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> # Max Mekhanikov - HW 12 - Okay to post

  # Question 1
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> $SFPe := \mathbf{proc}(f, x)$ **local** $f1, L, i, M$:
$f1 := normal(diff(f, x))$:
$L := [solve(numer(f-x), x)]$:
$M := [\ ]$:

for i **from** 1 **to** $nops(L)$ **do**
if $subs(x = L[i], denom(f1)) \neq 0$ **then**
$M := [op(M), [L[i], normal(subs(x = L[i], f1))]]$:
fi:
od:
M :

end:

> $SFPe(x^3 - 6 \cdot x^2 + 12 \cdot x - 6, x)$

$$[[1, 3], [2, 0], [3, 3]] \tag{1}$$

> $SFP := \mathbf{proc}(f, x)$ **local** $L, i, f1, pt, Ls$:
$L := FP(f, x)$: *#The list of fixed points (including complex ones)*

$Ls := [\ ]$: *#Ls is the list of stable fixed points, that starts out as the empty list*

$f1 := diff(f, x)$: *#The derivative of the function f w.r.t. x*

for i **from** 1 **to** $nops(L)$ **do**
$pt := L[i]$:

if $\text{abs}(subs(x = pt, f1)) < 1$ **then**

$Ls := [op(Ls), pt]$: *# if pt, is stable we add it to the list of stable points*

fi:

od:

Ls : *#The last line is the output*

end:

> $f1 := diff(x^3 - 6 \cdot x^2 + 12 \cdot x - 6, x)$

$$f1 := 3x^2 - 12x + 12 \tag{2}$$

> $eval(f1, x = 1)$

$$3 \tag{3}$$

> $eval(f1, x = 2)$

$$0 \tag{4}$$

> $eval(f1, x = 3)$

$$3 \tag{5}$$

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> # X=2 is a stable fixed point, -1<f1<1
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> $SFPe\left(x^4 - \frac{13\cdot x^2}{36} + x + \frac{1}{36}, x\right)$

$$\left[\left[\frac{1}{3}, \frac{49}{54}\right], \left[-\frac{1}{2}, \frac{31}{36}\right], \left[\frac{1}{2}, \frac{41}{36}\right], \left[-\frac{1}{3}, \frac{59}{54}\right]\right] \quad (6)$$

> $f2 := diff\left(x^4 - \frac{13\cdot x^2}{36} + x + \frac{1}{36}, x\right)$

$$f2 := 4\,x^3 - \frac{13}{18}\,x + 1 \quad (7)$$

> $eval\left(f2, x = \frac{1}{3}\right)$

$$\frac{49}{54} \quad (8)$$

> $eval\left(f2, x = -\frac{1}{2}\right)$

$$\frac{31}{36} \quad (9)$$

> $eval\left(f2, x = \frac{1}{2}\right)$

$$\frac{41}{36} \quad (10)$$

> $eval\left(f2, x = -\frac{1}{3}\right)$

$$\frac{59}{54} \quad (11)$$

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> # x = 1/3, x = -1/2 are stable fixed points

  # Question 2
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> $with(DynamicSystems):$

\# **(i)**

> $fx1 := diff(\mathrm{sqrt}(x + 4\cdot y),\ x)$

$$fx1 := \frac{1}{2\sqrt{x + 4\,y}} \quad (12)$$

> $fy1 := diff(\mathrm{sqrt}(x + 4\cdot y),\ y)$

$$fy1 := \frac{2}{\sqrt{x + 4\,y}} \quad (13)$$

> $lin1 := 1 + eval(fx1, x = 1) + eval(fy1, y = 2)$

$$lin1 := 1 + \frac{1}{2\sqrt{1 + 4\,y}} + \frac{2}{\sqrt{x + 8}} \quad (14)$$

> $lin1b := \mathrm{eval}(lin1, x = 0.95)$

$$lin1b := 1.668526271 + \frac{1}{2\sqrt{1 + 4y}} \quad \textbf{(15)}$$

> $lin1c := \mathrm{eval}(lin1b, y = 1.02)$

$$lin1c := 1.890365398 \quad \textbf{(16)}$$

> #(ii)

> $fx2 := \mathrm{diff}(x^3 \cdot y^4 \cdot z^5, x)$

$$fx2 := 3\,x^2\,y^4\,z^5 \quad \textbf{(17)}$$

> $fy2 := \mathrm{diff}(x^3 \cdot y^4 \cdot z^5, y)$

$$fy2 := 4\,x^3\,y^3\,z^5 \quad \textbf{(18)}$$

>

>

> $fz2 := \mathrm{diff}(x^3 \cdot y^4 \cdot z^5, z)$

$$fz2 := 5\,x^3\,y^4\,z^4 \quad \textbf{(19)}$$

> $lin2 := 1 + \mathrm{eval}(fx2, x = 1) + \mathrm{eval}(fy2, y = 1) + \mathrm{eval}(fz2, z = 1)$

$$lin2 := 3\,y^4\,z^5 + 4\,x^3\,z^5 + 5\,x^3\,y^4 + 1 \quad \textbf{(20)}$$

> $lin2b := \mathrm{eval}(lin2, x = 1.01)$

$$lin2b := 3\,y^4\,z^5 + 4.121204\,z^5 + 5.151505\,y^4 + 1 \quad \textbf{(21)}$$

> $lin2c := \mathrm{eval}(lin2b, y = 1.02)$

$$lin2c := 7.36850048\,z^5 + 6.576154684 \quad \textbf{(22)}$$

> $lin2d := \mathrm{eval}(lin2c, z = 0.99)$

$$lin2d := 13.58352532 \quad \textbf{(23)}$$

> #(iii)

