```
> #HW13 - Alan Ho
> # OK to post
>
> read("M13.txt")
> Help13( )
```

RT2(x,y,d,K), Orb2(F,x,y,pt0,K1,K2), FP2(F,x,y), SFP2(F,x,y), PlotOrb2(L), FP2drz(F,x,y), SFP2drz(F,x,y) **(1)**

```
> #2)
> #RUID: 191007691
```

$$> x \to \frac{1\cdot x(n-1)^2 + 9\cdot x(n-1) + 1}{1\cdot x(n-1)^2 + 0\cdot x(n-1) + 0}$$

$$x \mapsto \frac{x(n-1)^2 + 9\cdot x(n-1) + 1}{x(n-1)^2 + 0\cdot x(n-1)} \quad \textbf{(2)}$$

$$> y \to \frac{7\cdot x(n-1)^2 + 9\cdot x(n-1) + 1}{1\cdot x(n-1)^2 + 0\cdot x(n-1) + 6}$$

$$y \mapsto \frac{7\cdot x(n-1)^2 + 9\cdot x(n-1) + 1}{x(n-1)^2 + 6} \quad \textbf{(3)}$$

$$> F := \left[\frac{x^2 + 9\cdot x + 1}{x^2}, \frac{7\cdot y^2 + 9\cdot y + 1}{y^2 + 6}\right]$$

$$F := \left[\frac{x^2 + 9x + 1}{x^2}, \frac{7y^2 + 9y + 1}{y^2 + 6}\right] \quad \textbf{(4)}$$

```
> evalf(FP2(F, x, y))
```

[[3.586874512, 7.422337116]] **(5)**

```
> evalf(SFP2(F, x, y))
```

[[3.586874512, 7.422337116]] **(6)**

```
> evalf(Orb2(F, x, y, [9.5, 0.5], 1000, 1020));
```

[[3.586874511, 7.422337116], [3.586874512, 7.422337117], [3.586874513, 7.422337116], [3.586874511, 7.422337117], [3.586874512, 7.422337116], [3.586874513, 7.422337117], [3.586874511, 7.422337116], [3.586874512, 7.422337117], [3.586874513, 7.422337116], [3.586874511, 7.422337117], [3.586874512, 7.422337116], [3.586874513, 7.422337117], [3.586874511, 7.422337116], [3.586874512, 7.422337117], [3.586874513, 7.422337116], [3.586874511, 7.422337117], [3.586874512, 7.422337116], [3.586874513, 7.422337117], [3.586874511, 7.422337116], [3.586874512, 7.422337117]] **(7)**

```
> convert(%, set)
```

{[3.586874511, 7.422337116], [3.586874511, 7.422337117], [3.586874512, 7.422337116], [3.586874512, 7.422337117], [3.586874513, 7.422337116], [3.586874513, 7.422337117]} **(8)**

```
> # the stable fixed points for the system are 3.586874511 and 7.422337116
```

```
> #3)
> F1 := RT2(x, y, 1, 100)
```

$$F1 := \left[\frac{47 + 8\,y + 46\,x}{44 + 9\,y + 77\,x}, \frac{59 + 16\,y + x}{70 + 77\,y + 39\,x} \right] \tag{9}$$

```
> evalf(SFP2drz(F1, x, y))
```

$$[[0.7976878657, 0.4875444232]] \tag{10}$$

```
> convert(evalf(Orb2(F1, x, y, [3.12, 0.45], 1000, 1020)), set)
```

$$\{[0.7976878654, 0.4875444230], [0.7976878659, 0.4875444230]\} \tag{11}$$

```
> F2 := RT2(x, y, 1, 100)
```

$$F2 := \left[\frac{92 + 71\,y + 67\,x}{78 + 51\,y + 53\,x}, \frac{12 + 19\,y + 63\,x}{40 + 90\,y + 3\,x} \right] \tag{12}$$

```
> evalf(SFP2drz(F2, x, y))
```

$$[[1.259593357, 0.8791974895]] \tag{13}$$

```
> convert(evalf(Orb2(F2, x, y, [3.12, 0.45], 1000, 1020)), set)
```

