1458738654, 0.7212256460], [0.1458738654, 0.7212256460], [0.1458738654, 0.7212256460]]

$$F := \left[\frac{3 + 55\,y + 94\,x}{91 + 89\,y + 39\,x}, \frac{72 + 93\,y + 57\,x}{75 + 84\,y + 88\,x} \right]$$

Fixed points are

[[0.5383969571, 0.9445291617], [−1.192307121 − 1.264605545 I, 0.3272593440 + 0.5248977220 I], [0.7407772925, −1.480345145], [−1.192307121 + 1.264605545 I, 0.3272593440 − 0.5248977220 I]]

Stable fixed points are

[[0.5383969571, 0.9445291617]]

Confirm with Orb

[[0.5383969566, 0.9445291613], [0.5383969566, 0.9445291613], [0.5383969566, 0.9445291613], [0.5383969566, 0.9445291613], [0.5383969566, 0.9445291613], [0.5383969566, 0.9445291613], [0.5383969566, 0.9445291613], [0.5383969566, 0.9445291613], [0.5383969566, 0.9445291613]]

$$F := \left[\frac{59 + 54\,y + 89\,x}{14 + 31\,y + 55\,x}, \frac{41 + 18\,y + 25\,x}{34 + 12\,y + 40\,x} \right]$$

Fixed points are

[[1.891960832, 0.8651643182], [−0.7610532390 − 0.8889510620 I, 0.4186479524 + 1.669665161 I], [1.557888591, −7.423724591], [−0.7610532390 + 0.8889510620 I, 0.4186479524 − 1.669665161 I]]

Stable fixed points are

[[1.891960832, 0.8651643182]]

Confirm with Orb

[[1.891960831, 0.8651643180], [1.891960832, 0.8651643180], [1.891960831, 0.8651643180], [1.891960832, 0.8651643180], [1.891960831, 0.8651643180], [1.891960832, 0.8651643180], [1.891960831, 0.8651643180], [1.891960832, 0.8651643180], [1.891960831, 0.8651643180], [1.891960832, 0.8651643180]]

$$F := \left[\frac{17 + 99\,y + 39\,x}{74 + 38\,y + 38\,x}, \frac{9 + 12\,y + 46\,x}{62 + 78\,y + 14\,x} \right]$$

Fixed points are

[[0.7652976795, 0.4582648796], [−0.7742316814 − 2.411090435 I, −1.038327151 + 1.188415685 I], [−0.3303211590, −0.2188659757], [−0.7742316814 + 2.411090435 I, −1.038327151 − 1.188415685 I]]

Stable fixed points are

[[0.7652976795, 0.4582648796]]

Confirm with Orb

[[0.7652976794, 0.4582648796], [0.7652976797, 0.4582648797], [0.7652976792, 0.4582648799], [0.7652976798, 0.4582648796], [0.7652976797, 0.4582648799], [0.7652976793, 0.4582648794], [0.7652976794, 0.4582648796], [0.7652976797, 0.4582648797], [0.7652976792, 0.4582648799], [0.7652976798, 0.4582648796]]

$$F := \left[\frac{17 + 69\,y + 41\,x}{97 + 42\,y + 32\,x}, \frac{61 + 73\,y + 61\,x}{57 + 21\,y + 43\,x} \right]$$

Fixed points are

[[−3.941110650, 1.105772446], [0.8950963604, 1.871101208], [−0.6441299039, −0.4142939737], [3.220898144, −7.473766378]]

Stable fixed points are

[[0.8950963604, 1.871101208]]

Confirm with Orb

[[0.8950963628, 1.871101208], [0.8950963628, 1.871101208], [0.8950963628, 1.871101208], [0.8950963628, 1.871101208], [0.8950963628, 1.871101208], [0.8950963628, 1.871101208], [0.8950963628, 1.871101208], [0.8950963628, 1.871101208], [0.8950963628, 1.871101208], [0.8950963628, 1.871101208]]

$$F := \left[\frac{28 + 22\,y + 89\,x}{26 + 97\,y + 16\,x}, \frac{98 + 63\,y + 62\,x}{87 + 30\,y + 60\,x} \right]$$

