Hrudai B. Hw 12

1) i) $f(x) = x^3 - 6x^2 + 12x - 6 \to x$ $= x$ $= x^3 - 6x^2 + 11x - 6 = (x-1)(x-2)(x-3)$ $x = 1, 2, 3$

$f'(x) = 3x^2 - 12x + 12$. $f'(1) = 3$, $f'(2) = 0$, $f'(3) = 3$ $x = 2$ stable

ii) $x \to x^4 - \frac{13x^2}{36} + x + \frac{1}{36} - x = 36x^4 + 13x^2 + 1 = x = \pm\frac{1}{2}, \pm\frac{1}{3}$

$x = -1/2, -1/3$ stable f.p

$f'(x) = 4x^3 + \frac{13x}{18} + 1$ $f'(\frac{1}{2}) = \frac{41}{36}$ $f'(-\frac{1}{2}) = \frac{5}{36}$ $f'(\frac{1}{3}) = \frac{25}{18}$ $f'(\frac{-1}{3}) = \frac{11}{18}$

2) i) $f(x,y) = \sqrt{x+4y}$ $f_x(x,y) = \frac{1}{2}(x+4y)^{-1/2}$ $f_y(x,y) = 2(x+4y)^{-1/2}$ $(1, 2)$

$f(x,y) \approx 3 + \frac{1}{6}(x-1) + \frac{2}{3}(y-2)$

$f(0.95, 1.02) \approx 2.3833\ldots$ $f(0.95, 1.02) = 1.41$

ii) $f(x,y,z) = x^3y^4z^5$ $f_x = 3x^2y^4z^5$ $f_y = 4x^3y^3z^5$ $f_z = 5x^3y^4z^4$ $(1,1,1)$

$f(x,y,z) \approx 1 + 3(x-1) + 4(y-1) + 5(z-1)$

$f(1.01, 1.02, 0.99) \approx 1.06$ $f(\ldots) = 1.061$

iii) $f_{x_i}() = \frac{1}{2}(x_1 + x_2 + x_3 + x_4)^{-1/2}$ $\leftarrow$ for $x_1 \ldots x_4$ $(1,1,1,1)$

$f(x_1, x_2, x_3, x_4) \approx 2 + \frac{1}{4}(x_1 - 1) + \frac{1}{4}(x_2 - 1)) + \frac{1}{4}(x_3 - 1) + \frac{1}{4}(x_4 - 1)$

$f(1.01, 1.01, 0.99, 0.99) \approx 2$ $f(\ldots) = 2$

3) $(x,y) \Rightarrow \left(\frac{x}{y+1}, \frac{y}{x+1}\right)$ $\begin{matrix} f_{1x} & f_{1y} \\ f_{2x} & f_{2y} \end{matrix} = \begin{bmatrix} \frac{1}{y+1} & \frac{-1x}{(y+1)^2} \\ \frac{-y}{(x+1)^2} & \frac{1}{x+1} \end{bmatrix}$ $(1,1)$ $= \begin{bmatrix} \frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{2} \end{bmatrix}$ #

4) $(x,y,z) \to (x+y+z, x^2+y^2+z^2, x^3+y^3+z^3)$

$x = x+y+z$

$y = x^2+y^2+z^2$

$z = x^3+y^3+z^3$

$= \begin{bmatrix} 1 & 1 & 1 \\ 2x & 2y & 2z \\ 3x^2 & 3y^2 & 3z^2 \end{bmatrix}$ $(1,1,1)$ $= \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$ #

5) A fixed point such that $(x,y) \to (f(x,y), g(x,y))$ is a point in R^2 is a stable fixed point under the condition that the derivative of this function has a value $-1 < f'(x_0,y_0) < 1$ for the fixed point (x_0,y_0). For a jacobian matrix of a 2×2 matrix whose eigenvalues with absolute value less than 1 satisfies the criteria for a stable fixed point. Each item in the matrix is a derivative inputted with the point x_0,y_0. The eigen values for this matrix are a sufficient condition for stability.