$$\{[1.259593358, 0.8791974894]\} \tag{14}$$

```
> F3 := RT2(x, y, 1, 100)
```

$$F3 := \left[\frac{49 + 49\,y + 67\,x}{74 + 90\,y + 74\,x}, \frac{27 + 98\,y + 72\,x}{2 + 73\,y + 85\,x} \right] \tag{15}$$

```
> evalf(SFP2drz(F3, x, y))
```

$$[[0.6528216044, 1.321846941]] \tag{16}$$

```
> convert(evalf(Orb2(F3, x, y, [3.12, 0.45], 1000, 1020)), set)
```

$$\{[0.6528216034, 1.321846940]\} \tag{17}$$

```
> F4 := RT2(x, y, 1, 100)
```

$$F4 := \left[\frac{41 + 4\,y + 44\,x}{13 + 19\,y + 10\,x}, \frac{15 + 64\,y + 9\,x}{12 + 52\,y + 25\,x} \right] \tag{18}$$

```
> evalf(SFP2drz(F4, x, y))
```

$$[[3.169979666, 0.6897016763]] \tag{19}$$

```
> convert(evalf(Orb2(F4, x, y, [3.12, 0.45], 1000, 1020)), set)
```

$$\{[3.169979666, 0.6897016765]\} \tag{20}$$

```
> F5 := RT2(x, y, 1, 100)
```

$$F5 := \left[\frac{72 + 90\,y + 18\,x}{43 + 55\,y + 40\,x}, \frac{17 + 70\,y + 52\,x}{81 + 87\,y + 34\,x} \right] \tag{21}$$

```
> evalf(SFP2drz(F5, x, y))
```

$$[[1.208133500, 0.7042432306]] \tag{22}$$

```
> convert(evalf(Orb2(F5, x, y, [3.12, 0.45], 1000, 1020)), set)
```

$$\{[1.208133499, 0.7042432307], [1.208133499, 0.7042432312], [1.208133500, 0.7042432303], [1.208133501, 0.7042432303]\} \tag{23}$$

```
> F6 := RT2(x, y, 1, 100)
```

$$F6 := \left[\frac{85 + 9\,y + 68\,x}{83 + 63\,y + 100\,x}, \frac{70 + 36\,y + 36\,x}{10 + 40\,y + 66\,x} \right] \quad (24)$$

```
> evalf(SFP2drz(F6, x, y))
```

$$[[0.6049974289, 1.350612004]] \quad (25)$$

```
> convert(evalf(Orb2(F6, x, y, [3.12, 0.45], 1000, 1020)), set)
```

$$\{[0.6049974288, 1.350612003], [0.6049974288, 1.350612004]\} \quad (26)$$

```
> F7 := RT2(x, y, 1, 100)
```

$$F7 := \left[\frac{87 + 16\,y + 98\,x}{43 + 53\,y + 61\,x}, \frac{47 + 28\,y + 75\,x}{3 + 5\,y + 11\,x} \right] \quad (27)$$

```
> evalf(SFP2drz(F7, x, y))
```

$$[[0.5868505538, 6.506538819]] \quad (28)$$

```
> convert(evalf(Orb2(F7, x, y, [3.12, 0.45], 1000, 1020)), set)
```

$$\{[0.5868505549, 6.506538820]\} \quad (29)$$

```
> F8 := RT2(x, y, 1, 100)
```

$$F8 := \left[\frac{37 + 75\,y + 4\,x}{91 + 22\,y + 40\,x}, \frac{58 + 93\,y + 98\,x}{11 + 30\,y + 6\,x} \right] \quad (30)$$

```
> evalf(SFP2drz(F8, x, y))
```

$$[[1.459248295, 4.082633122]] \quad (31)$$

```
> convert(evalf(Orb2(F8, x, y, [3.12, 0.45], 1000, 1020)), set)
```

$$\{[1.459248294, 4.082633120], [1.459248294, 4.082633122]\} \quad (32)$$

```
> F9 := RT2(x, y, 1, 100)
```

