> #OK to post

> #Anne Somalwar, 10.8.2021, hw11

>

> **read** "C:/Users/aks238/OneDrive − Rutgers University/Documents/M11.txt"

> #2

> $SFPe(k \cdot x \cdot (1 - x), x)$

$$\left[[0, k], \left[\frac{k-1}{k}, -k+2\right]\right] \quad (1)$$

> #For $k>3$, $\frac{k-1}{k}$ is the only stable fixed point.

> (i)

> $Orb(3.1 \cdot x \cdot (1 - x), x, 0.5, 1000, 1010)$

[0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203] **(2)**

> $Orb(3.2 \cdot x \cdot (1 - x), x, 0.5, 1000, 1010)$

[0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906, 0.5130445091, 0.7994554906] **(3)**

> $Orb(3.3 \cdot x \cdot (1 - x), x, 0.5, 1000, 1010)$

[0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832, 0.4794270198, 0.8236032832] **(4)**

> $Orb(3.4 \cdot x \cdot (1 - x), x, 0.5, 1000, 1010)$

[0.4519632478, 0.8421543994, 0.4519632478, 0.8421543994, 0.4519632478, 0.8421543994, 0.4519632478, 0.8421543994, 0.4519632478, 0.8421543994, 0.4519632478, 0.8421543994] **(5)**

> $Orb(3.41 \cdot x \cdot (1 - x), x, 0.5, 1000, 1010)$

[0.4494639177, 0.8437912160, 0.4494639153, 0.8437912150, 0.4494639177, 0.8437912160, 0.4494639153, 0.8437912150, 0.4494639177, 0.8437912160, 0.4494639153, 0.8437912150] **(6)**

> $Orb(3.42 \cdot x \cdot (1 - x), x, 0.5, 1000, 1010)$

[0.4470032600, 0.8453944016, 0.4470032590, 0.8453944013, 0.4470032600, 0.8453944016, 0.4470032590, 0.8453944013, 0.4470032600, 0.8453944016, 0.4470032590, 0.8453944013] **(7)**

> $Orb(3.43 \cdot x \cdot (1 - x), x, 0.5, 1000, 1010)$

[0.4445800122, 0.8469651838, 0.4445800029, 0.8469651800, 0.4445800122, 0.8469651838, 0.4445800029, 0.8469651800, 0.4445800122, 0.8469651838, 0.4445800029, 0.8469651800] **(8)**

> $Orb(3.44 \cdot x \cdot (1 - x), x, 0.5, 1000, 1010)$

[0.4421929706, 0.8485047151, 0.4421929548, 0.8485047085, 0.4421929706, 0.8485047151, 0.4421929548, 0.8485047085, 0.4421929706, 0.8485047151, 0.4421929548, 0.8485047085] **(9)**

> $Orb(3.45 \cdot x \cdot (1-x), x, 0.5, 1000, 1010)$

[0.4462251607, 0.8525235201, 0.4337587289, 0.8473617243, 0.4462224221, 0.8525225037, 0.4337612012, 0.8473628542, 0.4462197139, 0.8525214988, 0.4337636455, 0.8473639713] **(10)**

> *# The second bifurcation point is around 3.45.*

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> #(*ii*)

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> $SFPe(Comp(k*x*(1-x), x), x)$

$$\left[[0, k^2], \left[\frac{k-1}{k}, k^2 - 4k + 4 \right], \left[\frac{\frac{k}{2} + \frac{1}{2} + \frac{\sqrt{k^2-2k-3}}{2}}{k}, -\frac{(k^2-2k-3)^{3/2}}{2} + \frac{\sqrt{k^2-2k-3}\, k^2}{2} - k^2 + 2k + 4 - \sqrt{k^2-2k-3}\, k - \frac{3\sqrt{k^2-2k-3}}{2} \right], \left[\frac{\frac{k}{2} + \frac{1}{2} - \frac{\sqrt{k^2-2k-3}}{2}}{k}, \frac{(k^2-2k-3)^{3/2}}{2} - \frac{\sqrt{k^2-2k-3}\, k^2}{2} - k^2 + 2k + 4 + \sqrt{k^2-2k-3}\, k + \frac{3\sqrt{k^2-2k-3}}{2} \right] \right] \quad \textbf{(11)}$$

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> $solve\left(-1 < -\frac{(k^2-2k-3)^{3/2}}{2} + \frac{\sqrt{k^2-2k-3}\, k^2}{2} - k^2 + 2k + 4 - \sqrt{k^2-2k-3}\, k - \frac{3\sqrt{k^2-2k-3}}{2} < 1, k \right)$

$$\left(-\frac{\left(1-\sqrt{6}+\sqrt{2}\right)^3}{8}+\frac{3\left(1-\sqrt{6}+\sqrt{2}\right)^2}{8}+\frac{3}{4}-\frac{17\sqrt{6}}{8}+\frac{17\sqrt{2}}{8},-1\right),\left(3,-\frac{\left(1+\sqrt{6}+\sqrt{2}\right)^3}{8}+\frac{3\left(1+\sqrt{6}+\sqrt{2}\right)^2}{8}+\frac{3}{4}+\frac{17\sqrt{6}}{8}+\frac{17\sqrt{2}}{8}\right) \quad \textbf{(12)}$$

