Homework 12

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You my post my answers.

1. $f(x) = \sin(x+x^2)$

$f'(x) = (2x+1)\cdot\cos(x+x^2)$

$f''(x) = 2\cos(x+x^2) - (2x+1)^2 \sin(x+x^2)$

$\frac{f(0)}{0!}\cancel{x^0} + \frac{f'(0)}{1!}\cdot\cancel{x^1} + \frac{f''(0)}{2!}\cancel{x^2}$

$= \frac{\sin(0)}{1}\cdot 1 + \frac{(1)\cos(0)}{1}\cdot x + \frac{2\cos(0)-(1)^2\sin(0)}{2}\cdot x^2$

$= 0 + x + x^2$

2. $\sin(z) = z - \frac{1}{3!}z^3 + \frac{1}{5!}z^5 - \frac{1}{7!}z^7 + \ldots$

$\sin(x+x^2) = (x+x^2) - \frac{1}{3!}(x+x^2)^3 + \frac{1}{5!}(x+x^2)^5 - \frac{1}{7!}(x+x^2)^7 \ldots$

$= (x+x^2) - \frac{1}{6}(x+x^2)^3 + \frac{1}{120}(x+x^2)^5 - \frac{1}{5040}(x+x^2)^7 \ldots$

$= x + x^2 - \frac{x^3}{6} - \frac{x^4}{2} + \ldots$

3. $\arctan(x) = \int_0^x \frac{1}{1+t^2}\,dt$; $\frac{1}{1-z} = \sum_{n=0}^{\infty} z^n$

$f'(x) = \frac{1}{1+x^2} = \frac{1}{1-(-x^2)} = \sum_{n=0}^{\infty}(-x^2)^n$

$= \sum_{n=0}^{\infty}(-1)^n x^{2n}$

$f(x) = \int \sum_{n=0}^{\infty}(-1)^n x^{2n}\,dx = \sum_{n=0}^{\infty}\int(-1)^n x^{2n}\,dx$

$= \sum_{n=1}^{\infty}(-1)^n\left(\frac{x^{2n+1}}{2n+1}\right) + C$

$f(0) = \sum_{n=1}^{\infty}(-1)^n\left(\frac{0^{2n+1}}{2n+1}\right) + C$; $c = 0$

$f(x) = \sum_{n=1}^{\infty}(-1)^n\left(\frac{x^{2n+1}}{2n+1}\right)$

4. $\tan(\alpha+\beta) = \tan(\alpha)+\tan(\beta) / 1-(\tan(\alpha)(\tan(\beta))$

$\arctan(x) = \alpha \quad , \quad x = \tan(\alpha)$

$\arctan(y) = \beta \quad , \quad y = \tan(\beta)$

$\tan(\alpha+\beta) = \tan(\alpha)+\tan(\beta) / 1-\tan(\alpha)\tan(\beta)$

$= x+y / 1-xy$

$\alpha+\beta = \arctan(x+y / 1-xy)$

$\arctan(x)+\arctan(y) = \arctan(x+y / 1-xy)$

5. Using above formula,

$\arctan(1/2)+\arctan(1/3) = \arctan(1/2 + 1/3 / 1-(1/2)(1/3))$

$= \arctan(5/6 / 1-1/6)$

$= \arctan(5/6 / 5/6)$

$= \arctan(1)$

$= \pi/4$