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1.i) $x \longrightarrow x^3 - 6x^2 + 12x - 6$

$f(x) = x^3 - 6x^2 + 12x - 6$

$x = x^3 - 6x^2 + 12x - 6$

$0 = x^3 - 6x^2 + 11x - 6$

$\pm\frac{6}{1}, \pm\frac{3}{1}, \pm\frac{2}{1}, \pm 1$

$0 = 1 - 6 + 11 - 6 = 0$

$0 = 8 - 24 + 22 - 6 = 0$

$0 = 27 - 54 + 33 - 6 = 0$

The fixed points are 1, 2, 3

$f'(x) = 3x^2 - 12x + 12$

$f'(1) = 2 - 12 + 12 = 2 > 1$

$f'(2) = 12 - 24 + 12 = 0 < 1$

$f'(3) = 27 - 36 + 12 = 3 > 1$

Only 2 is a stable fixed point

1.ii) $x \longrightarrow x^4 - \frac{13x^2}{36} + x + \frac{1}{36}$

$f(x) = x^4 - \frac{13x^2}{36} + x + \frac{1}{36}$

$x = x^4 - \frac{13x^2}{36} + x + \frac{1}{36}$

$0 = 36x^4 - 13x^2 + 1$

$0 = 36\left(\frac{1}{3}\right)^4 - 13\left(\frac{1}{3}\right)^2 + 1 = 0$

$0 = 36\left(\frac{1}{2}\right)^4 - 13\left(\frac{1}{2}\right)^2 + 1 = 0$

The fixed points are $\pm\frac{1}{3}, \pm\frac{1}{2}$

$f'(x) = 4x^3 - \frac{13}{18}x^2 + 1$

$f'\left(\frac{1}{3}\right) = \frac{49}{54} < 1$

$f'\left(-\frac{1}{3}\right) = \frac{59}{54} > 1$

$f'\left(\frac{1}{2}\right) = \frac{41}{36} > 1$

$f'\left(-\frac{1}{2}\right) = \frac{31}{36} < 1$

The stable fixed points are $\frac{1}{3}, -\frac{1}{2}$

2.i) $f(x,y) = \sqrt{x + 4y}$

$f_x = \frac{1}{2}(x+4y)^{-\frac{1}{2}} \cdot 1 = \frac{1}{2\sqrt{x+4y}}$

$f_y = \frac{1}{2}(x+4y)^{-\frac{1}{2}} \cdot 4 = \frac{2}{\sqrt{x+4y}}$

$f(1,2) \approx f(0.95, 1.02) + f_x(0.95, 1.02)(0.05) + f_y(0.95, 1.02)(0.98)$

≈ 3.1278

$f(1,2) = 3$

2.ii) $f(x,y,z) = x^3 y^4 z^5$

$f_x = 3x^2 y^4 z^5 \quad f_y = 4x^3 y^3 z^5 \quad f_z = 5x^3 y^4 z^4$

$f(1,1,1) \approx f(1.01, 1.02, 0.99) + f_x(1.01, 1.02, 0.99)(-0.01)$
$+ f_y(1.01, 1.02, 0.99)(-0.02) + f_z(1.01, 1.02, 0.99)(0.01)$

≈ 0.9994

$f(1,1,1) = 1$

2.iii) $f(x_1, x_2, x_3, x_4) = \sqrt{x_1 + x_2 + x_3 + x_4}$

$f_{x_1} = \frac{1}{2\sqrt{x_1+x_2+x_3+x_4}}$

$f_{x_2} = \frac{1}{2\sqrt{x_1+x_2+x_3+x_4}}$

$f_{x_3} = \frac{1}{2\sqrt{x_1+x_2+x_3+x_4}}$

$f_{x_4} = \frac{1}{2\sqrt{x_1+x_2+x_3+x_4}}$

$f(1,1,1,1) \approx f(1.01, 1.01, 0.99, 0.99) + \underbrace{(-\cancel{0.01} + -\cancel{0.01} + \cancel{0.01} + \cancel{0.01})}_{=0}\left(\frac{1}{2\sqrt{1.01+1.01+0.99+0.99}}\right)$

≈ 2

$f(1,1,1,1) = 2$

3. $(x,y) \longrightarrow \left(\frac{x}{y+1}, \frac{y}{x+1}\right)$

$$\text{Jac} = \begin{pmatrix} \frac{1}{y+1} & \frac{-x}{(y+1)^2} \\ -\frac{y}{(x+1)^2} & \frac{1}{x+1} \end{pmatrix}$$

$$\text{Jac}(1,1) = \begin{pmatrix} \frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{2} \end{pmatrix}$$

4. $(x,y,z) \rightarrow (x+y+z,\ x^2+y^2+z^2,\ +x^3+y^3+z^3)$

$$Jac = \begin{pmatrix} 1 & 1 & 1 \\ 2x & 2y & 2z \\ 3x^2 & 3y^2 & 3z^2 \end{pmatrix}$$

$$Jac(1,1,1) = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{pmatrix}$$

5. The eigenvalues are essentially the distance between the fixed point and the function, so if the eigenvalues are less than 1, it means the distance between the two is shrinking thus the function is stabilizing back to that point.