

> #OK to post homework
#Shreya Ghosh, 10-11-2021, Assignment 11 (1)

> read "/Users/shreyaghosh/Documents/M11.txt"
> Help11()
SFPe(f,x), Orbk(k,z,f,INI,K1,K2) (2)

> #1.
SFPe(k·x·(1 - x), x)
$$\left[[0, k], \left[\frac{k-1}{k}, -k+2 \right] \right]$$
 (3)

> #-1 <-k+2 < 1 --> -3 <-k < -1 --> 1 <k < 3 so fixed point and k = 2
SFPe(2·x·(1 - x), x)
$$\left[[0, 2], \left[\frac{1}{2}, 0 \right] \right]$$
 (4)

> Help9()
Orb(f,x,x0,K1,K2), Orb2D(f,x,x0,K), FP(f,x), SFP(f,x), Comp(f,x) (5)

> #2i.
evalf(Orb(3.1·x·(1 - x), x, 0.5, 1000, 1010))
[0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203] (6)

> evalf(Orb(3.2·x·(1 - x), x, 0.5, 1000, 1010))
[0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906] (7)

> evalf(Orb(3.3·x·(1 - x), x, 0.5, 1000, 1010))
[0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832] (8)

> evalf(Orb(3.4·x·(1 - x), x, 0.5, 1000, 1010))
[0.4519632478, 0.8421543994, 0.4519632478, 0.8421543994, 0.4519632478, 0.8421543994, 0.4519632478, 0.8421543994, 0.4519632478, 0.8421543994] (9)

> evalf(Orb(3.5·x·(1 - x), x, 0.5, 1000, 1010))
[0.5008842111, 0.8749972637, 0.3828196827, 0.8269407062, 0.5008842111, 0.8749972637, 0.3828196827, 0.8269407062, 0.5008842111, 0.8749972637, 0.3828196827, 0.8269407062] (10)

> evalf(Orb(3.49·x·(1 - x), x, 0.5, 1000, 1010))
[0.4944462264, 0.8723923530, 0.3885205345, 0.8291274275, 0.4944462264, 0.8723923530, 0.3885205345, 0.8291274275, 0.4944462264, 0.8723923530, 0.3885205345, 0.8291274275] (11)

> evalf(Orb(3.48·x·(1 - x), x, 0.5, 1000, 1010))
[0.4871592795, 0.8694262033, 0.3950644954, 0.8316801189, 0.4871592795, 0.8694262033, 0.3950644954, 0.8316801189, 0.4871592795, 0.8694262033, 0.3950644954, 0.8316801189] (12)

> $evalf(Orb(3.47 \cdot x \cdot (1 - x), x, 0.5, 1000, 1010))$
 $[0.4785612455, 0.8659051179, 0.4029136531, 0.8347926170, 0.4785612455, 0.8659051179,$ (13)
 $0.4029136531, 0.8347926170, 0.4785612455, 0.8659051179, 0.4029136531, 0.8347926170]$

> $evalf(Orb(3.46 \cdot x \cdot (1 - x), x, 0.5, 1000, 1010))$
 $[0.4674861810, 0.8613422665, 0.4132339117, 0.8389518948, 0.4674861810, 0.8613422665,$ (14)
 $0.4132339117, 0.8389518948, 0.4674861810, 0.8613422665, 0.4132339117, 0.8389518948]$

> $evalf(Orb(3.45 \cdot x \cdot (1 - x), x, 0.5, 1000, 1010))$
 $[0.4462251607, 0.8525235201, 0.4337587289, 0.8473617243, 0.4462224221, 0.8525225037,$ (15)
 $0.4337612012, 0.8473628542, 0.4462197139, 0.8525214988, 0.4337636455, 0.8473639713]$

> $evalf(Orb(3.44 \cdot x \cdot (1 - x), x, 0.5, 1000, 1010))$
 $[0.4421929706, 0.8485047151, 0.4421929548, 0.8485047085, 0.4421929706, 0.8485047151,$ (16)
 $0.4421929548, 0.8485047085, 0.4421929706, 0.8485047151, 0.4421929548, 0.8485047085]$

> #the second bifurcation point happens at approximately $k = 3.45$

> #3 (2).
 $SFPe\left(N \cdot \exp\left(r \cdot \left(1 - \frac{N}{K}\right)\right), N\right)$
 $[[0, e^r], [K, e^0 - r e^0]]$ (17)

> # $N = 0$, iff $-1 < K < 1$
 # $N = e^r$, iff $0 < r < 2$

> #3 (3).
 $SFPe(l \cdot N \cdot (1 + a \cdot N)^{-b}, N)$
 $\left[\left[0, l \right], \left[\frac{e^{\frac{\ln(l)}{b}} - 1}{a}, -\frac{l \left(e^{\frac{\ln(l)}{b}} \right)^{-b} \left(b e^{\frac{\ln(l)}{b}} - e^{\frac{\ln(l)}{b}} - b \right)}{e^{\frac{\ln(l)}{b}}} \right] \right]$ (18)

