```
> #Do not post
 #Nikita John, Assigment 11, October 11th, 2021
> #M11.txt: Maple code for Lecture 11 of Dynamical Models in Biology taught by Dr. Z.
Help11 :=proc( ) : print(`SFPe(f,x), Orbk(k,z,f,INI,K1,K2) `) :end:


    #SFPe(f,x): The set of fixed points of x->f(x) done exactly (and allowing symbolic
    parameters), followed by the condition of stability (if it is netween -1 and 1 it is stable)
#Try: FPe(k*x*(1-x),x);
SFPe :=proc( f, x ) local f1, L, i :
f1 := diff ( f, x ) :
L := [solve( f=x, x ) ] :
[seq( [L[i], normal(subs(x=L[i], f1 ) ) ], i=1 ..nops(L) ) ] :

end:


#Added after class

    #Orbk(k,z,f,INI,K1,K2): Given a positive integer k, a letter (symbol), z, an expression f of z
    [1], ..., z[k] (representing a multi-variable function of the variables z[1],...,z[k]

    #a vector INI representing the initial values [x[1],..., x[k]], and (in applications) positive
    integres K1 and K2, outputs the

    #values of the sequence starting at n=K1 and ending at n=K2. of the sequence satisfying the
    difference equation
##x[n]=f(x[n-1],x[n-2],..., x[n-k+1]):

    #This is a generalization to  higher-order difference equation of procedure Orb(f,x,x0,K1,K2)
    . For example
#Orbk(1,z,5/2*z[1]*(1-z[1]),[0.5],1000,1010);  should be the same as
#Orb(5/2*z[1]*(1-z[1]),z[1],[0,5],1000,1010);
#Try:
#Orbk(2,z,(5/4)*z[1]-(3/8)*z[2],[1,2],1000,1010);
Orbk :=proc(k, z, f, INI, K1, K2 ) local L, i, newguy :
L := INI : #We start out with the list of initial values

if not (type(k, integer) and type(z, symbol) and type(INI, list) and nops(INI) = k and type(K1,
    integer) and type(K2, integer) and K1 > 0 and K2 > K1 ) then
    #checking that the input is OK
 print( `bad input`) :
 RETURN(FAIL) :
fi:

while nops(L) < K2 do
 newguy := subs( {seq(z[i]=L[ -i], i=1 ..k) }, f ) :
    #Using what we know about the value yesterday, the day before yesterday, ... up to k days
    before yesterday we find the value of the sequence today
```

```
 L := [op(L), newguy] : #we append the new value to the running list of values of our sequence
od:

[op(K1 ..K2, L)] :

end:


####STAFT FROM M9.txt
#M9.txt: Maple Code for "Dynamical models in Biology" (Math 336) taught by Dr. Z., Lecture 9

Help9 :=proc( ) :
      print(`Orb(f,x,x0,K1,K2), Orb2D(f,x,x0,K) , FP(f,x) , SFP(f,x) , Comp(f,x)  `) :end:


    #Orb(f,x,x0,K1,K2): Inputs an expression f in x (desccribing) a function of x, an initial point,
    x0, and a positive integer K, outputs
#the values of x[n] from n=K1 to n=K2. Try: where x[n]=f(x[n-1]), . Try:
#Orb(2*x*(1-x),x,0.4,1000,2000);
Orb :=proc(f, x, x0, K1, K2) local x1, i, L :
x1 := x0 :
for i from 1 to K1 do
 x1 := subs(x=x1, f) :
    #we don't record the first values of K1, since we are interested in the long-time behavior of
    the orbit
od:

L := [x1] :

for i from K1 to K2 do
 x1 := subs(x=x1, f) :   #we compute the next member of the orbit
 L := [op(L), x1] : #we append it to the list
od:

L : #that's the output

end:

#Orb2D(f,x,x0,K): 2D version of Orb(f,x,x0,0,K), just for illustration
Orb2D :=proc(f, x, x0, K) local L, L1, i :
L := Orb(f, x, x0, 0, K) :
L1 := [[L[1], 0], [L[1], L[2]], [L[2], L[2]] ] :
for i from 3 to nops(L) do
 L1 := [op(L1), [L[i-1], L[i]], [L[i], L[i]]] :
od:
L1 :
end:
```

```
#FP(f,x): The list of fixed points of the map x->f where f is an expression in x. Try:
#FP(2*x*(1-x),x);
FP := proc(f, x)
evalf([solve(f=x)]) :
end:


#SFP(f,x): The list of stable fixed points of the map x->f where f is an expression in x. Try:
#SFP(2*x*(1-x),x);
SFP := proc(f, x) local L, i, f1, pt, Ls :
L := FP(f, x) : #The list of fixed points (including complex ones)

Ls := [ ] :        #Ls is the list of stable fixed points, that starts out as the empty list

f1 := diff(f, x) :  #The derivative of the function f w.r.t. x

for i from 1 to nops(L) do
pt := L[i] :

if abs(subs(x=pt, f1)) < 1 then

 Ls := [op(Ls), pt] :  # if pt,  is stable we add it to the list of stable points

fi:

od:

Ls :   #The last line is the output

end:

#Comp(f,x): f(f(x))
Comp := proc(f, x) : normal(subs(x=f, f)) :end:
```

```
> #1: prove that k·x·(1-x) has one stable fixed point for k < 3
  f := k·x·(1 − x) :
  SFPe(f, x);

      #solving the condition -1< -k + 2 < 3 we get that (k-1)\k is a stable fixed point for k < 3 (
      the work for this is provided separately)
```

