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> #Deven Singh
  # Assignment 12
  read `/Users/Deven/Desktop/Fall 2021/Dynamic Models of Biology/DMB.txt` :
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First Written: Nov. 2021

This is DMB.txt, A Maple package to explore Dynamical models in Biology (both discrete and continuous)
accompanying the class Dynamical Models in Biology, Rutgers University. Taught by Dr. Z. (Doron Zeilbeger)

The most current version is available on WWW at:
http://sites.math.rutgers.edu/~zeilberg/tokhniot/DMB.txt .
Please report all bugs to: DoronZeil at gmail dot com .

For general help, and a list of the MAIN functions,
type "Help();". For specific help type "Help(procedure_name);"

For a list of the supporting functions type: Help1();
For help with any of them type: Help(ProcedureName);

For a list of the functions that give examples of Discrete-time dynamical systems (some famous), type: HelpDDM();
For help with any of them type: Help(ProcedureName);

For a list of the functions continuous-time dynamical systems (some famous) type: HelpCDM();
For help with any of them type: Help(ProcedureName);

------------------------------ **(1)**

```
> #Q1
> #(i)
> evalf(FP([x^3 - 6·x^2 + 11·x - 5], [x]));
```

$\{[1.], [1.381966012], [3.618033988]\}$ **(2)**

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> SFP([x^3 - 6·x^2 + 11·x - 5], [x]);
```

$\{[1.381966012]\}$ **(3)**

```
> # Only 1.38 is a stable fixed point
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> #(ii)

> $evalf\left(FP\left(\left[x^4 - \frac{13\,x^2}{35} + x + \frac{1}{35}\right], [x]\right)\right);$

$$\{[-0.5124892307], [-0.3298232254], [0.3298232254], [0.5124892307]\} \quad \textbf{(4)}$$

> $SFP\left(\left[x^4 - \frac{13\,x^2}{35} + x + \frac{1}{35}\right], [x]\right);$

$$\{[-0.5124892307], [0.3298232254]\} \quad \textbf{(5)}$$

> # -.51 and .33 are stable fixed points

> #Q2

> #(i)

> $diff(\mathrm{sqrt}(x + 3\,y), x);$

$$\frac{1}{2\sqrt{x+3\,y}} \quad \textbf{(6)}$$

> $f := (x, y) \rightarrow \frac{1}{2\cdot \mathrm{sqrt}(x + 3\,y)};$

$$f := (x, y) \mapsto \frac{1}{2\cdot\sqrt{x+3\cdot y}} \quad \textbf{(7)}$$

> $f(1, 2);$

$$\frac{\sqrt{7}}{14} \quad \textbf{(8)}$$

> $diff(\mathrm{sqrt}(x + 3\,y), y);$

$$\frac{3}{2\sqrt{x+3\,y}} \quad \textbf{(9)}$$

> $g := (x, y) \rightarrow \frac{2}{\mathrm{sqrt}(x + 3\,y)};$

$$g := (x, y) \mapsto \frac{2}{\sqrt{x+3\cdot y}} \quad \textbf{(10)}$$

> $g(1, 2);$

$$\frac{2\sqrt{7}}{7} \quad \textbf{(11)}$$

> $h := (x, y) \rightarrow \mathrm{sqrt}(x + 3\,y);$

$$h := (x, y) \mapsto \sqrt{x+3\cdot y} \quad \textbf{(12)}$$

> $h(1, 2);$

$$\sqrt{7} \quad \textbf{(13)}$$

> $L1 := (x, y) \rightarrow \mathrm{sqrt}(7) + \frac{\mathrm{sqrt}(7)}{14}\cdot(x - 1) + \frac{2\cdot \mathrm{sqrt}(7)}{7}\cdot(y - 2);$

$$L1 := (x, y) \mapsto \sqrt{7} + \frac{\sqrt{7}\cdot(x-1)}{14} + \frac{2\cdot\sqrt{7}\cdot(y-2)}{7} \quad \textbf{(14)}$$

> $evalf(L1(.95, 1.02));$

$$1.895491832 \quad (15)$$

> $h(.95, 1.02);$

$$2.002498439 \quad (16)$$

> $\% - \%\%$

$$0.107006607 \quad (17)$$

> *# The value of the linearization at (.95,1.02) is approximately .11 less than the value of the original function*