$$F9 := \left[\frac{32 + 40\,y + 24\,x}{80 + 96\,y + 11\,x}, \frac{23 + 41\,y + 52\,x}{58 + 67\,y + 81\,x} \right] \quad (33)$$

```
> evalf(SFP2drz(F9, x, y))
```

$$[[0.4747723592, 0.5258468959]] \quad (34)$$

```
> convert(evalf(Orb2(F9, x, y, [3.12, 0.45], 1000, 1020)), set)
```

$$\{[0.4747723604, 0.5258468959]\} \quad (35)$$

```
> F10 := RT2(x, y, 1, 100)
```

$$F10 := \left[\frac{65 + 69\,y + 2\,x}{36 + 61\,y + 84\,x}, \frac{96 + 94\,y + 31\,x}{81 + 31\,y + 54\,x} \right] \quad (36)$$

```
> evalf(SFP2drz(F10, x, y))
```

$$[[0.8556835835, 1.523365570]] \quad (37)$$

```
> convert(evalf(Orb2(F10, x, y, [3.12, 0.45], 1000, 1020)), set)
```

$$\{[0.8556835824, 1.523365569], [0.8556835829, 1.523365570], [0.8556835833, 1.523365571]\} \quad (38)$$

```
> F11 := RT2(x, y, 1, 100)
```

$$F11 := \left[\frac{67 + 59\,y + 66\,x}{12 + 49\,y + 90\,x}, \frac{35 + 15\,y + 26\,x}{100 + 24\,y + 8\,x} \right] \quad (39)$$

```
> evalf(SFP2drz(F11, x, y))
```

$$[[1.212035727, 0.6085299018]] \quad (40)$$

```
> convert(evalf(Orb2(F11, x, y, [3.12, 0.45], 1000, 1020)), set)
```

$$\{[1.212035727, 0.6085299021]\} \quad (41)$$

```
> F12 := RT2(x, y, 1, 100)
```

$$F12 := \left[\frac{53 + 37\,y + 88\,x}{50 + 37\,y + 76\,x}, \frac{95 + 8\,y + 92\,x}{92 + 2\,y + 97\,x}\right] \quad (42)$$

```
> evalf(SFP2drz(F12, x, y))
```

$$[[1.094429423, 1.018166456]] \quad (43)$$

```
> convert(evalf(Orb12(F2, x, y, [3.12, 0.45], 1000, 1020)), set)
```

$$\left\{1000, 1020, x, y, [3.12, 0.45], \left[\frac{92 + 71\,y + 67\,x}{78 + 51\,y + 53\,x}, \frac{12 + 19\,y + 63\,x}{40 + 90\,y + 3\,x}\right]\right\} \quad (44)$$

```
> F13 := RT2(x, y, 1, 100)
```

$$F13 := \left[\frac{44 + 9\,y + 30\,x}{14 + 79\,y + 73\,x}, \frac{21 + 78\,y + 49\,x}{93 + 15\,y + 56\,x}\right] \quad (45)$$

```
> evalf(SFP2drz(F13, x, y))
```

$$[[0.5675431985, 0.8250663460]] \quad (46)$$

```
> convert(evalf(Orb2(F13, x, y, [3.12, 0.45], 1000, 1020)), set)
```

$$\{[0.5675431992, 0.8250663453]\} \quad (47)$$

```
> F14 := RT2(x, y, 1, 100)
```

$$F14 := \left[\frac{69 + 17\,y + 21\,x}{42 + 21\,y + 5\,x}, \frac{58 + 3\,y + 86\,x}{55 + 97\,y + 4\,x}\right] \quad (48)$$

```
> evalf(SFP2drz(F14, x, y))
```

$$[[1.652632624, 1.165682549]] \quad (49)$$

```
> convert(evalf(Orb2(F14, x, y, [3.12, 0.45], 1000, 1020)), set)
```

$$\{[1.652632625, 1.165682548], [1.652632625, 1.165682550]\} \quad (50)$$

```
> F15 := RT2(x, y, 1, 100)
```

