Hw 12 - Alan Ho

OK to post

1) i) $x \to x^3 - 6x^2 + 12x - 6$

$x = x^3 - 6x^2 + 12x - 6$

$x^3 - 6x^2 + 11x - 6$

$\boxed{x = 1 \quad \text{not stable}}$

$\boxed{\text{Fixed points } x = 1, 2, 3}$

$\boxed{f'(2) = 12 - 24 + 12 = 0 \text{ stable}}$

$\boxed{f'(3) = 3 > 1 \text{ not stable}}$

$f'(x) = 3x^2 - 12x + 12$

$f'(1) = 3 - 12 + 12 = 3 > 1$

ii) $x \to x^4 - \frac{13x^2}{36} + x + \frac{1}{36}$

$x = x^4 - \frac{13x^2}{36} + x + \frac{1}{36}$

$x^4 - \frac{13x^2}{36} + \frac{1}{36}$

$\boxed{x = \frac{1}{3} \quad \text{stable}}$

$f'(x) = 4x^3 - \frac{13x}{18} + 1$

$f'(\frac{1}{3}) = 4(\frac{1}{3})^3 - \frac{13}{18}(\frac{1}{3}) + 1 = \frac{49}{54} < 1$

2) i) $f(x,y) = \sqrt{x+4y}$ @ $(1,2)$

$f_x = \frac{1}{2\sqrt{x+4y}}$

$f_y = \frac{2}{\sqrt{x+4y}}$

$f(x,y) = f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$

$f(1,2) = f(1,2) + f_x(1,2)(1-0.95) + f_y(1,2)(2-1.02)$

$= \sqrt{9} + \frac{1}{2\cdot 3}(0.05) + \frac{2}{3}(0.98)$

$\approx \boxed{3.66}$

$f(1,2) = \boxed{3}$

$\boxed{f(0.95, 1.02) = 2.24}$

ii) $f(x,y,z) = x^3y^4z^5$

$f_x = 3x^2y^4z^5$

$f_y = 4x^3y^3z^5$

$f_z = 5x^3y^4z^4$

$f(1,1,1) \approx 1 + 3(1-1.01) + 4(1-1.02) + 5(1-0.99)$

$\approx \boxed{0.94}$

$f(1,1,1) = 1$

$f(1.01, 1.02, 0.99) = \boxed{1.06}$

iii) $f(x_1,x_2,x_3,x_4) = \sqrt{x_1+x_2+x_3+x_4}$ @ $(1,1,1,1)$

$f_x = f_y = f_z = f_p = \frac{1}{2\sqrt{x_1+x_2+x_3+x_4}}$

$f(1,1,1,1) \approx \sqrt{4} + \frac{1}{(2)(2)}(1-1.01) + \frac{1}{4}(1-1.01) + \frac{1}{4}(1-0.99) + \frac{1}{4}(1-0.99)$

$\approx \boxed{2}$

$f(1.01, 1.01, 0.99, 0.99) = \boxed{2}$

5) If all of it's eigenvalues from the Jacobian Matrix has absolute value < 1 it is stable because eigenvalues are the distance b/w each fixed point and the function, so if they are < 1 the distance is getting smaller i.e. stabilizing to that point.

3) $x = \frac{x}{y+1}$ $\quad y = \frac{y}{x+1}$

$dxx = \frac{1}{y+1}$ $\quad dxy = \frac{-x}{(y+1)^2}$ $\quad J = \begin{bmatrix} \frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{2} \end{bmatrix}$

$dyx = \frac{-y}{(x+1)^2}$ $\quad dyy = \frac{1}{x+1}$

4) $x = x+y+z$ $\quad y = x^2+y^2+z^2$ $\quad z = x^3+y^3+z^3$

$$J = \begin{bmatrix} 1 & 1 & 1 \\ 2x & 2y & 2z \\ 3x^2 & 3y^2 & 3z^2 \end{bmatrix} = J_{(1,1)} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$$