Fixed points are

[[0.7364644464, 1.329350758], [−2.126822227 − 1.665044471 I, 0.8418431366 + 0.3602793541 I], [0.08429139980, −2.401426545], [−2.126822227 + 1.665044471 I, 0.8418431366 − 0.3602793541 I]]

Stable fixed points are

[[0.7364644464, 1.329350758]]

Confirm with Orb

[[0.7364644511, 1.329350759], [0.7364644506, 1.329350758], [0.7364644511, 1.329350759], [0.7364644506, 1.329350758], [0.7364644511, 1.329350759], [0.7364644506, 1.329350758], [0.7364644511, 1.329350759], [0.7364644506, 1.329350758]]

$$F := \left[\frac{67 + 80\,y + 47\,x}{14 + 68\,y + 80\,x}, \frac{61 + 54\,y + 12\,x}{33 + 45\,y + 31\,x} \right]$$

Fixed points are

[[1.155223149, 1.135692573], [−3.143101331, 2.815669883], [0.1621512214, −1.018468366], [1.079111681, −1.427725786]]

Stable fixed points are

[[1.155223149, 1.135692573]]

Confirm with Orb

[[1.155223149, 1.135692573], [1.155223149, 1.135692573], [1.155223149, 1.135692573], [1.155223149, 1.135692573], [1.155223149, 1.135692573], [1.155223149, 1.135692573], [1.155223149, 1.135692573], [1.155223149, 1.135692573], [1.155223149, 1.135692573]]

$$F := \left[\frac{48 + 79\,y + 27\,x}{24 + 96\,y + 52\,x}, \frac{99 + 3\,y + 36\,x}{67 + 59\,y + 93\,x} \right]$$

Fixed points are

[[0.9217876610, 0.6933860171], [−1.263602407 − 1.358554454 I, 0.2164713739 + 0.7707072659 I], [0.7494050750, −2.982041150], [−1.263602407 + 1.358554454 I, 0.2164713739 − 0.7707072659 I]]

Stable fixed points are

[[0.9217876610, 0.6933860171]]

Confirm with Orb

[[0.9217876608, 0.6933860176], [0.9217876608, 0.6933860176], [0.9217876608,

```
0.6933860176], [0.9217876608, 0.6933860176], [0.9217876608, 0.6933860176],
[0.9217876608, 0.6933860176], [0.9217876608, 0.6933860176], [0.9217876608,
0.6933860176], [0.9217876608, 0.6933860176], [0.9217876608, 0.6933860176]]
```

(4)

$$F := \left[\frac{47 + 8\,y + 46\,x}{44 + 9\,y + 77\,x}, \frac{59 + 16\,y + x}{70 + 77\,y + 39\,x} \right] \tag{5}$$

```
>
```

```
>
>
>
>
> #4
>
>
>
> RT3 := proc(x, y, z, d, K) local ra, i, j, k, f, g, h :
  ra := rand(1..K) : #random integer from -K to K
  f := add(add(add(ra( ) * x^i * y^j * z^k, k = 0..d - i-j), j = 0..d - i), i = 0..d)
       / add(add(add(ra( ) * x^i * y^j * z^k, k = 0...d - i - j), j = 0..d - i), i = 0..d) :
  g := add(add(add(ra( ) * x^i * y^j * z^k, k = 0...d - i - j), j = 0..d - i), i = 0..d)
       / add(add(add(ra( ) * x^i * y^j * z^k, k = 0...d - i-j), j = 0..d - i), i = 0..d) :
  h := add(add(add(ra( ) * x^i * y^j * z^k, k = 0...d - i - j), j = 0..d - i), i = 0..d)
       / add(add(add(ra( ) * x^i * y^j * z^k, k = 0...d - i-j), j = 0..d - i), i = 0..d) :

  [f, g, h] :
  end:
> RT3(x, y, z, 1, 10)
```