> *evalf*(%)

$$(-1.449489744, -1.), (3., 3.449489733) \quad \textbf{(13)}$$

> *#The exact point is 3.449489733.*

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> *#3*

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> *#(a)*

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> $SFPe\left(\frac{l}{a}\cdot x^{-b}\cdot x, x\right)$

$$\left[[0,0],\left[e^{-\frac{\ln\left(\frac{a}{l}\right)}{b}}, -\frac{l\left(e^{-\frac{\ln\left(\frac{a}{l}\right)}{b}}\right)^{-b}(-1+b)}{a}\right]\right] \quad \textbf{(14)}$$

> *# I think this model is not really defined when x=0. As pointed out in the text, (Chapter 3, problem #1), for this model to make sense, the fraction of the population that survives each year* ($\frac{l}{a}\cdot x^{-b}$) *has* **to** *be less than* **or** *equal* **to** 1, *meaning x cannot get close* **to** 0. *i.e., this model doesn't acknowledge the situation where the population is* 0.

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> *# The fixed point* $\cdot\left(\frac{l}{a}\right)^{\frac{1}{b}}$ *is stable when* $0 < b < 2$.

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> #(b)

$$b \quad \textbf{(15)}$$

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> $SFPe\left(x\cdot\exp\left(r\cdot\left(1-\frac{x}{k}\right)\right), x\right)$

$$[[0, e^{r}], [k, e^{0} - r\,e^{0}]] \quad (16)$$

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> #The fixed point 0 is stable when -1< $e^{r} < 1$

> #The fixed point k is stable when 0<r<2

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> #(c)

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> $SFPe\left(l\cdot x\cdot(1+a\cdot x)^{-b}, x\right)$

$$\left[[0, l], \left[\frac{e^{\frac{\ln(l)}{b}}-1}{a}, -\frac{l\left(e^{\frac{\ln(l)}{b}}\right)^{-b}\left(b\,e^{\frac{\ln(l)}{b}}-e^{\frac{\ln(l)}{b}}-b\right)}{e^{\frac{\ln(l)}{b}}}\right]\right] \quad (17)$$

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> #The fixed point 0 is stable when -1<l<1

> #The fixed point $\frac{\left((l)^{\frac{1}{b}}-1\right)}{a}$ is stable when $-2 < bl^{-\frac{1}{b}} - b < 0.$

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> #4

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> #a=1, b=1

> $Orbk\left(2, z, \frac{z[1]+z[2]}{z[1]+z[2]}, [1.1, 5.3], 1000, 1010\right);$

$$[1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1] \tag{18}$$

> *#The equilibrium is 1.*

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> *#a=1, b=2*

> $Orbk\left(2, z, \frac{z[1]+z[2]}{2\cdot z[1]+z[2]}, [1.1, 5.3], 1000, 1010\right);$

$$[0.6666666665, 0.6666666665, 0.6666666665, 0.6666666665, 0.6666666665, 0.6666666665, 0.6666666665, 0.6666666665, 0.6666666665, 0.6666666665, 0.6666666665] \tag{19}$$

> *#The equilibrium is 0.667*

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> *#a=1, b=3*

> $Orbk\left(2, z, \frac{z[1]+z[2]}{3\cdot z[1]+z[2]}, [1.1, 5.3], 1000, 1010\right);$

$$[0.5000000002, 0.4999999998, 0.5000000002, 0.4999999998, 0.5000000002, 0.4999999998, 0.5000000002, 0.4999999998, 0.5000000002, 0.4999999998, 0.5000000002] \tag{20}$$

> *#There is no equilibrium here.*

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> *#a=1, b=4*

> $Orbk\left(2, z, \frac{z[1]+z[2]}{4\cdot z[1]+z[2]}, [1.1, 5.3], 1000, 1010\right);$

$$[0.4000000004, 0.3999999996, 0.4000000004, 0.3999999996, 0.4000000004, 0.3999999996, 0.4000000004, 0.3999999996, 0.4000000004, 0.3999999996, 0.4000000004] \tag{21}$$

> *#There is no equilibrium here.*

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> *#a=2, b=1*

> $Orbk\left(2, z, \frac{z[1]+2\cdot z[2]}{1\cdot z[1]+z[2]}, [1.1, 5.3], 1000, 1010\right);$

$$[1.500000000, 1.500000000, 1.500000000, 1.500000000, 1.500000000, 1.500000000, 1.500000000, 1.500000000, 1.500000000, 1.500000000, 1.500000000] \tag{22}$$

> *#The equilibrium is 1.5.*

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> *#a=2, b=2*

> $Orbk\left(2, z, \frac{z[1]+2\cdot z[2]}{2\cdot z[1]+z[2]}, [1.1, 5.3], 1000, 1010\right);$

$$[1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000, 1.000000000] \tag{23}$$

> #The equilibrium is 1.