$$\left[[0, k], \left[\frac{k-1}{k}, -k+2\right]\right] \quad (1)$$

```
> #2(i)
  f1 := 3.1·x·(1 − x) :
  Orb(f1, x, 0.5, 990, 1000);
```

[0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, **(2)** 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203]

> $f2 := 3.2 \cdot x \cdot (1 - x)$:
$Orb(f2, x, 0.5, 990, 1000)$;

[0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906] **(3)**

> $f3 := 3.3 \cdot x \cdot (1 - x)$:
$Orb(f3, x, 0.5, 990, 1000)$;

[0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832] **(4)**

> $f4 := 3.4 \cdot x \cdot (1 - x)$:
$Orb(f4, x, 0.5, 990, 1000)$;

[0.4519632478, 0.8421543994, 0.4519632478, 0.8421543994, 0.4519632478, 0.8421543994, 0.4519632478, 0.8421543994, 0.4519632478, 0.8421543994, 0.4519632478, 0.8421543994] **(5)**

> $f5 := 3.41 \cdot x \cdot (1 - x)$:
$Orb(f5, x, 0.5, 990, 1000)$;

[0.4494639153, 0.8437912150, 0.4494639177, 0.8437912160, 0.4494639153, 0.8437912150, 0.4494639177, 0.8437912160, 0.4494639153, 0.8437912150, 0.4494639177, 0.8437912160] **(6)**

> $f6 := 3.42 \cdot x \cdot (1 - x)$:
$Orb(f6, x, 0.5, 990, 1000)$;

[0.4470032590, 0.8453944013, 0.4470032600, 0.8453944016, 0.4470032590, 0.8453944013, 0.4470032600, 0.8453944016, 0.4470032590, 0.8453944013, 0.4470032600, 0.8453944016] **(7)**

> $f7 := 3.43 \cdot x \cdot (1 - x)$:
$Orb(f7, x, 0.5, 990, 1000)$;

[0.4445800029, 0.8469651800, 0.4445800122, 0.8469651838, 0.4445800029, 0.8469651800, 0.4445800122, 0.8469651838, 0.4445800029, 0.8469651800, 0.4445800122, 0.8469651838] **(8)**

> $f8 := 3.44 \cdot x \cdot (1 - x)$:
$Orb(f8, x, 0.5, 990, 1000)$;

[0.4421929548, 0.8485047085, 0.4421929706, 0.8485047151, 0.4421929548, 0.8485047085, 0.4421929706, 0.8485047151, 0.4421929548, 0.8485047085, 0.4421929706, 0.8485047151] **(9)**

> $f9 := 3.45 \cdot x \cdot (1 - x)$:
$Orb(f9, x, 0.5, 990, 1000)$;

[0.4337511427, 0.8473582567, 0.4462307332, 0.8525255874, 0.4337537002, 0.8473594259, **(10)** 0.4462279307, 0.8525245479, 0.4337562287, 0.8473605817, 0.4462251607, 0.8525235201]

> #Brownie points 2(ii)