$$\left[\frac{1 + 7\,z + 2\,y + 9\,x}{5 + 9\,z + 9\,y + 4\,x}, \frac{3 + 4\,z + 6\,y + 4\,x}{4 + 10\,z + 8\,y + 2\,x}, \frac{7 + 2\,z + 5\,y + 3\,x}{5 + 7\,z + 5\,y + 4\,x} \right] \tag{6}$$

```
>
>
>
>
> Orb3 := proc(F, x, y, z, pt0, K1, K2) local pt, L, i :
  pt := pt0 :

  for i from 1 to K1 do
  pt := subs( {x = pt[1], y = pt[2], z = pt[3]}, F) :
  od:
```

```
    L := [ ]:
    for i from K1 + 1 to K2 do
    L := [op(L), pt]:
    pt := subs( {x=pt[1], y=pt[2], z=pt[3]}, F) :

    od:
    L:
    end:
```

```
> evalf( Orb3( [ (1+7·z+2·y+9·x)/(5+9·z+9·y+4·x), (3+4·z+6·y+4·x)/(4+10·z+8·y+2·x), (7+2·z+5·y+3·x)/(5+7·z+5·y+4·x) ], x,
      y, z, [1.0, 1.0, 1.0], 1000, 1010 ) );
```

$$[[0.6655368677, 0.6846384024, 0.8327344879], [0.6655368677, 0.6846384024, 0.8327344879], [0.6655368677, 0.6846384024, 0.8327344879], [0.6655368677, 0.6846384024, 0.8327344879], [0.6655368677, 0.6846384024, 0.8327344879], [0.6655368677, 0.6846384024, 0.8327344879], [0.6655368677, 0.6846384024, 0.8327344879], [0.6655368677, 0.6846384024, 0.8327344879], [0.6655368677, 0.6846384024, 0.8327344879], [0.6655368677, 0.6846384024, 0.8327344879]] \quad (7)$$

```
> FP3 := proc(F, x, y, z) local L, i :
  L := [solve( {F[1]=x, F[2]=y, F[3]=z}, {x, y, z}) ] :

  [seq(subs(L[i], [x, y, z]), i=1..nops(L)) ] :
  end:
```

```
> evalf( FP3( [ (1+7·z+2·y+9·x)/(5+9·z+9·y+4·x), (3+4·z+6·y+4·x)/(4+10·z+8·y+2·x), (7+2·z+5·y+3·x)/(5+7·z+5·y+4·x) ], x,
      y, z ) )
```

$$[[8.027706, -3.7549129, 0.5994661946]] \quad (8)$$

```
> SFP3 := proc(F, x, y, z) local L, J, S, J0, i, pt, EV :

  L := evalf(FP3(F, x, y, z)) :
      #F is the list of ALL fixed points of the transformation [x,y]->F using the previous procedure
      FP2(F,x,y), but since we are interested in numbers we take the floating point version using
      evalf
```

```
J := Matrix(normal([[diff(F[1],x), diff(F[2],x), diff(F[3],x)], [diff(F[1],y), diff(F[2],
     y), diff(F[3],y)], [diff(F[1],z), diff(F[2],z), diff(F[3],z)]])):
     #J is the Jacobian matrix in general (in terms of the variables x and y). Note that J is a
     SYMBOLIC matrix featuring variables x and y
S := []:  #S is the list of stable fixed points that starts out empty

for i from 1 to nops(L) do  #we examime it case by case
pt := L[i]: #pt is the current fixed point to be examined

J0 := subs({x=pt[1], y=pt[2], z=pt[3]}, J):
     #J0 is the NUMERICAL matrix obtained by plugging-in the examined fixed pt

EV := Eigenvalues(J0):
     # We used Maple's command Eigenavalues to find the eigenvalues of this 3 by 3 matrix

if abs(EV[1]) < 1 and abs(EV[2]) < 1 and abs(EV[3]) < 1 then
 S := [op(S), pt]:
     #If all eigenvalues have absolute value less than 1 it means that they are stable, so we append
     the examined fixed point, pt, to the list of fixed points
fi:

od:
S: #the output is S
end:
```

```
> SFP3([ (1+7·z+2·y+9·x)/(5+9·z+9·y+4·x), (3+4·z+6·y+4·x)/(4+10·z+8·y+2·x), (7+2·z+5·y+3·x)/(5+7·z+5·y+4·x) ], x, y, z)
                                  [ ]                                                                    (9)
>
>
>
>
>
```

```
> for i from 1 to 10 by 1 do
  F := RT3(x, y, z, 1, 100);
  print('');
  print('Fixed points are:' );
  print(evalf(FP3(F, x, y, z)));
  print('');
  print('Stable fixed points are:');
  print(evalf(SFP3(F, x, y,  z)));
  print('');
  print('Confirm with Orb');
  print(Orb3(F, x, y, z, [0.5, 0.5, 0.5], 1000, 1010));
  print('');
  end do;
```