> *#(ii)*

> $f1 := (x, y, z) \rightarrow x^4 y^5 z^6$

$$f1 := (x, y, z) \mapsto x^4 \cdot y^5 \cdot z^6 \quad (18)$$

> $f1(1, 1, 1);$

$$1 \quad (19)$$

> $diff(x^4 y^5 z^6, x);$

$$4\, x^3\, y^5\, z^6 \quad (20)$$

> $f2 := (x, y, z) \rightarrow 4\, x^3 y^5 z^6$

$$f2 := (x, y, z) \mapsto 4 \cdot x^3 \cdot y^5 \cdot z^6 \quad (21)$$

> $f2(1, 1, 1);$

$$4 \quad (22)$$

> $diff(x^4 y^5 z^6, y);$

$$5\, x^4\, y^4\, z^6 \quad (23)$$

> $f3 := (x, y, z) \rightarrow 5\, x^4 y^4 z^6$

$$f3 := (x, y, z) \mapsto 5 \cdot x^4 \cdot y^4 \cdot z^6 \quad (24)$$

> $f3(1, 1, 1);$

$$5 \quad (25)$$

> $diff(x^4 y^5 z^6, z);$

$$6\, x^4\, y^5\, z^5 \quad (26)$$

> $f4 := (x, y, z) \rightarrow 6\, x^4 y^5 z^5$

$$f4 := (x, y, z) \mapsto 6 \cdot x^4 \cdot y^5 \cdot z^5 \quad (27)$$

> $f4(1, 1, 1);$

$$6 \quad (28)$$

> $L2 := (x, y, z) \rightarrow 1 + 4 \cdot (x - 1) + 5 \cdot (y - 1) + 6 \cdot (z - 1);$

$$L2 := (x, y, z) \mapsto -14 + 4 \cdot x + 5 \cdot y + 6 \cdot z \quad (29)$$

> $L2(1.01, 1.02, .99);$

$$1.08 \quad (30)$$

> $f1(1.01, 1.02, .99);$

$$1.081676816 \quad (31)$$

> $\% - \%\%$

$$0.001676816 \quad (32)$$

> *# The value of the linearization at (1.01,1.02,.99) is .002 less than the value of the original function at the same point*

> *#(iii)*

> $g1 := (a, b, c, d) \rightarrow \mathrm{sqrt}(a + b + c - d);$

$$g1 := (a, b, c, d) \mapsto \sqrt{a + b + c - d} \quad (33)$$

> $g1(1, 1, 1, 1);$

$$\sqrt{2} \quad (34)$$

> $\mathit{diff}(\mathrm{sqrt}(a + b + c - d), a);$

$$\frac{1}{2\sqrt{a + b + c - d}} \quad (35)$$

> $\mathit{diff}(\mathrm{sqrt}(a + b + c - d), b);$

$$\frac{1}{2\sqrt{a + b + c - d}} \quad (36)$$

> $\mathit{diff}(\mathrm{sqrt}(a + b + c - d), c);$

$$\frac{1}{2\sqrt{a + b + c - d}} \quad (37)$$

> $\mathit{diff}(\mathrm{sqrt}(a + b + c - d), d);$

$$-\frac{1}{2\sqrt{a + b + c - d}} \quad (38)$$

> $g2 := (a, b, c, d) \rightarrow \frac{1}{2 \cdot \mathrm{sqrt}(a + b + c - d)};$

$$g2 := (a, b, c, d) \mapsto \frac{1}{2 \cdot \sqrt{a + b + c - d}} \quad (39)$$

> $g2(1, 1, 1, 1);$

$$\frac{\sqrt{2}}{4} \quad (40)$$

> $g3 := (a, b, c, d) \rightarrow -\frac{1}{2 \cdot \mathrm{sqrt}(a + b + c - d)};$

$$g3 := (a, b, c, d) \mapsto -\frac{1}{2 \cdot \sqrt{a + b + c - d}} \quad (41)$$

> $g3(1, 1, 1, 1);$

$$-\frac{\sqrt{2}}{4} \quad (42)$$

> $L3 := (a, b, c, d) \rightarrow \mathrm{sqrt}(2) + \frac{\mathrm{sqrt}(2)}{4} \cdot (a - 1) + \frac{\mathrm{sqrt}(2)}{4} \cdot (b - 1) + \frac{\mathrm{sqrt}(2)}{4} \cdot (c - 1) - \frac{\mathrm{sqrt}(2)}{4} \cdot (d - 1);$

$$L3 := (a,b,c,d) \mapsto \sqrt{2} + \frac{\sqrt{2}\cdot(a-1)}{4} + \frac{\sqrt{2}\cdot(b-1)}{4} + \frac{\sqrt{2}\cdot(c-1)}{4} - \frac{\sqrt{2}\cdot(d-1)}{4} \quad \textbf{(43)}$$