$F := Comp(f, x);$

$$F := -k^2 x(-1+x)(k x^2 - k x + 1) \quad \textbf{(11)}$$

> $SFPe(F, x);$

$$\left[[0, k^2], \left[\frac{k-1}{k}, k^2 - 4k + 4\right], \left[\frac{\frac{k}{2} + \frac{1}{2} + \frac{\sqrt{k^2-2k-3}}{2}}{k}, -\frac{(k^2-2k-3)^{3/2}}{2} + \frac{\sqrt{k^2-2k-3}\, k^2}{2} - k^2 + 2k + 4 - \sqrt{k^2-2k-3}\, k - \frac{3\sqrt{k^2-2k-3}}{2}\right], \left[\frac{\frac{k}{2} + \frac{1}{2} - \frac{\sqrt{k^2-2k-3}}{2}}{k}, \frac{(k^2-2k-3)^{3/2}}{2} - \frac{\sqrt{k^2-2k-3}\, k^2}{2} - k^2 + 2k + 4 + \sqrt{k^2-2k-3}\, k + \frac{3\sqrt{k^2-2k-3}}{2}\right] \right] \quad \textbf{(12)}$$

> #It is assumed that the second bifurcation point is dependant on the fourth fixed point, and the work for determining the bounds for this fixed point is given on paper.

> #3 Model 1

$M1 := \left(\left(\frac{1}{a}\right)(x)^{-b}\right)\cdot(c\cdot x)$: #here c represents lambda

$SFPe(M1, x);$

$$\left[\left[0, \left(\frac{1}{a(0)}\right)^{-b} c\right], \left[RootOf\left(a(\_Z)\, e^{\frac{\ln(c)}{b}} - 1\right), \frac{1}{a\left(RootOf\left(a(\_Z)\, e^{\frac{\ln(c)}{b}} - 1\right)\right)} \left(\left(\frac{1}{a\left(RootOf\left(a(\_Z)\, e^{\frac{\ln(c)}{b}} - 1\right)\right)}\right)^{-b} c \left(\partial \Big/ \left(\partial RootOf\left(a(\_Z)\, e^{\frac{\ln(c)}{b}} - 1\right)\right) a\left(RootOf\left(a(\_Z)\, e^{\frac{\ln(c)}{b}} - 1\right)\right) RootOf\left(a(\_Z)\, e^{\frac{\ln(c)}{b}} - 1\right) b + a\left(RootOf\left(a(\_Z)\, e^{\frac{\ln(c)}{b}} - 1\right)\right)\right)\right)\right] \right] \quad \textbf{(13)}$$

> #3 Model 2

#In humanese, we have that 0 is a fixed point when -1 $< e^r <$ 1, which can be simplified **to** $r < 0$. K would be fixed point **if** $-1 < 1 - r < 1$, which can be simplified **to** $-2 < r < 0$.

$M2 := x\cdot\exp\left(r\cdot\left(1 - \frac{x}{K}\right)\right)$:

$SFPe(M2, x);$

$$[[0, e^r], [K, e^0 - r e^0]] \quad (14)$$

> #3 Model 3

$M3 := c \cdot x \cdot (1 + a \cdot x)^{-b}$: #Here, c is used to represent lambda

$SFPe(M3, x);$

#0 is a stable fixed point if -1 < c < 1, and $\frac{e^{\frac{\ln(c)}{b}} - 1}{a}$ is a fixed point **if** $-1 < \left(\frac{c^2 \cdot \left(b \cdot c^{\frac{1}{b}} - c^{\frac{1}{b}} - b \right)}{c^{\frac{1}{b}}} \right) < 1$

$$\left[[0, c], \left[\frac{e^{\frac{\ln(c)}{b}} - 1}{a}, -\frac{c \left(e^{\frac{\ln(c)}{b}} \right)^{-b} \left(b\, e^{\frac{\ln(c)}{b}} - e^{\frac{\ln(c)}{b}} - b \right)}{e^{\frac{\ln(c)}{b}}} \right] \right] \quad (15)$$

> #4 (i) a = 1, b = 1

$Orbk\left(2, z, \frac{(z[1] + 1 \cdot z[2])}{1 \cdot z[1] + z[2]}, [1.1, 5.3], 990, 1000 \right);$

$$[1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1] \quad (16)$$

> #4 (i) a = 2, b = 1

$evalf\left(Orbk\left(2, z, \frac{(z[1] + 2 \cdot z[2])}{1 \cdot z[1] + z[2]}, [1.1, 5.3], 990, 1000 \right) \right);$

