

```
> #Please do NOT post Homework!
#Jeton Hida, Assignment 11, October 11,2021
read "/Users/jeton/Desktop/Math 336/M9.txt"
> read "/Users/jeton/Desktop/Math 336/M11.txt"
```

```
> #Question 1
> print (SFPe)
```

```
proc(f,x) (1)
  local fl, L, i;
  fl := diff(f,x);
  L := [solve(f=x,x)];
  [seq([L[i], normal(subs(x=L[i],fl))], i=1..nops(L))]
end proc
```

```
> SFPe(k*x*(1-x), x)
```

$$\left[[0, k], \left[\frac{k-1}{k}, -k+2 \right] \right] \quad (2)$$

```
> #-1<-k+2<1 -> -3<-k<-1 -> 1<k<3 for this the stable fixed point is
(k-1)/k, and when -1<k<1 the stable fixed point is 0. Cannot be
both can only have 1 stable fixed point depending on the value of k
```

```
> #Question 2
> Help9()
```

```
Orb(f,x,x0,K1,K2), Orb2D(f,x,x0,K) , FP(f,x) , SFP(f,x) , Comp(f,x) (3)
```

```
> Orb(3.1*x*(1-x), x, .5, 1000, 1010)
```

```
[0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203,
0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245,
0.7645665203] (4)
```

```
> Orb(3.2*x*(1-x), x, .5, 1000, 1010)
```

```
[0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906,
0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091,
0.7994554906] (5)
```

```
> Orb(3.3*x*(1-x), x, .5, 1000, 1010)
```

```
[0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832,
0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198,
0.8236032832] (6)
```

```
> Orb(3.4*x*(1-x), x, .5, 1000, 1010)
```

```
[0.4519632478, 0.8421543994, 0.4519632478, 0.8421543994, 0.4519632478, 0.8421543994,
0.4519632478, 0.8421543994, 0.4519632478, 0.8421543994, 0.4519632478,
0.8421543994] (7)
```

```
> Orb(3.41*x*(1-x), x, .5, 1000, 1010)
```

```
[0.4494639177, 0.8437912160, 0.4494639153, 0.8437912150, 0.4494639177, 0.8437912160,
0.4494639153, 0.8437912150, 0.4494639177, 0.8437912160, 0.4494639153,
0.8437912150] (8)
```

```
> Orb(3.42*x*(1-x), x, .5, 1000, 1010)
```

```
(9)
```

[0.4470032600, 0.8453944016, 0.4470032590, 0.8453944013, 0.4470032600, 0.8453944016, 0.4470032590, 0.8453944013, 0.4470032600, 0.8453944016, 0.4470032590, 0.8453944013] (9)

> **Orb(3.43*x*(1-x), x, .5, 1000, 1010)**
[0.4445800122, 0.8469651838, 0.4445800029, 0.8469651800, 0.4445800122, 0.8469651838, 0.4445800029, 0.8469651800, 0.4445800122, 0.8469651838, 0.4445800029, 0.8469651800] (10)

> **Orb(3.44*x*(1-x), x, .5, 1000, 1010)**
[0.4421929706, 0.8485047151, 0.4421929548, 0.8485047085, 0.4421929706, 0.8485047151, 0.4421929548, 0.8485047085, 0.4421929706, 0.8485047151, 0.4421929548, 0.8485047085] (11)

> **Orb(3.45*x*(1-x), x, .5, 1000, 1010)**
[0.4462251607, 0.8525235201, 0.4337587289, 0.8473617243, 0.4462224221, 0.8525225037, 0.4337612012, 0.8473628542, 0.4462197139, 0.8525214988, 0.4337636455, 0.8473639713] (12)

> **Orb(3.441*x*(1-x), x, .5, 1000, 1010)**
[0.4419562161, 0.8486569930, 0.4419561944, 0.8486569840, 0.4419562161, 0.8486569930, 0.4419561944, 0.8486569840, 0.4419562161, 0.8486569930, 0.4419561944, 0.8486569840] (13)

> **Orb(3.442*x*(1-x), x, .5, 1000, 1010)**
[0.4417198110, 0.8488089703, 0.4417197842, 0.8488089592, 0.4417198110, 0.8488089703, 0.4417197842, 0.8488089592, 0.4417198110, 0.8488089703, 0.4417197842, 0.8488089592] (14)

> **Orb(3.443*x*(1-x), x, .5, 1000, 1010)**
[0.4414837469, 0.8489606450, 0.4414837300, 0.8489606381, 0.4414837469, 0.8489606450, 0.4414837300, 0.8489606381, 0.4414837469, 0.8489606450, 0.4414837300, 0.8489606381] (15)

> **Orb(3.444*x*(1-x), x, .5, 1000, 1010)**
[0.4412480486, 0.8491120290, 0.4412480049, 0.8491120115, 0.4412480469, 0.8491120284, 0.4412480063, 0.8491120118, 0.4412480462, 0.8491120280, 0.4412480073, 0.8491120125] (16)

