HW 10

Timothy [illegible]

1. $x=0$ is always a fixed point for

$x(n) = \frac{x(n-1)}{x(n-1)+c}$ since $x=0$ is always a solution to $x=f(x)$

the other fixed point will be $(1-c)$

For $x=0$ to be stable, $-1 < f'(x) < 1$ $\quad f' = \frac{1}{x+c} - \frac{x}{(x+c)^2}$

$-1 \leq \frac{1}{c} \leq 1 \quad -c \leq 1 \leq c \quad \pm 1 \leq c$

$\rightarrow$ c has to be greater than or equal to 1.

The other of value of c is a fixed point that is stable when

$f'(1-c) = c$ c is less than 1 and greater than 0

2. i

$$(x,y) = \left(-\frac{16}{3}x + 5y,\ -7x + \frac{13}{2}y\right) = \begin{bmatrix} -16/3 & 5 \\ -7 & 13/2 \end{bmatrix}$$

$$\lambda_1 = \frac{2}{3}, \begin{pmatrix} 5 \\ 6 \end{pmatrix} = \vec{v}_1 \quad \lambda_2 = \frac{1}{2}, \begin{pmatrix} 6 \\ 7 \end{pmatrix} = \vec{v}_2$$

$$(-16/3 - \lambda)(13/2 - \lambda) + 35 = 0$$

$$\cancel{\lambda^2 - \frac{104}{3} - \frac{7}{6}\lambda + 35 = 0}$$

$$\cancel{\lambda = \frac{2}{3} \text{ or } \frac{1}{2}}$$

$$x^2 - \frac{7x}{6} + \frac{1}{3} = 0 \quad x = \frac{1}{2}, \frac{2}{3}$$

$$\vec{V} = a\begin{pmatrix} 5 \\ 6 \end{pmatrix} \cdot e^{\frac{1}{2}x} + ae^{2/3x}\begin{pmatrix} 6 \\ 7 \end{pmatrix}$$

ii) $(x,y) \to \left(\frac{92}{3}x - 25y,\ 35x - \frac{57}{2}y\right)$

$$A = \begin{pmatrix} \frac{92}{3} & -25 \\ 35 & \frac{-57}{2} \end{pmatrix} \qquad A - I\lambda = 0$$

$$\left(\frac{92}{3} - \lambda\right)\left(\frac{-57}{2} - \lambda\right) + 25 \cdot 35 = \lambda^2 - \frac{13}{6}\lambda + 1 = 0$$

$$\lambda = \frac{3}{2} \qquad \vec{v} = \begin{pmatrix} 6 \\ 7 \end{pmatrix}$$

$$\vec{V} = a_1 e^{\frac{3}{2}t}\begin{pmatrix} 6 \\ 7 \end{pmatrix} + a_2 \begin{pmatrix} 5 \\ 6 \end{pmatrix} \cdot e^{2/3t} \qquad \lambda = \frac{2}{3} \qquad \vec{v} = \begin{pmatrix} 5 \\ 6 \end{pmatrix}$$

iii) $(x,y) \to \left(\frac{-177}{4}x + \frac{75}{2}y,\ \frac{-105}{2}x + \frac{89}{2}y\right)$

$$A = \begin{pmatrix} -177/4 & 75/2 \\ -105/2 & 89/2 \end{pmatrix} \qquad A - I\lambda = 0$$

$$x^2 - \frac{x}{4} - \frac{3}{8} = 0 \qquad \lambda = 3/4 \quad \begin{pmatrix} 5 \\ 6 \end{pmatrix}$$

$$\vec{V} = a_1 e^{\frac{3}{4}t}\begin{pmatrix} 5 \\ 6 \end{pmatrix} + a_2 e^{-\frac{1}{2}t}\begin{pmatrix} 6 \\ 7 \end{pmatrix} \qquad \lambda = 1/2 \quad \begin{pmatrix} 6 \\ 7 \end{pmatrix}$$

Unsure of where to go, however every eigenvector has been the same?