> $evalf(L3(1.01, 1.01, .99, .99));$

$$1.421284630 \quad \textbf{(44)}$$

> $g1(1.01, 1.01, .99, .99);$

$$1.421267040 \quad \textbf{(45)}$$

> $\% - \%\%$

$$-0.000017590 \quad \textbf{(46)}$$

> *#The value of the linearization at (1.01,1.01,.99,.99) is .00002 greater than the value of the original function at the same point*

> #Q3

> $(x,y) \rightarrow \left(\frac{y}{x+1}, \frac{x}{y+1}\right);$

$$(x,y) \mapsto \left(\frac{y}{x+1}, \frac{x}{y+1}\right) \quad \textbf{(47)}$$

> $diff\left(\frac{y}{x+1}, x\right)$

$$-\frac{y}{(x+1)^2} \quad \textbf{(48)}$$

> $h11 := (x,y) \rightarrow -\frac{y}{(x+1)^2}; h11(1,1);$

$$h11 := (x,y) \mapsto -\frac{y}{(x+1)^2}$$

$$-\frac{1}{4} \quad \textbf{(49)}$$

> $diff\left(\frac{y}{x+1}, y\right);$

$$\frac{1}{x+1} \quad \textbf{(50)}$$

> $h12 := (x,y) \rightarrow \frac{1}{x+1}; h12(1,1);$

$$h12 := (x,y) \mapsto \frac{1}{x+1}$$

$$\frac{1}{2} \quad \textbf{(51)}$$

> $diff\left(\frac{x}{y+1}, x\right);$

$$\frac{1}{y+1} \quad \textbf{(52)}$$

> $h21 := (x, y) \to \frac{1}{y+1}; h21(1, 1);$

$$h21 := (x, y) \mapsto \frac{1}{y+1}$$

$$\frac{1}{2} \quad (53)$$

> $diff\left(\frac{x}{y+1}, y\right);$

$$-\frac{x}{(y+1)^2} \quad (54)$$

> $h22 := (x, y) \to -\frac{x}{(y+1)^2}; h22(1, 1);$

$$h22 := (x, y) \mapsto -\frac{x}{(y+1)^2}$$

$$-\frac{1}{4} \quad (55)$$

> *#This is the Jacobian Matrix at (1,1)*
with(LinearAlgebra) :
$J := Matrix\left(\left[\left[-\frac{1}{4}, \frac{1}{2}\right], \left[\frac{1}{2}, -\frac{1}{4}\right]\right]\right);$

$$J := \begin{bmatrix} -\frac{1}{4} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{4} \end{bmatrix} \quad (56)$$

> *#Q4*

> $(x, y, z) \to (x^3 + y^3 + z^3, x^2 + y^2 + z^2, x + y + z);$

$$(x, y, z) \mapsto (x^3 + y^3 + z^3, x^2 + y^2 + z^2, x + y + z) \quad (57)$$

> $diff(x^3 + y^3 + z^3, x);$

$$3\,x^2 \quad (58)$$

> $diff(x^3 + y^3 + z^3, y);$

$$3\,y^2 \quad (59)$$

> $diff(x^3 + y^3 + z^3, z);$

$$3\,z^2 \quad (60)$$

> *# The partial derivatives at (1,1,1) are 3,3, and 3 respectively*

> $diff(x^2 + y^2 + z^2, x);$

$$2\,x \quad (61)$$

> $diff(x^2 + y^2 + z^2, y);$

$$2\,y \quad (62)$$

> $diff(x^2 + y^2 + z^2, z);$

$$2\,z \tag{63}$$

> *# The partial derivatives at (1,1,1) are 2,2, and 2 respectively*

> $diff(x + y + z, x);$

$$1 \tag{64}$$

> $diff(x + y + z, y);$

$$1 \tag{65}$$

> $diff(x + y + z, z);$

$$1 \tag{66}$$

> *#This is the Jacobian Matrix*
> $Matrix([[3, 3, 3], [2, 2, 2], [1, 1, 1]]);$

$$\begin{bmatrix} 3 & 3 & 3 \\ 2 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix} \tag{67}$$

> *#Q5*

> *#We can solve for the Jacobian Matrix J's eigenvalues (λ) such that :*
> *#* $J \cdot v = \lambda \cdot v$
> *# where v is [x-x0,y-y0], some linearization of (x,y) -> (f(x,y),g(x,y)) at the fixed point*

> *# If the absolute value of all eigenvalues of J is less than 1, then an initial condition in the environment of the fixed point does not make the value of the function stray too far from the fixed point. Therefore, the fixed point is stable.*

>