$$F := \left[\frac{59 + 44\,z + 100\,y + 38\,x}{69 + 27\,z + 96\,y + 17\,x}, \frac{90 + 34\,z + 18\,y + 52\,x}{56 + 43\,z + 83\,y + 25\,x}, \frac{90 + 93\,z + 60\,y + 93\,x}{14 + 50\,z + 47\,y + 8\,x} \right]$$

Fixed points are

[[1.254534792, 0.9408767631, 2.554740902]]

Stable fixed points are

[[1.254534792, 0.9408767631, 2.554740902]]

Confirm with Orb

[[1.254534792, 0.9408767629, 2.554740901], [1.254534792, 0.9408767632, 2.554740901], [1.254534791, 0.9408767632, 2.554740901], [1.254534792, 0.9408767629, 2.554740901], [1.254534792, 0.9408767632, 2.554740901], [1.254534791, 0.9408767632, 2.554740901], [1.254534792, 0.9408767629, 2.554740901], [1.254534792, 0.9408767632, 2.554740901], [1.254534791, 0.9408767632, 2.554740901], [1.254534792, 0.9408767629, 2.554740901]]

$$F := \left[\frac{46 + 44\,z + 9\,y + 77\,x}{59 + 16\,z + y + 70\,x}, \frac{77 + 39\,z + 92\,y + 71\,x}{67 + 78\,z + 51\,y + 53\,x}, \frac{12 + 19\,z + 63\,y + 40\,x}{90 + 3\,z + 49\,y + 49\,x} \right]$$

Fixed points are

[[0.6317083865, −1.057434086, −0.607445584]]

Stable fixed points are

[ ]

Confirm with Orb

[[1.162115979, 1.215214421, 0.7121565102], [1.162115979, 1.215214421, 0.7121565102], [1.162115979, 1.215214421, 0.7121565102], [1.162115979, 1.215214421, 0.7121565102], [1.162115979, 1.215214421, 0.7121565102], [1.162115979, 1.215214421, 0.7121565102], [1.162115979, 1.215214421, 0.7121565102], [1.162115979, 1.215214421, 0.7121565102], [1.162115979, 1.215214421, 0.7121565102], [1.162115979, 1.215214421, 0.7121565102]]

$$F := \left[\frac{67 + 74\,z + 90\,y + 74\,x}{27 + 98\,z + 72\,y + 2\,x}, \frac{73 + 85\,z + 41\,y + 4\,x}{44 + 13\,z + 19\,y + 10\,x}, \frac{15 + 64\,z + 9\,y + 12\,x}{52 + 25\,z + 72\,y + 90\,x} \right]$$

Fixed points are

[[2.45740726, 1.617085528, 0.1788798404]]

Stable fixed points are

[[2.45740726, 1.617085528, 0.1788798404]]

Confirm with Orb

[[2.457407575, 1.617085521, 0.1788798405], [2.457407576, 1.617085521, 0.1788798404], [2.457407574, 1.617085521, 0.1788798405], [2.457407575, 1.617085521, 0.1788798405], [2.457407576, 1.617085521, 0.1788798404], [2.457407574, 1.617085521, 0.1788798405], [2.457407575, 1.617085521, 0.1788798405], [2.457407576, 1.617085521, 0.1788798404], [2.457407574, 1.617085521, 0.1788798405], [2.457407575, 1.617085521, 0.1788798405]]

$$F := \left[\frac{18 + 43\,z + 55\,y + 40\,x}{17 + 70\,z + 52\,y + 81\,x}, \frac{87 + 34\,z + 85\,y + 9\,x}{68 + 83\,z + 63\,y + 100\,x}, \frac{70 + 36\,z + 36\,y + 10\,x}{40 + 66\,z + 87\,y + 16\,x} \right]$$