[1.500000000, 1.500000000, 1.500000000, 1.500000000, 1.500000000, 1.500000000, 1.500000000, 1.500000000, 1.500000000, 1.500000000, 1.500000000] (17)

> #4 (i) a = 3, b = 1

$Orbk\left(2, z, \frac{(z[1] + 3 \cdot z[2])}{1 \cdot z[1] + z[2]}, [1.1, 5.3], 990, 1000 \right);$

[2.000000000, 2.000000000, 2.000000000, 2.000000000, 2.000000000, 2.000000000, 2.000000000, 2.000000000, 2.000000000, 2.000000000, 2.000000000] (18)

> #4 (i) a = 4, b = 1

$Orbk\left(2, z, \frac{(z[1] + 4 \cdot z[2])}{1 \cdot z[1] + z[2]}, [1.1, 5.3], 990, 1000 \right);$

[2.500000002, 2.499999998, 2.500000002, 2.499999998, 2.500000002, 2.499999998, 2.500000002, 2.499999998, 2.500000002, 2.499999998, 2.500000002] (19)

> #4 (i) a = 1, b = 2

$Orbk\left(2, z, \frac{(z[1] + 1 \cdot z[2])}{2 \cdot z[1] + z[2]}, [1.1, 5.3], 990, 1000 \right);$

[0.6666666665, 0.6666666665, 0.6666666665, 0.6666666665, 0.6666666665, 0.6666666665, 0.6666666665, 0.6666666665, 0.6666666665, 0.6666666665, 0.6666666665] (20)

> #4 (i) a = 1, b = 3

$$Orbk\left(2, z, \frac{(z[1] + 1\cdot z[2])}{3\cdot z[1] + z[2]}, [1.1, 5.3], 990, 1000\right);$$

[0.5000000002, 0.4999999998, 0.5000000002, 0.4999999998, 0.5000000002, 0.4999999998, 0.5000000002, 0.4999999998, 0.5000000002, 0.4999999998, 0.5000000002] **(21)**

> *#4 (i) a = 1, b = 4*

$$Orbk\left(2, z, \frac{(z[1] + 1\cdot z[2])}{4\cdot z[1] + z[2]}, [1.1, 5.3], 990, 1000\right);$$

[0.4000000004, 0.3999999996, 0.4000000004, 0.3999999996, 0.4000000004, 0.3999999996, 0.4000000004, 0.3999999996, 0.4000000004, 0.3999999996, 0.4000000004] **(22)**

> *#4 (i) a = 2, b = 2*

$$Orbk\left(2, z, \frac{(z[1] + 2\cdot z[2])}{2\cdot z[1] + z[2]}, [1.1, 5.3], 990, 1000\right);$$

[1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000] **(23)**

> *#4 (i) a = 2, b = 3*

$$Orbk\left(2, z, \frac{(z[1] + 2\cdot z[2])}{3\cdot z[1] + z[2]}, [1.1, 5.3], 990, 1000\right);$$

[0.7500000002, 0.7499999997, 0.7500000002, 0.7499999994, 0.7500000002, 0.7499999997, 0.7500000002, 0.7499999994, 0.7500000002, 0.7499999997, 0.7500000002] **(24)**

> *#4 (i) a = 2, b = 4*

$$Orbk\left(2, z, \frac{(z[1] + 2\cdot z[2])}{4\cdot z[1] + z[2]}, [1.1, 5.3], 990, 1000\right);$$

[0.6000000033, 0.5999999967, 0.6000000033, 0.5999999967, 0.6000000033, 0.5999999967, 0.6000000033, 0.5999999967, 0.6000000033, 0.5999999967, 0.6000000033] **(25)**

> *#4 (i) a = 3, b = 2*

$$Orbk\left(2, z, \frac{(z[1] + 3\cdot z[2])}{2\cdot z[1] + z[2]}, [1.1, 5.3], 990, 1000\right);$$

[1.333333335, 1.333333332, 1.333333335, 1.333333332, 1.333333335, 1.333333332, 1.333333335, 1.333333332, 1.333333335, 1.333333332, 1.333333335] **(26)**

> *#4 (i) a = 3, b = 3*

$$Orbk\left(2, z, \frac{(z[1] + 3\cdot z[2])}{3\cdot z[1] + z[2]}, [1.1, 5.3], 990, 1000\right);$$

