2) Find the Linearizations

i) $f(x,y) = \sqrt{x+4y}$ at $(1,2)$ at $(0.95, 1.02)$

$L(x,y) = f(a,b) + (x-a) f_x(a,b) + (y-b) f_y(a,b)$

$= \sqrt{1+8} + (0.95)-(1) \cdot \frac{1}{2\sqrt{1+8}} + (1.02)-(2) \cdot \frac{2}{\sqrt{1+8}}$

$= 3 - 0.008333 - 0.653333$

$= \boxed{2.33834}$ -> approximation

Actual Value => $\boxed{2.24277}$

ii) $f(x,y,z) = x^3y^4z^5$ at $(1,1,1)$ at values $(1.01, 1.02, 0.99)$

$L(x,y,z) = f(a,b,c) + (x-a) f_x(a,b,c) + (y-b) f_y(a,b,c) + (z-c) f_z(a,b,c)$

$= 1 + (1.01-1) \cdot 3x^2y^4z^5 + (1.02-1) \cdot 4y^3x^3z^5 + (0.99-1) \cdot 5z^4x^3y^4$

$= 1 + (0.01 \cdot 3) + (0.02 \cdot 4) - (0.01 \cdot 5)$

$= 1 + 0.03 + 0.08 - 0.05$

$= \boxed{1.06}$ -> approximation

Actual value -> $\boxed{1.06057}$

iii) $f(x_1, x_2, x_3, x_4) = \sqrt{x_1 + x_2 + x_3 + x_4}$ at $(1,1,1,1)$ at value $(1.01, 1.01, 0.99, 0.99)$

$L(x_1, x_2, x_3, x_4) = 2 + (0.01 \cdot \frac{1}{2}) + (0.01 \cdot \frac{1}{2}) - (0.01 \cdot \frac{1}{2}) (0.01 \cdot \frac{1}{2})$

$= 2 + 0.005 + 0.005 - 0.005 - 0.005$

$= \boxed{2}$ -> approximation

actual value -> $\boxed{2}$

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3) $(x,y) \rightarrow \left(\frac{x}{y+1}, \frac{y}{x+1}\right)$

We set up the Jacobian matrix for 2 varibles

$$J = \begin{bmatrix} \frac{\partial}{\partial x}\frac{x}{y+1} & \frac{\partial}{\partial y}\frac{x}{y+1} \\ \frac{\partial}{\partial x}\frac{y}{x+1} & \frac{\partial}{\partial y}\frac{y}{x+1} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{y+1} & -\frac{x}{(y+1)^2} \\ -\frac{y}{(x+1)^2} & \frac{1}{x+1} \end{bmatrix}$$

4) $(x,y,z) \rightarrow (x+y+z,\ x^2+y^2+z^2,\ x^3+y^3+z^3)$

$$J = \begin{bmatrix} \frac{\partial}{\partial x} x+y+z & \frac{\partial}{\partial y} x+y+z & \frac{\partial}{\partial z} x+y+z \\ \frac{\partial}{\partial x} x^2+y^2+z^2 & \frac{\partial}{\partial y} x^2+y^2+z^2 & \frac{\partial}{\partial z} x^2+y^2+z^2 \\ \frac{\partial}{\partial x} x^3+y^3+z^2 & \frac{\partial}{\partial y} x^3+y^3+z^3 & \frac{\partial}{\partial z} x^3+y^3+z^3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 2x & 2y & 2z \\ 3x^2 & 3y^2 & 3z^2 \end{bmatrix}$$

5) The reason it makes sense for a fixed point to be a stable fixed point if its Jacobian has its eigenvalues with abs < 1

Because the way we normally find stable fixed points is by taking the derivative and plugging in the fixed point at the derivative to find if it is abs < 1

for our Jacobian, we are doing the same thing but we do it for 2 variables

But we find the stable fixed point in the same way.