OK to post

1.

i. $x \to x^3 - 6x^2 + 12x - 6$

$f'(x) = 3x^2 - 18x + 12$

$x = x^3 - 6x^2 + 12x - 6$

$0 = x^3 - 6x^2 + 11x - 6$

$0 = (x-1)(x-2)(x-3)$

$x = 1, 2, 3$

$f'(1) = -3$ stable

$f'(2) = 6$ unstable

$f'(3) = 21$ unstable

ii. $x \to x^4 - \frac{13}{36}x^2 + x + \frac{1}{36}$

$f'(x) = 4x^3 - \frac{13}{18}x + 1$

$x = x^4 - \frac{13}{36}x^2 + x + \frac{1}{36}$

$0 = x^4 - \frac{13}{36}x^2 + \frac{1}{36}$

$0 = 36x^4 - 13x^2 + 1$

$x = \pm\frac{1}{2}, \pm\frac{1}{3}$

$f'(\frac{1}{2}) = \frac{41}{36}$ unstable

$f'(-\frac{1}{2}) = \frac{31}{36}$ stable

$f'(\frac{1}{3}) = \frac{49}{54}$ stable

$f'(-\frac{1}{3}) = \frac{59}{54}$ unstable

2. i. $f(x,y) = (x+4y)^{\frac{1}{2}}$ at $(1,2)$

$f_x(x,y) = \frac{1}{2}(x+4y)^{-\frac{1}{2}}$

$f_y(x,y) = 2(x+4y)^{-\frac{1}{2}}$

$f(x,y) \approx f(1,2) + f_x(1,2)(x-1) + f_y(1,2)(y-2)$

$f(x,y) \approx 3 + \frac{1}{6}(x-1) + \frac{2}{3}(y-2)$

$f(x,y) \approx \frac{1}{6}x + \frac{2}{3}y + \frac{3}{2}$

$f(0.95, 1.02) \approx \frac{1}{6}(0.95) + \frac{2}{3}(1.02) + \frac{3}{2}$

$f(0.95, 1.02) \approx 2.338\overline{3}$

ii. $f(x,y,z) = x^3y^4z^5$ at $(1,1,1)$

$f_x(x,y,z) = 3x^2y^4z^5$

$f_y(x,y,z) = 4x^3y^3z^5$

$f_z(x,y,z) = 5x^3y^4z^4$

$f(x,y,z) \approx 1 + 3(x-1) + 4(y-1) + 5(z-1)$

$f(1.01, 1.02, 0.99) \approx 1.06$

iii. $f(x_1, x_2, x_3, x_4) = (x_1 + x_2 + x_3 + x_4)^{\frac{1}{2}}$ at $(1,1,1,1)$

$f_{x_{1234}}(x_1, x_2, x_3, x_4) = \frac{1}{2}(x_1 + x_2 + x_3 + x_4)^{-\frac{1}{2}}$

$f(x_1, x_2, x_3, x_4) \approx 2 + \frac{1}{8}(x_1 - 1) + \frac{1}{8}(x_2 - 1) + \frac{1}{8}(x_3 - 1) + \frac{1}{8}(x_4 - 1)$

$f(x_1, x_2, x_3, x_4) \approx \frac{1}{8}x_1 + \frac{1}{8}x_2 + \frac{1}{8}x_3 + \frac{1}{8}x_4 + \frac{3}{2}$

$f(1.01, 1.01, 0.99, 0.99) \approx 0.5$

3. $(x, y) \to \left(\frac{x}{y+1}, \frac{y}{x+1}\right)$

$x = \frac{x}{y+1} \qquad y = \frac{y}{x+1}$

$$\begin{bmatrix} \frac{1}{y+1} & -\frac{x}{(y+1)^2} \\ -\frac{y}{(x+1)^2} & \frac{1}{x+1} \end{bmatrix}$$

4. $x = x + y + z \qquad y = x^2 + y^2 + z^2 \qquad z = x^3 + y^3 + z^3$

$$\begin{bmatrix} \frac{\partial x}{\partial x} & \frac{\partial x}{\partial y} & \frac{\partial x}{\partial z} \\ \frac{\partial y}{\partial x} & \frac{\partial y}{\partial y} & \frac{\partial y}{\partial z} \\ \frac{\partial z}{\partial x} & \frac{\partial z}{\partial y} & \frac{\partial z}{\partial z} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 2x & 2y & 2z \\ 3x^2 & 3y^2 & 3z^2 \end{bmatrix}$$

at $(1,1,1)$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$$

5. For a fixed point to be stable, the value of the derivative at that point needs to be Less than 1. Since the Jacobian matrix deals with derivatives, it makes sense that finding the eigenvalues of a stable fixed point would be Less than 1.