# HW12

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1. i $x \to x^3 - 6x^2 + 12x - 6$

$f(z) = z^3 - 6z^2 + 12z - 6$

$f(z) = z \to z^3 - 6z^2 + 12z - 6 = z$

$z^3 - 6z^2 + 11z - 6 = 0$

$(z-1)(z-2)(z-3) = 0$

$z = 1, 2, 3$

$\boxed{\text{Fixed points are } (x = 1, 2, 3)}$

$\frac{df}{dz} = 3z^2 - 12z + 12$

$\frac{df}{dz}(1) = 3$ x $\quad \frac{df}{dz}(2) = 0$ ✓ $\quad \frac{df}{dz}(3) = 3$ x

$\boxed{\text{Stable fixed point is } x = 2 \text{ only.}}$

ii $x \to x^4 - \frac{13}{36}x^2 + x + \frac{1}{36}$

$f(z) = z^4 - \frac{13}{36}z^2 + z + \frac{1}{36}$

$z = z^4 - \frac{13}{36}z^2 + z + \frac{1}{36}$

$0 = z^4 - \frac{13}{36}z^2 + \frac{1}{36} = \left(z^2 - \frac{1}{4}\right)\left(z^2 - \frac{1}{9}\right)$

$z = \pm 1/2, \pm 1/3$

$\boxed{\text{Fixed points are } x = \frac{1}{2}, \frac{-1}{2}, \frac{1}{3}, \frac{-1}{3}}$

$\frac{df}{dz} = 4z^3 - \frac{13}{18}z + 1$

$\frac{df}{dz}\left(\frac{1}{2}\right) = 1\frac{5}{36}$ x $\quad \frac{df}{dz}\left(\frac{-1}{2}\right) = \frac{31}{36}$ ✓

$\frac{df}{dz}\left(\frac{1}{3}\right) = \frac{49}{54}$ ✓ $\quad \frac{df}{dz}\left(\frac{-1}{3}\right) = 1\frac{5}{54}$

$\boxed{\text{Stable fixed points } x = \frac{-1}{2}, \frac{1}{3}}$

i $f(x,y) = \sqrt{x+4y}$ at $(1,2)$ $f_x = \frac{1}{2}(x+4y)^{-1/2}$ $f_y = 2(x+4y)^{-1/2}$

$f(1,2) = 3$

$f_x(1,2) = \frac{1}{6}$ $f_y(1,2) = \frac{2}{3}$

Linearization: $\boxed{f(x,y) \approx 3 + \frac{1}{6}(x-1) + \frac{2}{3}(y-2) = \frac{3}{2} + \frac{1}{6}x + \frac{2}{3}y}$

Approx of $f(0.95, 1.02) = 2.338$

Actual value is $2.242$, pretty close!

ii $f(x,y,z) = x^3y^4z^5$ $f_x = 3x^2y^4z^5$ $f_y = 4x^3y^3z^5$ $f_z = 5x^3y^4z^4$

at $(1,1,1)$ $f = 1$ $f_x = 3$ $f_y = 4$ $f_z = 5$

Linearization: $f(x,y,z) \approx 1 + 3(x-1) + 4(y-1) + 5(z-1) \approx 3x + 4y + 5z - 11$

Approx of $f(1.01, 1.02, .99) = 1.06$ Actual value is $1.061$ very close!

iii $f(x_1, x_2, x_3, x_4) = \sqrt{x_1 + x_2 + x_3 + x_4}$ $f_{x_1} = f_{x_2} = f_{x_3} = f_{x_4} = \frac{1}{2}(x_1 + x_2 + x_3 + x_4)^{-1/2}$

$f(x_1, x_2, x_3, x_4) = 2$ $f_{x_1} = f_{x_2} = f_{x_3} = f_{x_4}$ at $(1,1,1,1) = \frac{1}{4}$

Linearization $f(x_1, x_2, x_3, x_4) \approx 2 + \frac{1}{4}(x_1 - 1) + \frac{1}{4}(x_2 - 1) + \frac{1}{4}(x_3 - 1) + \frac{1}{4}(x_4 - 1)$

$\approx 1 + \frac{1}{4}x_1 + \frac{1}{4}x_2 + \frac{1}{4}x_3 + \frac{1}{4}x_4$

Approx of $f(1.01, 1.01, 0.99, 0.99) = 2$

Actual value $= 2$ exactly correct at this point

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Jacobian Matrix of

$$(x,y) = \left(\frac{x}{y+1}, \frac{y}{x+1}\right) \text{ at } (1,1)$$

$$\begin{bmatrix} f_x & f_y \\ g_x & g_y \end{bmatrix} = \begin{bmatrix} \frac{1}{y+1} & \frac{-x}{(y+1)^2} \\ \frac{-y}{(x+1)^2} & \frac{1}{x+1} \end{bmatrix} (1,1) = \begin{bmatrix} \frac{1}{2} & \frac{-1}{4} \\ \frac{-1}{4} & \frac{1}{2} \end{bmatrix}$$

4 Jacobian of $(x,y,z) \Rightarrow (x+y+z,\ x^2+y^2+z^2,\ x^3+y^3+z^3)$

$$\begin{bmatrix} f_x & f_y & f_z \\ g_x & g_y & g_z \\ h_x & h_y & h_z \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 2x & 2y & 2z \\ 3x^2 & 3y^2 & 3z^2 \end{bmatrix} (1,1,1) = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$$

5.

Eigenvalues in general tell us the future of our matrix which is made of a transformation. The eigenvalues tell us the change that can happen by this matrix, which in this situation of the Jacobian Matrix determines changes in the derivative of the transformation at each point.

Therefore, if the eigenvalue is less than 1 (with absolute value) then the derivatives over time of the matrix would approach 0 and therefore the point of the linear transformation (i.e, $(x_0, y_0)$) would not change and be a stable fixed point.