HW ~~10~~ (Alan Ho)

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1) $x(n) = \dfrac{x(n-1)}{x(n-1)+c}$

$x = \dfrac{x}{x+c}$

$x^2 + xc = x$

$x^2 + x(c-1) = 0$

$x(c-1) = -x^2$

$c-1 = -x$

$c = 1 - x \quad (x=0)$

$\boxed{c = 1}$ ~~x=0 will be a stable~~ $f'(c-1) = \dfrac{c}{(1-c+c)^2} = c$

$f'(x) = \dfrac{c}{(x+c)^2} = \dfrac{1}{(0+1)^2} = 1 \qquad \dfrac{c}{c^2} < 1 \qquad \dfrac{1}{c} < 1 \qquad c > 1$

$\boxed{c > 1 \quad x = 0 \text{ will be a stable fixed point}}$ $\boxed{-1 < c < 1}$

2) i) $x = -\frac{16}{3}x + 5y \qquad = \frac{19x}{3} = 5y \qquad y = \frac{19x}{15}$

$y = -7x + \frac{13}{2}y \qquad 7x = \frac{11y}{2}$

$7x = \frac{11}{2}\left(\frac{19x}{15}\right) \quad = \quad 7x = \frac{209x}{30} \qquad x = 0, y = 0$

$\therefore$ (0,0) is a fixed point

~~ii) $x = \frac{92x}{3} - 25y \quad = 25y = \frac{89x}{3} \quad > x = 2y = 0$~~

~~$y = 35x - \frac{57}{2}y \quad = 35x = \frac{59}{2}y$~~

$\det\begin{bmatrix} -\frac{16}{3} & 5 \\ -7 & \frac{13}{2} \end{bmatrix} = \begin{vmatrix} -\frac{16}{3}-\lambda & 5 \\ -7 & \frac{13}{2}-\lambda \end{vmatrix} = \lambda^2 - \frac{7}{6}\lambda + \frac{1}{3} = 0$

$\boxed{\lambda = \frac{2}{3}, \frac{1}{3} \text{ stable}} \qquad \frac{2}{3} < 1 \qquad \frac{1}{3} < 1$

ii) $\det\begin{bmatrix} \frac{92}{3}-\lambda & -25 \\ 35 & -\frac{57}{2}-\lambda \end{bmatrix} = \lambda^2 - \frac{13}{6}\lambda + 1 = 0$

$\boxed{\lambda = \frac{3}{2}, \frac{2}{3} \text{ not stable}} \qquad \frac{3}{2} > 1$

iii) $\det \begin{bmatrix} -\frac{177}{4}-\lambda & \frac{75}{2} \\ -\frac{105}{2} & \frac{89}{2}-\lambda \end{bmatrix} = \lambda^2 - \frac{1}{4}\lambda - \frac{3}{8} = \boxed{\lambda = \frac{3}{4}, -\frac{1}{2} \quad \text{stable}}$

$\frac{3}{4} < 1, \quad -\frac{1}{2} < 1$