> # $N = 0$, iff $(1-a)^b < l < (1+a)^b$, b is **not equal to 0**, a is **not equal to 0**

> #4i.
 $Orbk\left(1, z, \frac{5}{2} \cdot z[1] \cdot (1 - z[1]), [0.5], 1000, 1010\right)$
 $[0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000,$ (19)
 $0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000]$

> $Orbk\left(2, z, \frac{z[1] + z[2]}{z[1] + z[2]}, [1.1, 5.3], 1000, 1010\right)$
 $[1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]$ (20)

> $Orbk\left(2, z, \frac{z[1] + z[2]}{2 \cdot z[1] + z[2]}, [1.1, 5.3], 1000, 1010\right)$
 $[0.6666666665, 0.6666666665, 0.6666666665, 0.6666666665, 0.6666666665, 0.6666666665,$ (21)

0.6666666665, 0.6666666665, 0.6666666665, 0.6666666665, 0.6666666665]

$$\begin{aligned} &> \text{Orbk}\left(2, z, \frac{z[1] + z[2]}{3 \cdot z[1] + z[2]}, [1.1, 5.3], 1000, 1010\right) \\ &[0.5000000002, 0.4999999998, 0.5000000002, 0.4999999998, 0.5000000002, 0.4999999998, \\ &0.5000000002, 0.4999999998, 0.5000000002, 0.4999999998, 0.5000000002] \end{aligned} \quad (22)$$

$$\begin{aligned} &> \text{Orbk}\left(2, z, \frac{z[1] + z[2]}{4 \cdot z[1] + z[2]}, [1.1, 5.3], 1000, 1010\right) \\ &[0.4000000004, 0.3999999996, 0.4000000004, 0.3999999996, 0.4000000004, 0.3999999996, \\ &0.4000000004, 0.3999999996, 0.4000000004, 0.3999999996, 0.4000000004] \end{aligned} \quad (23)$$

$$\begin{aligned} &> \text{Orbk}\left(2, z, \frac{z[1] + 2 \cdot z[2]}{z[1] + z[2]}, [1.1, 5.3], 1000, 1010\right) \\ &[1.5000000000, 1.5000000000, 1.5000000000, 1.5000000000, 1.5000000000, 1.5000000000, \\ &1.5000000000, 1.5000000000, 1.5000000000, 1.5000000000, 1.5000000000] \end{aligned} \quad (24)$$

$$\begin{aligned} &> \text{Orbk}\left(2, z, \frac{z[1] + 2 \cdot z[2]}{2 \cdot z[1] + z[2]}, [1.1, 5.3], 1000, 1010\right) \\ &[1.0000000000, 1.0000000000, 1.0000000000, 1.0000000000, 1.0000000000, 1.0000000000, \\ &1.0000000000, 1.0000000000, 1.0000000000, 1.0000000000, 1.0000000000] \end{aligned} \quad (25)$$

$$\begin{aligned} &> \text{Orbk}\left(2, z, \frac{z[1] + 2 \cdot z[2]}{3 \cdot z[1] + z[2]}, [1.1, 5.3], 1000, 1010\right) \\ &[0.7500000002, 0.7499999994, 0.7500000002, 0.7499999997, 0.7500000002, 0.7499999994, \\ &0.7500000002, 0.7499999997, 0.7500000002, 0.7499999994, 0.7500000002] \end{aligned} \quad (26)$$

$$\begin{aligned} &> \text{Orbk}\left(2, z, \frac{z[1] + 2 \cdot z[2]}{4 \cdot z[1] + z[2]}, [1.1, 5.3], 1000, 1010\right) \\ &[0.6000000033, 0.5999999967, 0.6000000033, 0.5999999967, 0.6000000033, 0.5999999967, \\ &0.6000000033, 0.5999999967, 0.6000000033, 0.5999999967, 0.6000000033] \end{aligned} \quad (27)$$

$$\begin{aligned} &> \text{Orbk}\left(2, z, \frac{z[1] + 3 \cdot z[2]}{z[1] + z[2]}, [1.1, 5.3], 1000, 1010\right) \\ &[2.0000000000, 2.0000000000, 2.0000000000, 2.0000000000, 2.0000000000, 2.0000000000, \\ &2.0000000000, 2.0000000000, 2.0000000000, 2.0000000000, 2.0000000000] \end{aligned} \quad (28)$$