Fixed points are

[[0.7170657983, 0.7249749609, 0.7901526826]]

Stable fixed points are

[[0.7170657983, 0.7249749609, 0.7901526826]]

Confirm with Orb

[[0.7170657978, 0.7249749585, 0.7901526826], [0.7170657986, 0.7249749588, 0.7901526831], [0.7170657978, 0.7249749585, 0.7901526826], [0.7170657986, 0.7249749588, 0.7901526831], [0.7170657978, 0.7249749585, 0.7901526826], [0.7170657986, 0.7249749588, 0.7901526831], [0.7170657978, 0.7249749585, 0.7901526826], [0.7170657986, 0.7249749588, 0.7901526831], [0.7170657978, 0.7249749585, 0.7901526826], [0.7170657986, 0.7249749588, 0.7901526831]]

$$F := \left[\frac{98 + 43\,z + 53\,y + 61\,x}{47 + 28\,z + 75\,y + 3\,x}, \frac{5 + 11\,z + 37\,y + 75\,x}{4 + 91\,z + 22\,y + 40\,x}, \frac{58 + 93\,z + 98\,y + 11\,x}{30 + 6\,z + 32\,y + 40\,x} \right]$$

Fixed points are

[[2.150927164, 0.6603458634, 2.479116249]]

Stable fixed points are

[[2.150927164, 0.6603458634, 2.479116249]]

Confirm with Orb

[[2.150927164, 0.6603458633, 2.479116254], [2.150927164, 0.6603458633, 2.479116254], [2.150927164, 0.6603458633, 2.479116254], [2.150927164, 0.6603458633, 2.479116254], [2.150927164, 0.6603458633, 2.479116254], [2.150927164, 0.6603458633, 2.479116254], [2.150927164, 0.6603458633, 2.479116254], [2.150927164, 0.6603458633, 2.479116254], [2.150927164, 0.6603458633, 2.479116254]]

$$F := \left[\frac{24 + 80\,z + 96\,y + 11\,x}{23 + 41\,z + 52\,y + 58\,x}, \frac{67 + 81\,z + 65\,y + 69\,x}{2 + 36\,z + 61\,y + 84\,x}, \frac{96 + 94\,z + 31\,y + 81\,x}{31 + 54\,z + 67\,y + 59\,x} \right]$$

Fixed points are

[[0.9140405380, −8.045807733, 10.00380501]]

Stable fixed points are

[ ]

Confirm with Orb

[[1.257408951, 1.458632901, 1.340624954], [1.257408950, 1.458632900, 1.340624953], [1.257408950, 1.458632900, 1.340624954], [1.257408951, 1.458632901, 1.340624954], [1.257408950, 1.458632900, 1.340624953], [1.257408950, 1.458632900, 1.340624954], [1.257408951, 1.458632901, 1.340624954], [1.257408950, 1.458632900, 1.340624953], [1.257408950, 1.458632900, 1.340624954], [1.257408951, 1.458632901, 1.340624954]]

$$F := \left[\frac{66 + 12\,z + 49\,y + 90\,x}{35 + 15\,z + 26\,y + 100\,x}, \frac{24 + 8\,z + 63\,y + 78\,x}{23 + 73\,z + 22\,y + 32\,x}, \frac{98 + 9\,z + 53\,y + 3\,x}{98 + 69\,z + 3\,y + 73\,x} \right]$$

Fixed points are

[[0.7581797577, −1.715996617, 0.064913963]]

Stable fixed points are

[ ]

Confirm with Orb

[[1.239378867, 1.456465894, 0.7576953106], [1.239378867, 1.456465894, 0.7576953106], [1.239378867, 1.456465894, 0.7576953106], [1.239378867, 1.456465894, 0.7576953106], [1.239378867, 1.456465894, 0.7576953106], [1.239378867,