$$F15 := \left[\frac{92 + 46\,y + 88\,x}{34 + 68\,y + 49\,x}, \frac{61 + 21\,y + 86\,x}{42 + 5\,y + 33\,x}\right] \quad (51)$$

```
> evalf(SFP2drz(F15, x, y))
```

$$[[1.222514770, 2.283353847]] \quad (52)$$

```
> convert(evalf(Orb2(F15, x, y, [3.12, 0.45], 1000, 1020)), set)
```

$$\{[1.222514771, 2.283353847]\} \quad (53)$$

```
> F16 := RT2(x, y, 1, 100)
```

$$F16 := \left[\frac{77 + 98\,y + 58\,x}{98 + 29\,y + 65\,x}, \frac{29 + 35\,y + 29\,x}{34 + 66\,y + 44\,x}\right] \quad (54)$$

```
> evalf(SFP2drz(F16, x, y))
```

$$[[1.092079965, 0.6664393702]] \quad (55)$$

```
> convert(evalf(Orb2(F16, x, y, [3.12, 0.45], 1000, 1020)), set)
```

$$\{[1.092079965, 0.6664393704], [1.092079966, 0.6664393702]\} \quad (56)$$

```
> F17 := RT2(x, y, 1, 100)
```

$$F17 := \left[\frac{60 + 83\,y + 32\,x}{85 + 100\,y + 68\,x}, \frac{59 + 40\,y + 76\,x}{92 + 39\,y + 17\,x}\right] \quad (57)$$

```
> evalf(SFP2drz(F17, x, y))
```

$$[[0.7136147135, 1.069749065]] \quad (58)$$

```
> convert(evalf(Orb2(F17, x, y, [3.12, 0.45], 1000, 1020)), set)
```

$$\{[0.7136147138, 1.069749066], [0.7136147139, 1.069749065]\} \quad (59)$$

```
> F18 := RT2(x, y, 1, 100)
```

$$F18 := \left[\frac{50 + 78\,y + 20\,x}{18 + 18\,y + 51\,x}, \frac{34 + 78\,y + 10\,x}{52 + 100\,y + 13\,x}\right] \quad (60)$$

```
> evalf(SFP2drz(F18, x, y))
```

$$[[1.343941710, 0.7327254243]] \quad (61)$$

```
> convert(evalf(Orb2(F18, x, y, [3.12, 0.45], 1000, 1020)), set)
```

$$\{[1.343941686, 0.7327254244], [1.343941687, 0.7327254249]\} \quad (62)$$

```
> F19 := RT2(x, y, 1, 100)
```

$$F19 := \left[\frac{87 + 13\,y + 37\,x}{92 + 97\,y + 69\,x}, \frac{62 + 38\,y + 60\,x}{46 + 78\,y + 61\,x}\right] \quad (63)$$

```
> evalf(SFP2drz(F19, x, y))
```

$$[[0.5537572255, 0.8690944951]] \quad (64)$$

```
> convert(evalf(Orb2(F19, x, y, [3.12, 0.45], 1000, 1020)), set)
```

$$\{[0.5537572256, 0.8690944954]\} \quad (65)$$

```
> F20 := RT2(x, y, 1, 100)
```

$$F20 := \left[\frac{12 + 52\,y + 25\,x}{72 + 90\,y + 18\,x}, \frac{43 + 55\,y + 40\,x}{17 + 70\,y + 52\,x}\right] \quad (66)$$

```
> evalf(SFP2drz(F20, x, y))
```

$$[[0.4449874645, 1.044140881]] \quad (67)$$

```
> convert(evalf(Orb2(F20, x, y, [3.12, 0.45], 1000, 1020)), set)
```