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> #a=2, b=3

> $Orbk\left(2, z, \frac{z[1] + 2\cdot z[2]}{3\cdot z[1] + z[2]}, [1.1, 5.3], 1000, 1010\right);$

[0.7500000002, 0.7499999994, 0.7500000002, 0.7499999997, 0.7500000002, 0.7499999994, 0.7500000002, 0.7499999997, 0.7500000002, 0.7499999994, 0.7500000002] **(24)**

> #There is no equilibrium here.

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> #a=2, b=4

> $Orbk\left(2, z, \frac{z[1] + 2\cdot z[2]}{4\cdot z[1] + z[2]}, [1.1, 5.3], 1000, 1010\right);$

[0.6000000033, 0.5999999967, 0.6000000033, 0.5999999967, 0.6000000033, 0.5999999967, 0.6000000033, 0.5999999967, 0.6000000033, 0.5999999967, 0.6000000033] **(25)**

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> #There is no equilibrium here.

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> #a=3, b=1

> $Orbk\left(2, z, \frac{z[1] + 3\cdot z[2]}{1\cdot z[1] + z[2]}, [1.1, 5.3], 1000, 1010\right);$

[2.000000000, 2.000000000, 2.000000000, 2.000000000, 2.000000000, 2.000000000, 2.000000000, 2.000000000, 2.000000000, 2.000000000, 2.000000000] **(26)**

> #The equilibrium is 2.

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> #a=3, b=2

> $Orbk\left(2, z, \frac{z[1] + 3\cdot z[2]}{2\cdot z[1] + z[2]}, [1.1, 5.3], 1000, 1010\right);$

[1.333333335, 1.333333332, 1.333333335, 1.333333332, 1.333333335, 1.333333332, 1.333333335, 1.333333332, 1.333333335, 1.333333332, 1.333333335] **(27)**

> #The equilibrium is 1.33.

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> #a=3, b=3

> $Orbk\left(2, z, \frac{z[1] + 3 \cdot z[2]}{3 \cdot z[1] + z[2]}, [1.1, 5.3], 1000, 1010\right);$

[1.056238496, 0.9467817065, 1.056180901, 0.9468332585, 1.056123481, 0.9468846597, 1.056066236, 0.9469359103, 1.056009163, 0.9469870121, 1.055952263] **(28)**

> #There is no equilibrium here.

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> #a=3, b=4

> $Orbk\left(2, z, \frac{z[1] + 3 \cdot z[2]}{4 \cdot z[1] + z[2]}, [1.1, 5.3], 1000, 1010\right);$

[1.577350271, 0.4226497305, 1.577350271, 0.4226497306, 1.577350271, 0.4226497305, 1.577350271, 0.4226497306, 1.577350271, 0.4226497305, 1.577350271] **(29)**

> #There is no equilibrium here.

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> #a=4, b=1

> $Orbk\left(2, z, \frac{z[1] + 4 \cdot z[2]}{1 \cdot z[1] + z[2]}, [1.1, 5.3], 1000, 1010\right);$

[2.500000002, 2.499999998, 2.500000002, 2.499999998, 2.500000002, 2.499999998, 2.500000002, 2.499999998, 2.500000002, 2.499999998, 2.500000002] **(30)**

> #There is no equilibrium here.

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> #a=4, b=2

> $Orbk\left(2, z, \frac{z[1] + 4 \cdot z[2]}{2 \cdot z[1] + z[2]}, [1.1, 5.3], 1000, 1010\right);$

[1.666666671, 1.666666662, 1.666666671, 1.666666662, 1.666666671, 1.666666662, 1.666666671, 1.666666662, 1.666666671, 1.666666662, 1.666666671] **(31)**

> #The equilibrium is 1.667.

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> #a=4, b=3

> $Orbk\left(2, z, \frac{z[1] + 4 \cdot z[2]}{3 \cdot z[1] + z[2]}, [1.1, 5.3], 1000, 1010\right);$

[2.366025406, 0.6339745957, 2.366025405, 0.6339745958, 2.366025406, 0.6339745957, 2.366025405, 0.6339745958, 2.366025406, 0.6339745957, 2.366025405] **(32)**

> #There is no equilibrium here.

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> *#a=4, b=4*

> $Orbk\left(2, z, \frac{z[1] + 4 \cdot z[2]}{4 \cdot z[1] + z[2]}, [1.1, 5.3], 1000, 1010\right);$

[2.618033990, 0.3819660111, 2.618033989, 0.3819660110, 2.618033990, 0.3819660111, 2.618033989, 0.3819660110, 2.618033990, 0.3819660111, 2.618033989] **(33)**

> *#There is no equilibrium here.*