>

> $fa3 := \mathrm{diff}(\mathrm{sqrt}(a + b + c + d), a)$

$$fa3 := \frac{1}{2\sqrt{a + b + c + d}} \quad \textbf{(24)}$$

> $fb3 := \mathrm{diff}(\mathrm{sqrt}(a + b + c + d), b)$

$$fb3 := \frac{1}{2\sqrt{a + b + c + d}} \quad \textbf{(25)}$$

> $fc3 := \mathrm{diff}(\mathrm{sqrt}(a + b + c + d), c)$

$$fc3 := \frac{1}{2\sqrt{a + b + c + d}} \quad \textbf{(26)}$$

> $fd3 := \mathrm{diff}(\mathrm{sqrt}(a + b + c + d), d)$

$$fd3 := \frac{1}{2\sqrt{a + b + c + d}} \quad \textbf{(27)}$$

> $lin3 := 1 + \mathrm{eval}(fa3, a = 1) + \mathrm{eval}(fb3, b = 1) + \mathrm{eval}(fc3, c = 1) + \mathrm{eval}(fd3, d = 1)$

(28)

$$lin3 := 1 + \frac{1}{2\sqrt{1+b+c+d}} + \frac{1}{2\sqrt{a+1+c+d}} + \frac{1}{2\sqrt{a+b+1+d}} + \frac{1}{2\sqrt{a+b+c+1}} \quad \textbf{(28)}$$

> lin3b := eval(lin3, a = 1.01)

$$lin3b := 1 + \frac{1}{2\sqrt{1+b+c+d}} + \frac{1}{2\sqrt{2.01+c+d}} + \frac{1}{2\sqrt{2.01+b+d}} + \frac{1}{2\sqrt{2.01+b+c}} \quad \textbf{(29)}$$

> lin3c := eval(lin3b, b = 1.01)

$$lin3c := 1 + \frac{1}{\sqrt{2.01+c+d}} + \frac{1}{2\sqrt{3.02+d}} + \frac{1}{2\sqrt{3.02+c}} \quad \textbf{(30)}$$

> lin3d := eval(lin3c, c = 0.99)

$$lin3d := 1.249688085 + \frac{1}{\sqrt{3.00+d}} + \frac{1}{2\sqrt{3.02+d}} \quad \textbf{(31)}$$

> lin3e := eval(lin3d, d = 0.99)

$$lin3e := 2.000002344 \quad \textbf{(32)}$$

> **# Question 3**

> $eq3f := \frac{x}{y+1}$

$$eq3f := \frac{x}{y+1} \quad \textbf{(33)}$$

> $eq3g := \frac{y}{x+1}$

$$eq3g := \frac{y}{x+1} \quad \textbf{(34)}$$

> with(LinearAlgebra) :

> JacMatrix := Matrix([[diff(eq3f, x), diff(eq3f, y)], [diff(eq3g, x), diff(eq3g, y)]])

$$JacMatrix := \begin{bmatrix} \frac{1}{y+1} & -\frac{x}{(y+1)^2} \\ -\frac{y}{(x+1)^2} & \frac{1}{x+1} \end{bmatrix} \quad \textbf{(35)}$$

> eval(JacMatrix, [x = 1, y = 1])

$$\begin{bmatrix} \frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{2} \end{bmatrix} \quad \textbf{(36)}$$

> **# Question 4**

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> eq4f := x + y + z
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$$eq4f := x + y + z \tag{37}$$

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> eq4g := x^2 + y^2 + z^2
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$$eq4g := x^2 + y^2 + z^2 \tag{38}$$

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> eq4h := x^3 + y^3 + z^3
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$$eq4h := x^3 + y^3 + z^3 \tag{39}$$

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> JacMatrix2 := Matrix([[diff(eq4f, x), diff(eq4f, y), diff(eq4f, z)], [diff(eq4g, x), diff(eq4g,
    y), diff(eq4g, z)], [diff(eq4h, x), diff(eq4h, y), diff(eq4h, z)]])
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$$JacMatrix2 := \begin{bmatrix} 1 & 1 & 1 \\ 2x & 2y & 2z \\ 3x^2 & 3y^2 & 3z^2 \end{bmatrix} \tag{40}$$

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> eval(JacMatrix2, [x = 1, y = 1, z = 1])
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$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix} \tag{41}$$

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> # Question 5
> # By nature of eigenvalues, a fixed point must be stable if all
  eigenvalues of the Jacobian matrix have an absolute value less than
  one. Thinking about this graphically, all points would shrink
  towards and stabilize on the fixed point rather than increasing out
  of control caused by an eigenvalue greater than one.
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Question 5

$$f(x,y) \approx x_0 + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$$

↳ for (x,y) near (x_0,y_0)

$$g(x,y) \approx y_0 + g_x(x_0,y_0)(x-x_0) + g_y(x_0,y_0)(y-y_0)$$

$$x_0 = f(x_0,y_0)$$
$$y_0 = g(x_0,y_0)$$

*

$$\begin{bmatrix} f_x(x_0,y_0) & f_y(x_0,y_0) \\ g_x(x_0,y_0) & g_y(x_0,y_0) \end{bmatrix}$$

↳ stable if all $\lambda < 1$