$$\{[0.4449874742, 1.044140881], [0.4449874742, 1.044140882]\} \quad (68)$$

```
> #4i)
> print(RT2)
```

```
proc(x, y, d, K)                                                    (69)
    local ra, i, j, f, g;
    ra := rand(1..K);
    f := add(add(ra( ) * x^i * y^j, j = 0..d - i), i = 0..d) / add(add(ra( ) * x^i * y^j, j = 0
     ..d - i), i = 0..d);
    g := add(add(ra( ) * x^i * y^j, j = 0..d - i), i = 0..d) / add(add(ra( ) * x^i * y^j, j = 0
     ..d - i), i = 0..d);
    [f, g]
end proc
```

```
> RT3 := proc(x, y, z, d, K) local ra, i, j, k, f, g, h :
  ra := rand(1..K) : #random integer from -K to K
  f := add(add(add(ra( ) * x^i * y^j * z^k, k = 0..d - j - i), j = 0..d - i), i = 0..d)
       / add(add(add(ra( ) * x^i * y^j * z^k, j = 0..d - i), i = 0..d), k = 0..d) :
```

```
g := add(add(add(ra( ) * x^i * y^j * z^k, k = 0 ..d - j - i), j = 0 ..d - i), i = 0 ..d)
     / add(add(add(ra( ) * x^i * y^j * z^k, j = 0 ..d - i), i = 0 ..d), k = 0 ..d) :
h := add(add(add(ra( ) * x^i * y^j * z^k, k = 0 ..d - j - i), j = 0 ..d - i), i = 0 ..d)
     / add(add(add(ra( ) * x^i * y^j * z^k, j = 0 ..d - i), i = 0 ..d), k = 0 ..d) :
[f, g, h] :
end:
```

```
> RT3(x, y, z, 1, 100)
```

$$\left[\frac{8+63z+78y+23x}{53xz+9yz+32x+22y+98z+73}, \frac{3+98z+69y+3x}{52xz+94yz+37x+88y+60z+73}, \frac{16+29z+51y+3x}{49xz+74yz+40x+67y+71z+45}\right] \quad (70)$$

```
> #4ii)
> print(Orb2)
```

```
proc(F, x, y, pt0, K1, K2)                                                    (71)
    local pt, L, i;
    pt := pt0;
    for i to K1 do pt := subs({x = pt[1], y = pt[2]}, F) end do;
    L := [ ];
    for i from K1 + 1 to K2 do
        L := [op(L), pt]; pt := subs({x = pt[1], y = pt[2]}, F)
    end do;
    L
end proc
```

```
> Orb3 := proc(F, x, y, z, pt0, K1, K2) local pt, L, i :
  pt := pt0 :

  for i from 1 to K1 do
  pt := subs({x = pt[1], y = pt[2], z = pt[3]}, F) :
  od:

  L := [ ] :
  for i from K1 + 1 to K2 do
  L := [op(L), pt] :
  pt := subs({x = pt[1], y = pt[2], z = pt[3]}, F) :

  od:
  L :
  end:
```

```
> T := RT3(x, y, z, 1, 100)
```

$$T := \left[\frac{39+28z+16y+40x}{49xz+77yz+100x+17y+56z+94}, \frac{54+35z+57y+44x}{50xz+91yz+30x+15y+16z+36}, \frac{88+98z+98y+55x}{66xz+55yz+38x+30y+23z+15}\right] \quad (72)$$

```
> convert(evalf(Orb3(T, x, y, z, [3.12, 0.45, 1.4], 1000, 1020)), set)
```

$$\{[0.3093261714, 0.7416716619, 1.772427324], [0.3093261716, 0.7416716614, 1.772427324]\} \quad (73)$$

```
> #4iv)
> print(FP2)
```

```
proc(F, x, y)
    local L, i;
    L := [solve({F[1]=x, F[2]=y}, {x, y})]; [seq(subs(L[i], [x, y]), i=1..nops(L))]
end proc
```

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```
> FP3 := proc(F, x, y, z) local L, i:
  L := [solve({F[1]=x, F[2]=y, F[3]=z}, {x, y, z})]:

  [seq(subs(L[i], [x, y, z]), i=1..nops(L))]:
  end:

> evalf(FP3(T, x, y, z))
```

$$[[0.3929129337, -1.439298387, 0.2637220026]] \quad (75)$$