> **Orb(3.445*x*(1-x), x, .5, 1000, 1010)**
[0.4410128914, 0.8492631948, 0.4410124357, 0.8492630098, 0.4410128808, 0.8492631907, 0.4410124453, 0.8492630136, 0.4410128714, 0.8492631869, 0.4410124546, 0.8492630174] (17)

> **Orb(3.446*x*(1-x), x, .5, 1000, 1010)**
[0.4407801674, 0.8494149173, 0.4407751570, 0.8494128724, 0.4407800813, 0.8494148821, 0.4407752417, 0.8494129069, 0.4407799982, 0.8494148484, 0.4407753227, 0.8494129400] (18)

> **Orb(3.4451*x*(1-x), x, .5, 1000, 1010)**
[0.4409894343, 0.8492783112, 0.4409888586, 0.8492780773, 0.4409894216, 0.8492783060, 0.4409888586, 0.8492780773, 0.4409894216, 0.8492783060, 0.4409888586, 0.8492780773] (19)

0.4409888714, 0.8492780825, 0.4409894092, 0.8492783012, 0.4409888828,
0.8492780869]

> #The second bifurcation point is somewhere around 3.445 as that is when we start to get away from period 2 behavior!

> #Question 3

> SFPe((1/a)*x^(-b)*L*x, x)

$$\left[[0, 0], \left[e^{-\frac{\ln\left(\frac{a}{L}\right)}{b}}, -\frac{\left(e^{-\frac{\ln\left(\frac{a}{L}\right)}{b}}\right)^{-b} L (-1 + b)}{a} \right] \right] \quad (20)$$

> #0<b<2 -a<L<a a>1 & a<-1 all these points to be met for a stable fixed point when x=e^(-ln(a/L)/b). No such parameter bounds for when x=0

> SFPe(x*exp(r*(1-x/K)), x)

$$[[0, e^r], [K, e^0 - r e^0]] \quad (21)$$

> #Stability for x=0 is reached when r<0 and stability for x=K is reached when 0<r<2

> SFPe(L*x*(1+a*x)^(-b), x)

$$\left[[0, L], \left[\frac{e^{\frac{\ln(L)}{b}} - 1}{a}, -\frac{L \left(e^{\frac{\ln(L)}{b}}\right)^{-b} \left(b e^{\frac{\ln(L)}{b}} - e^{\frac{\ln(L)}{b}} - b\right)}{e^{\frac{\ln(L)}{b}}}\right] \right] \quad (22)$$

> #Stability for x=0 is reached when -1<L<1, stability for x=(e^(ln(L)/b)-1)/a is reached when b>1 & b<-1 and e^-b<L<e^b

> #Question 4

print (Orbk)

proc(k, z, f, INI, K1, K2) (23)

local L, i, newguy;

L := INI;

if not (type(k, integer) and type(z, symbol) and type(INI, list) and nops(INI) = k and type(K1, integer) and type(K2, integer) and 0 < K1 and K1 < K2) then

print('bad input'); RETURN(FAIL)

end if;

while nops(L) < K2 do

newguy := subs({seq(z[i] = L[-i], i = 1..k)}, f); L := [op(L), newguy]

end do;

[op(K1..K2, L)]

end proc

> Orbk(2, z, (z[1]+1*z[2])/(1*z[1]+z[2]), [1.1, 5.3], 1000, 1010) (24)

[1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]

