Ok to post.

Anne Somalwar, 10/11/21, hw10

1. $f(x) = \frac{x}{x+c}$

$$f'(x) = \frac{x+c-x}{(x+c)^2}$$

$$f'(0) = \frac{c}{c^2} = \frac{1}{c}$$

0 is stable when $-1 < \frac{1}{c} < 1$

i.e. when $\boxed{c>1 \text{ or } c<-1}$.

2.

(i) $\begin{bmatrix} \frac{-16}{3} & 5 \\ -7 & 13/2 \end{bmatrix}$

$$\det \begin{bmatrix} \frac{-16}{3} - \lambda & 5 \\ -7 & 13/2 - \lambda \end{bmatrix}$$

$$= \left(\frac{-16}{3} - \lambda\right)\left(\frac{13}{2} - \lambda\right) + 35$$

$$= \frac{-104}{3} + \frac{16}{3}\lambda - 13/2\lambda + \lambda^2 + 35$$

$$= \lambda^2 - \frac{7\lambda}{6} + 1/3$$

$$= \lambda^2 - \frac{3}{6}\lambda - \frac{4}{6}\lambda + 1/3$$

$$= (\lambda - 3/6)(\lambda - 4/6)$$

$$= (\lambda - 1/2)(\lambda - 2/3) = 0$$

$$\lambda = 1/2,\ 2/3$$

$$\boxed{\begin{array}{l} 1/2,\ 2/3 < 1 \quad \text{so} \\ 0 \text{ is stable} \end{array}}$$

(ii) $\begin{bmatrix} \frac{92}{3} & -25 \\ 35 & -57/2 \end{bmatrix}$

$$\det \begin{bmatrix} \frac{92}{3} - \lambda & -25 \\ 35 & -57/2 - \lambda \end{bmatrix}$$

$$= \left(\frac{92}{3} - \lambda\right)\left(\frac{-57}{2} - \lambda\right) + (25)(35)$$

$$= -874 - 92/3\,\lambda + 57/2\,\lambda + \lambda^2 + 875$$

$$= \lambda^2 - \frac{13}{6}\lambda + 1 = 0$$

$$= \lambda^2 - \tfrac{4}{6}\lambda - \tfrac{9}{6}\lambda + 1 = 0$$

$$= (\lambda - \tfrac{4}{6})(\lambda - \tfrac{9}{6})$$

$$\lambda = \tfrac{2}{3}, \tfrac{3}{2}$$

$$\boxed{\tfrac{3}{2} > 1 \quad \text{so} \quad 0 \text{ is not stable.}}$$

(iii) $\begin{bmatrix} \frac{-177}{4} & \frac{75}{2} \\ \frac{-105}{2} & \frac{89}{2} \end{bmatrix}$

$$\det \begin{bmatrix} \frac{-177}{4} - \lambda & \frac{75}{2} \\ \frac{-105}{2} & \frac{89}{2} - \lambda \end{bmatrix}$$

$= \left(\frac{-177}{4} - \lambda\right)\left(\frac{89}{2} - \lambda\right) + \frac{105}{2} \cdot \frac{75}{2}$

$= \lambda^2 - \frac{\lambda}{4} - \frac{3}{8}$

$= \lambda^2 - \frac{4}{8}\lambda + \frac{6}{8} - \frac{3}{8}$

$= \left(\lambda + \frac{1}{2}\right)\left(\lambda - \frac{3}{4}\right) = 0$

$\lambda = -\frac{1}{2}, \frac{3}{4}$

$-1 < -\frac{1}{2}, \frac{3}{4} < 1$

so 0 is stable.