```
> #4v)
> print(SFP2)
```

```
proc(F, x, y)
    local L, J, S, J0, i, pt, EV;
    L := evalf(FP2(F, x, y));
    J := Matrix(normal([[diff(F[1], x), diff(F[2], x)], [diff(F[1], y), diff(F[2], y)]]));
    S := [ ];
    for i to nops(L) do
        pt := L[i];
        J0 := subs({x=pt[1], y=pt[2]}, J);
        EV := LinearAlgebra:-Eigenvalues(J0);
        if abs(EV[1]) < 1 and abs(EV[2]) < 1 then S := [op(S), pt] end if
    end do;
    S
end proc
```

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```
> SFP3 := proc(F, x, y, z) local L, J, S, J0, i, pt, EV:

  L := evalf(FP3(F, x, y, z)):
      #F is the list of ALL fixed points of the transformation [x,y]->F using the previous procedure
      FP2(F,x,y), but since we are interested in numbers we take the floating point version using
      evalf

  J := Matrix(normal([[diff(F[1], x), diff(F[2], x), diff(F[3], x)], [diff(F[1], y), diff(F[2],
      y), diff(F[3], y)], [diff(F[1], z), diff(F[2], z), diff(F[3], z)]])):
```

```
S := [ ]:  #S is the list of stable fixed points that starts out empty

for i from 1 to nops(L) do  #we examime it case by case
pt := L[i]: #pt is the current fixed point to be examined

J0 := subs( {x=pt[1], y=pt[2], z=pt[3]}, J ):
    #J0 is the NUMERICAL matrix obtained by plugging-in the examined fixed pt

EV := Eigenvalues(J0):
    # We used Maple's command Eigenavalues to find the eigenvalues of this 2 by 2 matrix

if abs(EV[1]) < 1 and abs(EV[2]) < 1  and abs(EV[3]) < 1 then
 S := [op(S), pt]:
    #If both eigenvalues have absolute value less than 1 it means that they are stable, so we
    append the examined fixed point, pt, to the list of fixed points
fi:
od:
S: #the output is S
end:
```

```
> SFP3(T, x, y, z)
```

$$[\ ] \tag{77}$$

```
> #5)
> H1 := RT3(x, y, z, 1, 100)
```

$$H1 := \left[\frac{51 + 47\,z + 33\,y + 77\,x}{19\,x\,z + 16\,y\,z + 10\,x + 83\,y + 40\,z + 20}, \frac{40 + 40\,z + 78\,y + 91\,x}{23\,x\,z + 29\,y\,z + 5\,x + 75\,y + 78\,z + 71}, \frac{81 + 21\,z + 6\,y + 36\,x}{44\,x\,z + 91\,y\,z + 69\,x + 42\,y + 73\,z + 90} \right] \tag{78}$$

```
> evalf(SFP3(H1, x, y, z))
```

$$[[1.251205, 1.1614046, 0.4349838414]] \tag{79}$$

```
> convert(evalf(Orb3(H1, x, y, z, [3.12, 0.45, 2.7], 1000, 1020)), set)
```

$$\{[1.251205132, 1.161404634, 0.4349838412], [1.251205132, 1.161404635, 0.4349838415], [1.251205133, 1.161404635, 0.4349838414], [1.251205134, 1.161404635, 0.4349838415]\} \tag{80}$$

```
>
> H2 := RT3(x, y, z, 1, 100)
```

$$H2 := \left[\frac{51 + 52\,z + 23\,y + 44\,x}{34\,x\,z + 85\,y\,z + 96\,x + 23\,y + 95\,z + 80}, \frac{69 + 29\,z + 70\,y + 20\,x}{65\,x\,z + 95\,y\,z + 95\,x + 11\,y + 32\,z + 49}, \frac{20 + 60\,z + 60\,y + 90\,x}{31\,x\,z + 18\,y\,z + 60\,x + 76\,y + 18\,z + 39} \right] \tag{81}$$

```
> evalf(SFP3(H2, x, y, z))
```

$$[\ ] \tag{82}$$

```
> convert(evalf(Orb3(H2, x, y, z, [3.12, 0.45, 2.7], 1000, 1020)), set)
```

$$\{[0.4507814160, 0.6879298886, 0.9942748125], [0.4507814160, 0.6879298889, \tag{83}$$