$$\begin{aligned} &> \text{Orbk}\left(2, z, \frac{z[1] + 3 \cdot z[2]}{2 \cdot z[1] + z[2]}, [1.1, 5.3], 1000, 1010\right) \\ &[1.333333335, 1.333333332, 1.333333335, 1.333333332, 1.333333335, 1.333333332, \\ &1.333333335, 1.333333332, 1.333333335, 1.333333332, 1.333333335] \end{aligned} \quad (29)$$

$$\begin{aligned} &> \text{Orbk}\left(2, z, \frac{z[1] + 3 \cdot z[2]}{3 \cdot z[1] + z[2]}, [1.1, 5.3], 1000, 1010\right) \\ &[1.056238496, 0.9467817065, 1.056180901, 0.9468332585, 1.056123481, 0.9468846597, \\ &1.056066236, 0.9469359103, 1.056009163, 0.9469870121, 1.055952263] \end{aligned} \quad (30)$$

$$\begin{aligned} &> \text{Orbk}\left(2, z, \frac{z[1] + 3 \cdot z[2]}{4 \cdot z[1] + z[2]}, [1.1, 5.3], 1000, 1010\right) \\ &[1.577350271, 0.4226497305, 1.577350271, 0.4226497306, 1.577350271, 0.4226497305, \\ &1.577350271, 0.4226497305, 1.577350271, 0.4226497306, 1.577350271, 0.4226497305] \end{aligned} \quad (31)$$

1.577350271, 0.4226497306, 1.577350271, 0.4226497305, 1.577350271]

$$\begin{aligned} &> \text{Orbk}\left(2, z, \frac{z[1] + 4 \cdot z[2]}{z[1] + z[2]}, [1.1, 5.3], 1000, 1010\right) \\ &[2.500000002, 2.499999998, 2.500000002, 2.499999998, 2.500000002, 2.499999998, \\ &2.500000002, 2.499999998, 2.500000002, 2.499999998, 2.500000002] \end{aligned} \quad (32)$$

$$\begin{aligned} &> \text{Orbk}\left(2, z, \frac{z[1] + 4 \cdot z[2]}{2 \cdot z[1] + z[2]}, [1.1, 5.3], 1000, 1010\right) \\ &[1.666666671, 1.666666662, 1.666666671, 1.666666662, 1.666666671, 1.666666662, \\ &1.666666671, 1.666666662, 1.666666671, 1.666666662, 1.666666671] \end{aligned} \quad (33)$$

$$\begin{aligned} &> \text{Orbk}\left(2, z, \frac{z[1] + 4 \cdot z[2]}{3 \cdot z[1] + z[2]}, [1.1, 5.3], 1000, 1010\right) \\ &[2.366025406, 0.6339745957, 2.366025405, 0.6339745958, 2.366025406, 0.6339745957, \\ &2.366025405, 0.6339745958, 2.366025406, 0.6339745957, 2.366025405] \end{aligned} \quad (34)$$

$$\begin{aligned} &> \text{Orbk}\left(2, z, \frac{z[1] + 4 \cdot z[2]}{4 \cdot z[1] + z[2]}, [1.1, 5.3], 1000, 1010\right) \\ &[2.618033990, 0.3819660111, 2.618033989, 0.3819660110, 2.618033990, 0.3819660111, \\ &2.618033989, 0.3819660110, 2.618033990, 0.3819660111, 2.618033989] \end{aligned} \quad (35)$$

> #4ii.

$$\#x = \frac{(x + ax)}{(bx + x)} = \frac{x(1 + a)}{x(1 + b)}$$

$$\#x^2 = \frac{1 + a}{1 + b}x$$

$$\#x^2 - \frac{1 + a}{1 + b}x = 0$$

$$\#x\left(x - \frac{1 + a}{1 + b}\right) = 0 \rightarrow x = \frac{1 + a}{1 + b}$$

$$\#a = 1, b = 1 \rightarrow \frac{2}{2} = 1$$

$$\#a = 1, b = 2 \rightarrow \frac{2}{3} = 0.666667$$

$$\#a = 2, b = 1 \rightarrow \frac{3}{2} = 1.5$$

$$\#a = 2, b = 2 \rightarrow \frac{2}{2} = 1$$

$$\#a = 3, b = 1 \rightarrow \frac{4}{2} = 2$$

$$\#a = 3, b = 2 \rightarrow \frac{4}{3} = 1.33337$$

> #3 (1).

$$SFPe\left(\frac{l}{a} \cdot N \cdot N^{-b}, N\right)$$



$$\left[[0, 0], \left[e^{-\frac{\ln\left(\frac{a}{l}\right)}{b}}, -\frac{l \left(e^{-\frac{\ln\left(\frac{a}{l}\right)}{b}} \right)^{-b}}{a} (-1 + b) \right] \right]$$

(36)