1.

i. $x \to x^3 - 6x^2 + 12x - 6$

$f'(x) = 3x^2 - 18x + 12$

$x = x^3 - 6x^2 + 12x - 6$

$0 = x^3 - 6x^2 + 11x - 6$

$0 = (x-1)(x-2)(x-3)$

$x = 1 \quad x = 2 \quad x = 3$

$f'(1) = -3 \to$ Stable

$f'(2) = 6 \to$ Unstable

$f'(3) = 21 \to$ Unstable

ii. $x \to x^4 - \frac{13}{36}x^2 + x + \frac{1}{36}$

$g'(x) = 4x^3 - 13/18x + 1$

$x = x^4 - 13/36x^2 + x + \frac{1}{36}$

$0 = x^4 - 13/36x^2 + \frac{1}{36}$

$= 36x^4 - 13x^2 + 1$

$x = \pm\frac{1}{2} \quad x = \pm\frac{1}{3}$

$f'(\frac{1}{2}) = 41/36 \to$ Unstable

$f'(-\frac{1}{2}) = 31/36 \to$ Stable

$f'(\frac{1}{3}) = 49/54 \to$ Stable

$f'(-\frac{1}{3}) = 59/54 \to$ unstable

2.

i. $f(x,y) = (x+4y)^{\frac{1}{2}}$ at $(1,2)$

$f_x(x,y) = \frac{1}{2}(x+4y)^{-\frac{1}{2}}$

$f_y(x,y) = 2(x+4y)^{-\frac{1}{2}}$

$f(x,y) \approx f(1,2) + f_x(1,2)(x-1) + f_y(1,2)(y-2)$

$f(x,y) = 3 + \frac{1}{6}(x-1) + \frac{2}{3}(y-2)$

$f(0.95, 1.02) \approx 3 + \frac{1}{6}(-0.05) + \frac{2}{3}(-0.98) \approx 2.338$

ii. $f(x,y,z) = x^3 y^4 z^5$ at $(1,1,1)$

$f_x(x,y,z) = 3x^2 y^4 z^5$

$f_y(x,y,z) = 4x^3 y^3 z^5$

$f_z(x,y,z) = 5x^3 y^4 z^4$

$f(x,y,z) \approx 1 + 3(x-1) + 4(y-1) + 5(z-1)$

$f(1.01, 1.02, 0.99) \approx 1.05$

iii. $f(x_1, x_2, x_3, x_4) = (x_1 + x_2 + x_3 + x_4)^{\frac{1}{2}}$ at $(1,1,1,1)$

$f_x = \frac{1}{2}(x_1 + x_2 + x_3 + x_4)^{-\frac{1}{2}}$

$f(x_1, x_2, x_3, x_4) = 2 + \frac{1}{8}(x_1 - 1) + \frac{1}{8}(x_2 - 1) + \frac{1}{8}(x_3 - 1) + \frac{1}{8}(x_4 -$

$f(1.01, 1.01, 0.99, 0.99) \approx 0.5$

3. a. $(x, y) \to \left(\frac{x}{y+1}, \frac{y}{x+1}\right)$ $\quad x = \frac{x}{y+1} \quad y = \frac{y}{x+1}$

$$\begin{bmatrix} \frac{1}{y+1} & -\frac{x}{(y+1)^2} \\ -\frac{y}{(x+1)^2} & \frac{1}{x+1} \end{bmatrix}$$

4. $x = x + y + z \quad y = x^2 + y^2 + z^2 \quad z = x^3 + y^3 + z^3$

$$\begin{bmatrix} \frac{dx}{dx} & \frac{dx}{dy} & \frac{dx}{dz} \\ \frac{dy}{dx} & \frac{dy}{dy} & \frac{dy}{dz} \\ \frac{dz}{dx} & \frac{dz}{dy} & \frac{dz}{dz} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 2x & 2y & 2z \\ 3x^2 & 3y^2 & 3z^2 \end{bmatrix} \text{ at } (1,1,1) = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$$

5. A stable fixed point needs to have a derivative of less than 1 to be stable, thus the eigenvalues of the Jacobian would be less than 1.