0.9942748119], [0.4507814160, 0.6879298889, 0.9942748125], [0.4507814163, 0.6879298889, 0.9942748125], [0.4507814163, 0.6879298892, 0.9942748113], [0.4507814163, 0.6879298892, 0.9942748119]}

> $H3 := RT3(x, y, z, 1, 100)$

$$H3 := \left[\frac{23 + 65z + y + 73x}{33xz + 4yz + 35x + 2y + 60z + 50}, \frac{15 + 22z + 45y + 36x}{8xz + 63yz + 20x + 62y + 79z + 25}, \frac{14 + 69z + 100y + 60x}{29xz + 25yz + 12x + 88y + 43z + 20}\right] \quad (84)$$

> $evalf(SFP3(H3, x, y, z))$

$$[\,] \quad (85)$$

> $convert(evalf(Orb3(H3, x, y, z, [3.12, 0.45, 2.7], 1000, 1020)), set)$

{[0.8700877308, 0.4514264348, 1.176022537], [0.8700877309, 0.4514264348, 1.176022536], [0.8700877314, 0.4514264347, 1.176022536], [0.8700877314, 0.4514264349, 1.176022536], [0.8700877314, 0.4514264349, 1.176022537]} (86)

> $H4 := RT3(x, y, z, 1, 100)$

$$H4 := \left[\frac{6 + 29z + 36y + 27x}{88xz + 47yz + 77x + 20y + 67z + 96}, \frac{65 + 78z + 68y + 46x}{89xz + 92yz + 19x + 90y + 59z + 28}, \frac{62 + 32z + y + 69x}{81xz + 19yz + 6x + 89y + 42z + 24}\right] \quad (87)$$

> $evalf(SFP3(H4, x, y, z))$

[[0.286235722, 0.85881048, 0.6645286347]] (88)

> $convert(evalf(Orb3(H4, x, y, z, [3.12, 0.45, 2.7], 1000, 1020)), set)$

{[0.2862357306, 0.8588104959, 0.6645286343], [0.2862357307, 0.8588104962, 0.6645286352]} (89)

> $H5 := RT3(x, y, z, 1, 100)$

$$H5 := \left[\frac{39 + 94z + 14y + 85x}{43xz + 54yz + 8x + 100y + 65z + 28}, \frac{31 + 14z + 87y + 16x}{30xz + 9yz + 68x + 64y + 57z + 100}, \frac{1 + 98z + 84y + 72x}{54xz + 14yz + 20x + 62y + 59z + 48}\right] \quad (90)$$

> $evalf(SFP3(H5, x, y, z))$

[[1.19684311, 0.308174216, 0.9676115555]] (91)

> $convert(evalf(Orb3(H5, x, y, z, [3.12, 0.45, 2.7], 1000, 1020)), set)$

{[1.196843083, 0.3081742156, 0.9676115554], [1.196843084, 0.3081742156, 0.9676115554]} (92)

>

> $H6 := RT3(x, y, z, 1, 100)$

$$H6 := \left[\frac{92 + 45z + 64y + 24x}{47xz + 4yz + 86x + 83y + 96z + 72}, \frac{56 + 59z + 78y + 21x}{80xz + 57yz + 71x + 29y + 88z + 68}, \frac{36 + 45z + 29y + 27x}{73xz + 10yz + 27x + 69y + 99z + 3}\right] \quad (93)$$

```
> evalf(SFP3(H6, x, y, z))
```

$$[\,] \tag{94}$$

```
> convert(evalf(Orb3(H6, x, y, z, [3.12, 0.45, 2.7], 1000, 1020)), set)
```

$$\{[0.6710121695, 0.6390718783, 0.6236595914], [0.6710121695, 0.6390718787, 0.6236595914], [0.6710121698, 0.6390718786, 0.6236595918], [0.6710121700, 0.6390718788, 0.6236595918]\} \tag{95}$$

```
>
> H7 := RT3(x, y, z, 1, 100)
```