[1.056529125, 0.9465216579, 1.056470642, 0.9465739756, 1.056412338, 0.9466261387, 1.056354213, 0.9466781477, 1.056296266, 0.9467300035, 1.056238496] **(27)**

> *#4 (i) a = 3, b = 4*

$$Orbk\left(2, z, \frac{(z[1] + 3\cdot z[2])}{4\cdot z[1] + z[2]}, [1.1, 5.3], 990, 1000\right);$$

[1.577350271, 0.4226497306, 1.577350271, 0.4226497305, 1.577350271, 0.4226497306, 1.577350271, 0.4226497305, 1.577350271, 0.4226497306, 1.577350271] **(28)**

> *#4 (i) a = 4, b = 2*

$$Orbk\left(2, z, \frac{(z[1] + 4\cdot z[2])}{2\cdot z[1] + z[2]}, [1.1, 5.3], 990, 1000\right);$$

[1.666666671, 1.666666662, 1.666666671, 1.666666662, 1.666666671, 1.666666662, 1.666666671, 1.666666662, 1.666666671, 1.666666662, 1.666666671] **(29)**

> *#4 (i) a = 4, b = 3*

$$Orbk\left(2, z, \frac{(z[1] + 4\cdot z[2])}{3\cdot z[1] + z[2]}, [1.1, 5.3], 990, 1000\right);$$

[2.366025405, 0.6339745958, 2.366025406, 0.6339745957, 2.366025405, 0.6339745958, 2.366025406, 0.6339745957, 2.366025405, 0.6339745958, 2.366025406] **(30)**

> *#4 (i) a = 4, b = 4*

$$Orbk\left(2, z, \frac{(z[1] + 4\cdot z[2])}{4\cdot z[1] + z[2]}, [1.1, 5.3], 990, 1000\right);$$

[2.618033989, 0.3819660110, 2.618033990, 0.3819660111, 2.618033989, 0.3819660110, 2.618033990, 0.3819660111, 2.618033989, 0.3819660110, 2.618033990] **(31)**

>

HW 11 - Do Not Post

1) $-1 < -k + 2 < 1$
$\quad -2 \qquad\quad -2 \quad -2$

$-3 < -k < -1$

$\quad -1$

$1 < k < 3$ → $\boxed{k < 3}$

2) (ii)

$$\left(-1 < \frac{(k^2-2k-3)^{3/2}}{2} - \frac{k^2(k^2-2k-3)^{1/2}}{2} - k^2 + 2k + 4 + k(k^2-2k-3)^{1/2} + \frac{3}{2}(k^2-2k-3)^{1/2} < 1\right)^2$$

$$-1 < \frac{[(k-3)(k+1)]^3}{4} - \frac{k^4(k-3)(k+1)}{4} - k^4 + 4k^2 + 16 + k^2(k-3)(k+1) + \frac{9}{4}(k-3)(k+1) < 1$$

3) Model 1:

(i) $-1 < c\left(\frac{1}{a(0)}\right)^{-b} < 1$

↳ no further way to simplify

(ii) Maple couldn't fully solve for the second possible fixed point

Model 2:

(i) $-1 < e^r < 1$ (ln ln ln) $\rightarrow$ $\boxed{r < 0}$

(ii) $-1 < e^0 - re^0 < 1$

$-1 < 1 - r < 1$

$-1 \quad +1 \quad -1$

$0 < -r < 2$

-1

$-2 < r < 0$

Model 3:

(i) $\boxed{-1 < c < 1}$

(ii) $-1 < \frac{c\left(e^{\frac{\ln c}{b}}\right)^{-b}\left(be^{\frac{\ln c}{b}} - e^{\frac{\ln c}{b}} - b\right)}{e^{\frac{\ln c}{b}}} < 1$

$-1 < \frac{c\left(e^{\ln c}\right)\left(b\left(e^{\ln c}\right)^{1/b} - \left(e^{\ln c}\right)^{1/b} - b\right)}{\left(e^{\ln c}\right)^{1/b}} < 1$

$-1 < \frac{(c^2)\left(b(c^{1/b}) - c^{1/b} - b\right)}{(c^{1/b})} < 1$

4) (ii) $f(z) = \dfrac{z - az}{bz + z}$

$$z = \frac{z(1-a)}{z(b+1)}$$

$$\frac{z^2(b+1) = z(1-a)}{z}$$

$$z(b+1) = 1-a$$

$$\boxed{z = \frac{1-a}{b+1}}$$