- > **Orbk(2, z, (z[1]+1*z[2])/(2*z[1]+z[2]), [1.1, 5.3], 1000, 1010)**
 [0.6666666665, 0.6666666665, 0.6666666665, 0.6666666665, 0.6666666665, 0.6666666665, 0.6666666665, 0.6666666665, 0.6666666665, 0.6666666665] (25)
- > **Orbk(2, z, (z[1]+1*z[2])/(3*z[1]+z[2]), [1.1, 5.3], 1000, 1010)**
 [0.5000000002, 0.4999999998, 0.5000000002, 0.4999999998, 0.5000000002, 0.4999999998, 0.5000000002, 0.4999999998, 0.5000000002, 0.4999999998] (26)
- > **Orbk(2, z, (z[1]+1*z[2])/(4*z[1]+z[2]), [1.1, 5.3], 1000, 1010)**
 [0.4000000004, 0.3999999996, 0.4000000004, 0.3999999996, 0.4000000004, 0.3999999996, 0.4000000004, 0.3999999996, 0.4000000004, 0.3999999996] (27)
- > **Orbk(2, z, (z[1]+2*z[2])/(1*z[1]+z[2]), [1.1, 5.3], 1000, 1010)**
 [1.500000000, 1.500000000, 1.500000000, 1.500000000, 1.500000000, 1.500000000, 1.500000000, 1.500000000, 1.500000000, 1.500000000] (28)
- > **Orbk(2, z, (z[1]+2*z[2])/(2*z[1]+z[2]), [1.1, 5.3], 1000, 1010)**
 [1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000] (29)
- > **Orbk(2, z, (z[1]+2*z[2])/(3*z[1]+z[2]), [1.1, 5.3], 1000, 1010)**
 [0.7500000002, 0.7499999994, 0.7500000002, 0.7499999997, 0.7500000002, 0.7499999994, 0.7500000002, 0.7499999997, 0.7500000002, 0.7499999994] (30)
- > **Orbk(2, z, (z[1]+2*z[2])/(4*z[1]+z[2]), [1.1, 5.3], 1000, 1010)**
 [0.6000000033, 0.5999999967, 0.6000000033, 0.5999999967, 0.6000000033, 0.5999999967, 0.6000000033, 0.5999999967, 0.6000000033, 0.5999999967] (31)
- > **Orbk(2, z, (z[1]+3*z[2])/(1*z[1]+z[2]), [1.1, 5.3], 1000, 1010)**
 [2.000000000, 2.000000000, 2.000000000, 2.000000000, 2.000000000, 2.000000000, 2.000000000, 2.000000000, 2.000000000, 2.000000000] (32)
- > **Orbk(2, z, (z[1]+3*z[2])/(2*z[1]+z[2]), [1.1, 5.3], 1000, 1010)**
 [1.333333335, 1.333333332, 1.333333335, 1.333333332, 1.333333335, 1.333333332, 1.333333335, 1.333333332, 1.333333335, 1.333333332] (33)
- > **Orbk(2, z, (z[1]+3*z[2])/(3*z[1]+z[2]), [1.1, 5.3], 1000, 1010)**
 [1.056238496, 0.9467817065, 1.056180901, 0.9468332585, 1.056123481, 0.9468846597, 1.056066236, 0.9469359103, 1.056009163, 0.9469870121, 1.055952263] (34)
- > **Orbk(2, z, (z[1]+3*z[2])/(4*z[1]+z[2]), [1.1, 5.3], 1000, 1010)**
 [1.577350271, 0.4226497305, 1.577350271, 0.4226497306, 1.577350271, 0.4226497305, 1.577350271, 0.4226497306, 1.577350271, 0.4226497305] (35)
- > **Orbk(2, z, (z[1]+4*z[2])/(1*z[1]+z[2]), [1.1, 5.3], 1000, 1010)**
 [2.500000002, 2.499999998, 2.500000002, 2.499999998, 2.500000002, 2.499999998, 2.500000002, 2.499999998, 2.500000002, 2.499999998] (36)
- > **Orbk(2, z, (z[1]+4*z[2])/(2*z[1]+z[2]), [1.1, 5.3], 1000, 1010)**
 [1.666666671, 1.666666662, 1.666666671, 1.666666662, 1.666666671, 1.666666662, 1.666666671, 1.666666662, 1.666666671, 1.666666662] (37)
- > **Orbk(2, z, (z[1]+4*z[2])/(3*z[1]+z[2]), [1.1, 5.3], 1000, 1010)**
 [2.366025406, 0.6339745957, 2.366025405, 0.6339745958, 2.366025406, 0.6339745957, 2.366025405, 0.6339745958, 2.366025406, 0.6339745957] (38)

```
> Orbk(2,z,(z[1]+4*z[2])/(4*z[1]+z[2]),[1.1,5.3],1000,1010)
[2.618033990, 0.3819660111, 2.618033989, 0.3819660110, 2.618033990, 0.3819660111,
 2.618033989, 0.3819660110, 2.618033990, 0.3819660111, 2.618033989]
```

(39)

HW 11

01 WH

4. (ii) $X_n = \frac{X_{n-1} + aX_{n-2}}{bX_{n-1} + X_{n-2}}$

$X_n = \frac{X_{n-1} + aX_{n-2}}{bX_{n-1} + X_{n-2}}$

$X_n(bX_{n-1} + X_{n-2}) = X_{n-1} + aX_{n-2}$ $X_n = r^2$

$bX_n X_{n-1} + X_n X_{n-2} = X_{n-1} + aX_{n-2}$ $X_{n-1} = r$

$br^3 + r^2 = r + a$ $X_{n-2} = 1$

$br^3 + r^2 - r - a = 0$

$br^3 + r^2 - r = a$

$r(br^2 + r - 1) = a$

$r = a$ or

$br^2 + r - 1 - a = 0$

$r = \frac{-1 \pm \sqrt{1 - 4b(-1-a)}}{2b} = \frac{-1 \pm \sqrt{1 - 4b(-1-a)}}{2b}$

$\frac{-1 + 3}{2} = 1$ ✓

obtain ~~...~~

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = (I - A) \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ (iii)

$(1 - 1) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = (I - A) \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

obtain $(0, 0)$ or $|x|, |y| < 1$ fixed point