$$H7 := \left[\frac{68 + 66\,z + 73\,y + 56\,x}{32\,x\,z + 44\,y\,z + 58\,x + 69\,y + 54\,z + 82}, \frac{49 + 37\,z + 90\,y + 18\,x}{29\,x\,z + 49\,y\,z + 100\,x + 38\,y + 29\,z + 74}, \frac{84 + 30\,z + 36\,y + 65\,x}{83\,x\,z + 51\,y\,z + 45\,x + 93\,y + 94\,z + 22}\right] \tag{96}$$

```
> evalf(SFP3(H7, x, y, z))
```

$$[[0.828409914, 0.606448513, 0.7133476995]] \tag{97}$$

```
> convert(evalf(Orb3(H7, x, y, z, [3.12, 0.45, 2.7], 1000, 1020)), set)
```

$$\{[0.8284099273, 0.6064485601, 0.7133476996]\} \tag{98}$$

```
> H8 := RT3(x, y, z, 1, 100)
```

$$H8 := \left[\frac{30 + 2\,z + 3\,y + 56\,x}{15\,x\,z + 59\,y\,z + 50\,x + 91\,y + 23\,z + 63}, \frac{26 + 92\,z + 62\,y + 5\,x}{76\,x\,z + 94\,y\,z + 63\,x + 5\,y + 88\,z + 38}, \frac{45 + 78\,z + y + 77\,x}{80\,x\,z + 67\,y\,z + 20\,x + 34\,y + 47\,z + 39}\right] \tag{99}$$

```
> evalf(SFP3(H8, x, y, z))
```

$$[[0.2377912817, 0.747574241, 0.7817417724]] \tag{100}$$

```
> convert(evalf(Orb3(H8, x, y, z, [3.12, 0.45, 2.7], 1000, 1020)), set)
```

$$\{[0.2377913904, 0.7475746092, 0.7817417723], [0.2377913904, 0.7475746096, 0.7817417728], [0.2377913904, 0.7475746097, 0.7817417723], [0.2377913904, 0.7475746097, 0.7817417724], [0.2377913904, 0.7475746097, 0.7817417729]\} \tag{101}$$

```
> H9 := RT3(x, y, z, 1, 100)
```

$$H9 := \left[\frac{30 + 52\,z + 18\,y + 53\,x}{88\,x\,z + 33\,y\,z + 84\,x + 84\,y + 80\,z + 84}, \frac{56 + 41\,z + 51\,y + 13\,x}{92\,x\,z + 65\,y\,z + 14\,x + 95\,y + 84\,z + 29}, \frac{57 + 82\,z + 4\,y + 81\,x}{43\,x\,z + 18\,y\,z + 99\,x + 37\,y + 60\,z + 45}\right] \tag{102}$$

```
> evalf(SFP3(H9, x, y, z))
```

$$[\,] \tag{103}$$

```
> convert(evalf(Orb3(H9, x, y, z, [3.12, 0.45, 2.7], 1000, 1020)), set)
```

$$\{[0.3803544514, 0.5554849897, 0.9056269178]\} \tag{104}$$

```
> H10 := RT3(x, y, z, 1, 100)
```

$$H10 := \left[\frac{76 + 99\,z + 24\,y + 7\,x}{91\,x\,z + 45\,y\,z + 89\,x + 42\,y + 25\,z + 1}, \right. \tag{105}$$

$$\frac{69 + 92\,z + 6\,y + 36\,x}{41\,x\,z + 47\,y\,z + 41\,x + 74\,y + 97\,z + 39}, \frac{75 + 85\,z + 17\,y + 97\,x}{72\,x\,z + 80\,y\,z + 21\,x + 73\,y + 72\,z + 31}\Bigg]$$

```
> evalf(SFP3(H10, x, y, z))
```

$$[[0.8385572429, 0.685090881, 0.9116487373]] \quad \textbf{(106)}$$

```
> convert(evalf(Orb3(H10, x, y, z, [3.12, 0.45, 2.7], 1000, 1020)), set)
```

$$\{[0.8385574296, 0.6850906183, 0.9116487373], [0.8385574300, 0.6850906183, 0.9116487373]\} \quad \textbf{(107)}$$

```
>
```