1.456465894, 0.7576953106], [1.239378867, 1.456465894, 0.7576953106], [1.239378867, 1.456465894, 0.7576953106], [1.239378867, 1.456465894, 0.7576953106], [1.239378867, 1.456465894, 0.7576953106]]

$$F := \left[\frac{88 + 37\,z + 60\,y + 94\,x}{52 + 16\,z + 29\,y + 51\,x}, \frac{3 + 45\,z + 67\,y + 40\,x}{71 + 74\,z + 49\,y + 60\,x}, \frac{69 + 33\,z + 30\,y + x}{83 + 9\,z + 64\,y + 43\,x} \right]$$

Fixed points are

[[1.8412178, 0.5552576636, 0.516649]]

Stable fixed points are

[[1.8412178, 0.5552576636, 0.516649]]

Confirm with Orb

[[1.841217881, 0.5552576638, 0.5166486817], [1.841217881, 0.5552576638, 0.5166486817], [1.841217881, 0.5552576638, 0.5166486817], [1.841217881, 0.5552576638, 0.5166486817], [1.841217881, 0.5552576638, 0.5166486817], [1.841217881, 0.5552576638, 0.5166486817], [1.841217881, 0.5552576638, 0.5166486817], [1.841217881, 0.5552576638, 0.5166486817], [1.841217881, 0.5552576638, 0.5166486817], [1.841217881, 0.5552576638, 0.5166486817]]

$$F := \left[\frac{57 + 52\,z + 62\,y + 46\,x}{76 + 9\,z + 53\,y + 37\,x}, \frac{88 + 50\,z + 37\,y + 76\,x}{95 + 8\,z + 92\,y + 92\,x}, \frac{2 + 97\,z + 44\,y + 9\,x}{30 + 14\,z + 79\,y + 73\,x} \right]$$

Fixed points are

[[0.3801042908, 0.4499335101, −1.21622087]]

Stable fixed points are

[ ]

Confirm with Orb

[[1.133155672, 0.8294348918, 0.5476544536], [1.133155672, 0.8294348915, 0.5476544536], [1.133155672, 0.8294348918, 0.5476544536], [1.133155672, 0.8294348915, 0.5476544536], [1.133155672, 0.8294348918, 0.5476544536], [1.133155672, 0.8294348915, 0.5476544536], [1.133155672, 0.8294348918, 0.5476544536], [1.133155672, 0.8294348915, 0.5476544536], [1.133155672, 0.8294348918, 0.5476544536], [1.133155672, 0.8294348915, 0.5476544536]]

$$F := \left[\frac{21 + 78\,z + 49\,y + 93\,x}{15 + 56\,z + 69\,y + 17\,x}, \frac{21 + 42\,z + 21\,y + 5\,x}{58 + 3\,z + 86\,y + 55\,x}, \frac{97 + 4\,z + 92\,y + 46\,x}{88 + 34\,z + 68\,y + 49\,x} \right]$$

Fixed points are

[[2.612774991, 0.3435507475, 0.931598400]]

Stable fixed points are

[[2.612774991, 0.3435507475, 0.931598400]]

Confirm with Orb

[[2.612774994, 0.3435507474, 0.9315983994], [2.612774994, 0.3435507474, 0.9315983994], [2.612774994, 0.3435507474, 0.9315983994], [2.612774994, 0.3435507474, 0.9315983994], [2.612774994, 0.3435507474, 0.9315983994], [2.612774994, 0.3435507474, 0.9315983994], [2.612774994, 0.3435507474, 0.9315983994], [2.612774994, 0.3435507474, 0.9315983994], [2.612774994, 0.3435507474, 0.9315983994], [2.612774994, 0.3435507474, 0.9315983994]]

(10)

```
> #It definitely missed a few fixed points.
>
>
>
>
>
> #5
>
>
> NB := proc(lambda, a, c) :
  print([ lambda · ln(lambda) / ((lambda - 1)·a·c), ln(lambda) / a ]);
  print("It is not stable.");
  end:
> NB(3, 1, 2)
```

$$\left[\frac{3\ln(3)}{4}, \ln(3) \right]$$

"It is not stable."

(11)

```
>
```