Max Mekhanikov – HW 10 – okay to post

1) $X(n) = \frac{x(n-1)}{x(n-1)+C}$

$X(n) = f(x_{n-1})$

$f(x) = \frac{X}{X+C}$

$f(x) = x \rightarrow \frac{X}{X+C} = X$

$X = X(X+C)$

$X = X^2 + XC$

$0 = X^2 + XC - X$

$0 = X(X+C-1)$

$X = 0,\ 1-C$

$f'(x) = \frac{1}{X+C} - \frac{X}{(X+C)^2} = \frac{C}{(X+C)^2}$

$f'(0) = \frac{0}{C^2} = 0$

$f'(1-C) = \frac{C}{(1-C+C)^2} = C$

x = 0 is a stable fixed point for all values c =/= 0 because f'(x) will always equal 0 and therefore be less than 1 unless c=0 and the fraction becomes undefined (0/0). The other point is stable for -1≤c≤1.

2) (i) $(x,y) \to (-16/3\, x + 5y,\ -7x + 13/2\, y)$

$$\vec{x}(n) = \begin{bmatrix} -16/3 & 5 \\ -7 & 13/2 \end{bmatrix} \vec{x}(n-1)$$

$$\det(A - \lambda I) = \begin{vmatrix} -16/3 - \lambda & 5 \\ -7 & 13/2 - \lambda \end{vmatrix} = 0$$

$$\frac{-208 - 7\lambda + 6\lambda^2}{6} + 35 = 0$$

$\lambda = 2/3,\ 1/2$

↳ $(0,0)$ is <u>stable</u>

(ii) $(x,y) \to (92/3\, x - 25y,\ 35x - 57/2\, y)$

$$\begin{vmatrix} 92/3 - \lambda & -25 \\ 35 & -57/2 - \lambda \end{vmatrix} = 0$$

$$\frac{-5244 - 13\lambda + 6\lambda^2}{6} + 875 = 0$$

$\lambda = 2/3,\ 3/2$

$(0,0)$ is <u>unstable</u>

(iii) $(x,y) \to \left(-\frac{177}{4}x + \frac{75}{2}y, -\frac{105}{2}x + \frac{89}{2}y\right)$

$$\begin{vmatrix} -177/4 & 75/2 \\ -105/2 & 89/2 \end{vmatrix} = 0$$

$$\frac{-15753 - 2\lambda + 8\lambda^2}{8} + \frac{7875}{4} = 0$$

$\lambda = 3/4, -1/2$

$(0,0)$ is <u>unstable</u>

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> # (i)

> $with(LinearAlgebra):$
$A := Matrix\left(\left[\left[-\frac{16}{3}, 5\right], \left[-7, \frac{13}{2}\right]\right]\right)$

$$A := \begin{bmatrix} -\frac{16}{3} & 5 \\ -7 & \frac{13}{2} \end{bmatrix} \quad (1)$$

> $B := Matrix(2, 1, [0.3, 0.6])$

$$B := \begin{bmatrix} 0.3 \\ 0.6 \end{bmatrix} \quad (2)$$

> $C := A \cdot B$

$$C := \begin{bmatrix} 1.40000000000000 \\ 1.80000000000000 \end{bmatrix} \quad (3)$$

> **for** i **from** 995 **to** 1000 **do**
$C := A \cdot C;$
end do;

$$C := \begin{bmatrix} 3.12331538145075 \times 10^{-178} \\ 3.74797845774089 \times 10^{-178} \end{bmatrix}$$

$$C := \begin{bmatrix} 2.08221025430046 \times 10^{-178} \\ 2.49865230516055 \times 10^{-178} \end{bmatrix}$$

$$C := \begin{bmatrix} 1.38814016953363 \times 10^{-178} \\ 1.66576820344035 \times 10^{-178} \end{bmatrix}$$

$$C := \begin{bmatrix} 9.25426779689068 \times 10^{-179} \\ 1.11051213562688 \times 10^{-178} \end{bmatrix}$$

$$C := \begin{bmatrix} 6.16951186459366 \times 10^{-179} \\ 7.40341423751237 \times 10^{-179} \end{bmatrix}$$

$$C := \begin{bmatrix} 4.11300790972902 \times 10^{-179} \\ 4.93560949167481 \times 10^{-179} \end{bmatrix} \quad (4)$$

> $E := Matrix\left(\left[\left[\frac{92}{3}, -25\right], \left[35, -\frac{57}{2}\right]\right]\right)$

(5)

$$E := \begin{bmatrix} \frac{92}{3} & -25 \\ 35 & -\frac{57}{2} \end{bmatrix} \quad (5)$$

> $F := E \cdot B$

$$F := \begin{bmatrix} -5.80000000000000 \\ -6.60000000000000 \end{bmatrix} \quad (6)$$

> **for** i **from** 995 **to** 1000 **do**
$F := E \cdot B$;
end do;

$$F := \begin{bmatrix} -5.80000000000000 \\ -6.60000000000000 \end{bmatrix}$$

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$$F := \begin{bmatrix} -5.80000000000000 \\ -6.60000000000000 \end{bmatrix} \quad (7)$$

> $G := Matrix\left(\left[\left[-\frac{177}{4}, \frac{75}{2}\right], \left[-\frac{105}{2}, \frac{89}{2}\right]\right]\right)$

$$G := \begin{bmatrix} -\frac{177}{4} & \frac{75}{2} \\ -\frac{105}{2} & \frac{89}{2} \end{bmatrix} \quad (8)$$

> $H := G \cdot B$

$$H := \begin{bmatrix} 9.22500000000000 \\ 10.9500000000000 \end{bmatrix} \quad (9)$$

> **for** i **from** 995 **to** 1000 **do**
$H := G \cdot B$;
end do;

$$H := \begin{bmatrix} 9.22500000000000 \\ 10.9500000000000 \end{bmatrix}$$

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